Partonic structure of the nucleon from Lattice QCD

Krzysztof Cichy
Adam Mickiewicz University, Poznań, Poland

NATIONAL SCIENCE CENTRE POLAND

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Outline of the talk

1. Introduction
2. Quasi-PDFs and pseudo-PDFs
3. Results – pseudo-PDFs
4. Lattice impact on pheno?
5. New directions – twist-3, GPDs
6. Conclusions and prospects

Collaborators:

- C. Alexandrou (Cyprus)
- M. Bhat (Poznań)
- S. Bhattacharya (Temple)
- M. Constantinou (Temple)
- L. Del Debbio (Edinburgh)
- T. Giani (Edinburgh)
- K. Hadjiyiannakou (Cyprus)
- K. Jansen (DESY)
- A. Metz (Temple)
- A. Scapellato (Poznań)
- F. Steffens (Bonn)

Based on:

- M. Bhat, K. Cichy, M. Constantinou, A. Scapellato, "Parton distribution functions from lattice QCD at physical quark masses via the pseudo-distribution approach", arXiv:2005.02102
- S. Bhattacharya, K. Cichy, M. Constantinou, A. Metz, A. Scapellato, F. Steffens, "New insights on proton structure from lattice QCD: the twist-3 parton distribution function $g_T(x)$", arXiv:2004.04130, "One-loop matching for the twist-3 parton distribution $g_T(x)$", arXiv:2005.10939 (accepted in PRD), "The role of zero-mode contributions in the matching for the twist-3 PDFs $e(x)$ and $h_L(x)$", arXiv:2006.12347
- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, A. Scapellato, F. Steffens, in preparation (GPDs)
- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, A. Scapellato, F. Steffens, "Systematic uncertainties in parton distribution functions from lattice QCD simulations at the physical point", Phys. Rev. D99 (2019) 114504
- K. Cichy, L. Del Debbio, T. Giani, "Parton distributions from lattice data: the nonsinglet case", JHEP 10 (2019) 137
- C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens, "Light-Cone Parton Distribution Functions from Lattice QCD", Phys. Rev. Lett. 121 (2018) 112001,"Transversity parton distribution functions from lattice QCD", Phys. Rev. D98 (2018) 091503 (Rapid Communications)

Review of the field:

- K. Cichy, M. Constantinou, "A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results", invited review article for a special issue of Advances in High Energy Physics, Adv. High Energy Phys. 2019 (2019) 3036904, arXiv: 1811.07248 [hep-lat]

Nucleon structure

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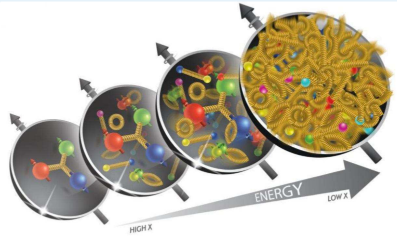

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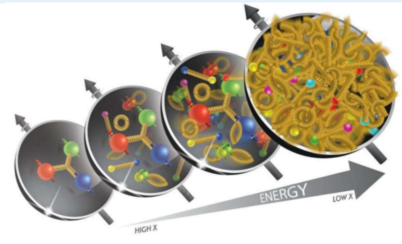

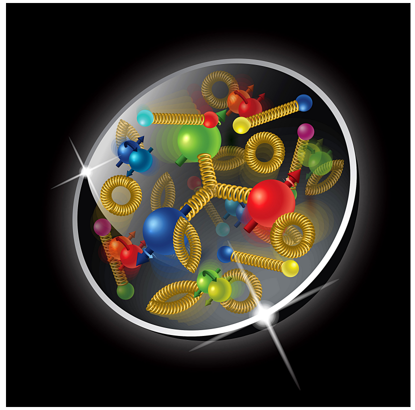

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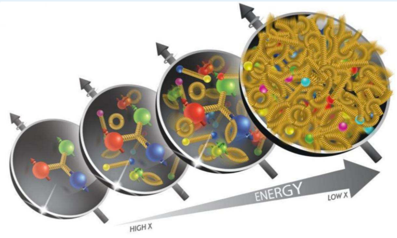

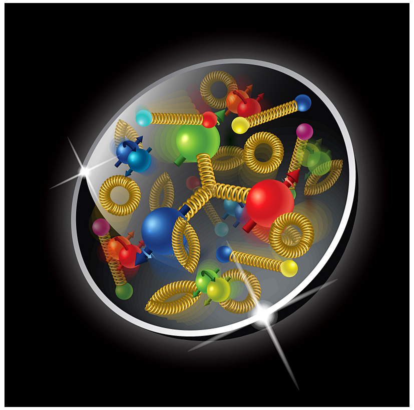

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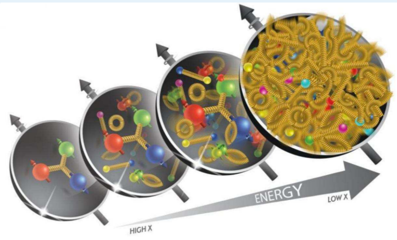

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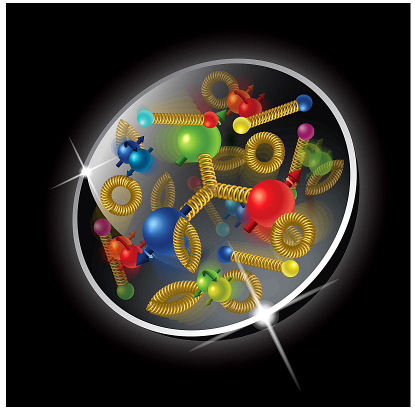

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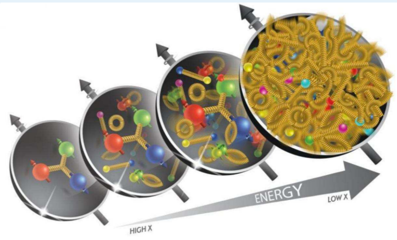

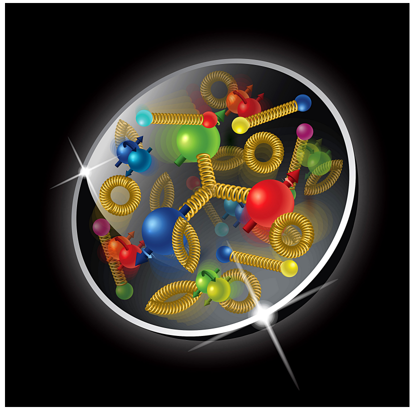

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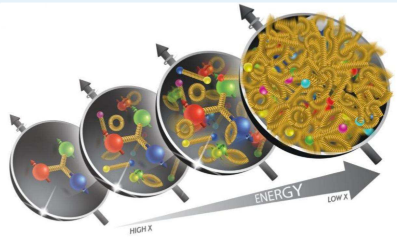

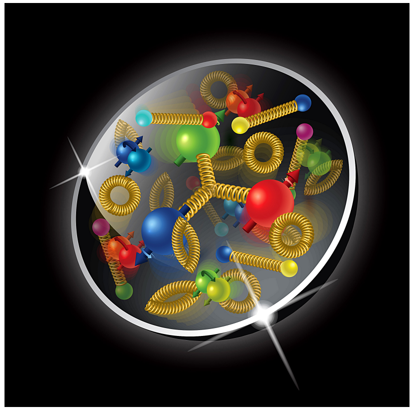

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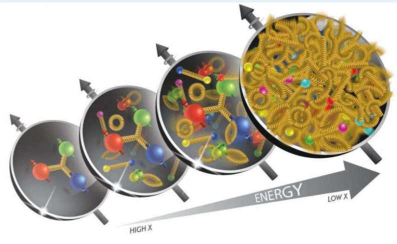

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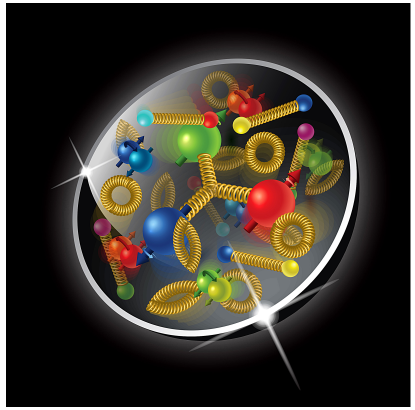

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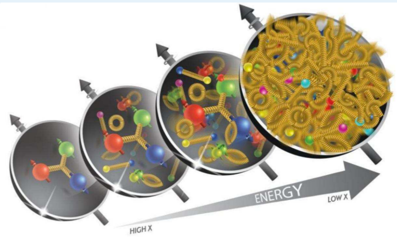

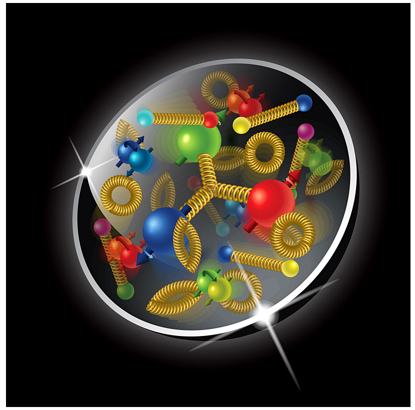

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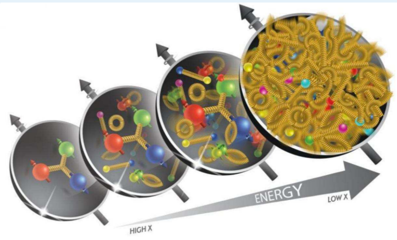

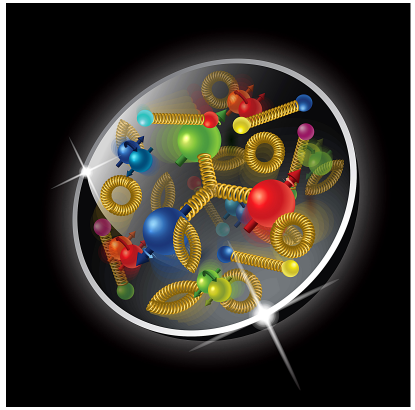

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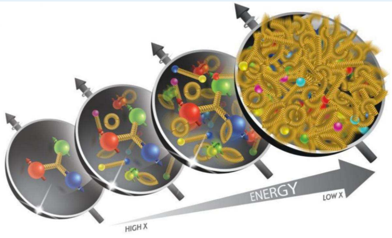

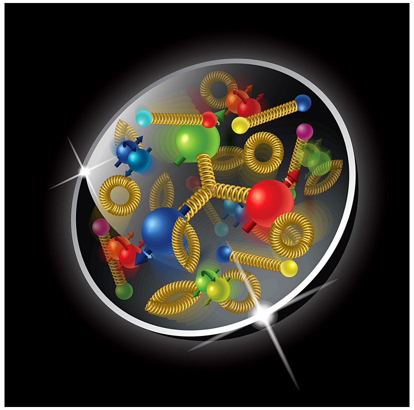

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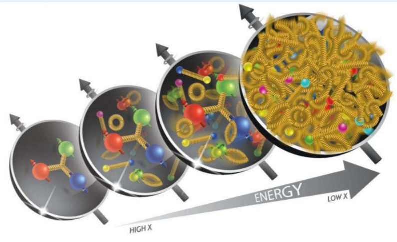

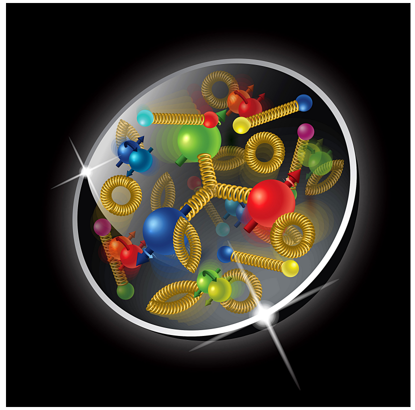

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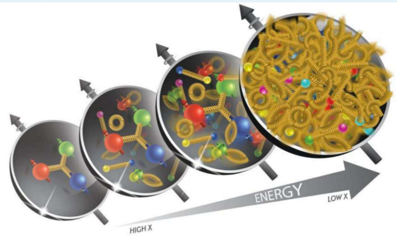

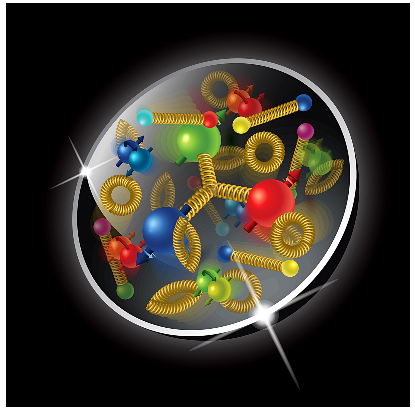

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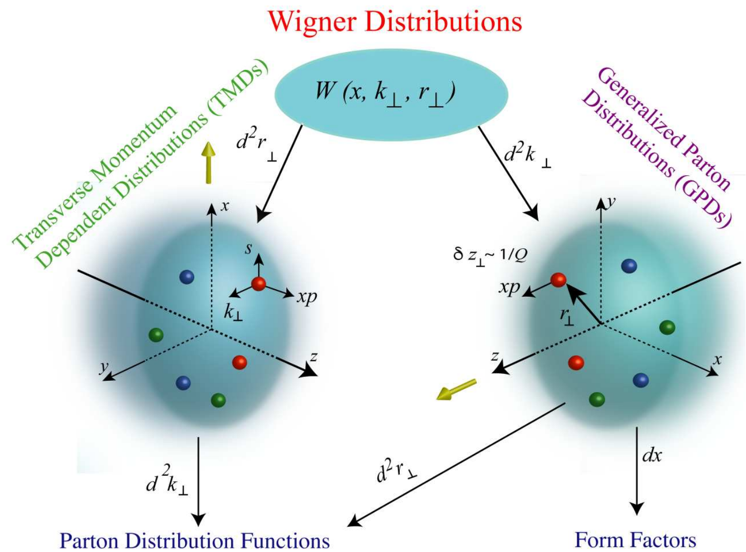

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Recently: new direct approaches to get x-dependence.

Approaches to light-cone PDFs

- The common feature of all the approaches is that they rely to some extent on the factorization framework:

$$Q(x,\mu_R) = \int_{-1}^{1} \frac{dy}{y} C\left(\frac{x}{y},\mu_F,\mu_R\right) q(y,\mu_F),$$

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- Matrix elements: $\langle N|\bar{\psi}(z)\Gamma F(z)\Gamma'\psi(0)|N\rangle$ with different choices of Γ, Γ' Dirac structures and objects $F(z)$.
  - ⋆ **hadronic tensor** – K.-F. Liu, S.-J. Dong, 1993
  - ⋆ **auxiliary scalar quark** – U. Aglietti et al., 1998
  - ⋆ **auxiliary heavy quark** – W. Detmold, C.-J. D. Lin, 2005
  - ⋆ **auxiliary light quark** – V. Braun, D. Müller, 2007
  - ⋆ **quasi-distributions** – X. Ji, 2013
  - ⋆ **"good lattice cross sections"** – Y.-Q. Ma, J.-W. Qiu, 2014,2017
  - ⋆ **pseudo-distributions** – A. Radyushkin, 2017
  - ⋆ **"OPE without OPE"** – QCDSF, 2017

Overview of results from different approaches

results @ physical pion mass
results extrapolated to physical pion mass
results @ non-physical pion mass

- **Quasi-distributions**
  - Nucleon twist-2 PDFs
    - unpolarized
      - ETMC, PRL121(2018)112001
      - ETMC, PRD99(2019)114504
      - LP3, arXiv:1803.04393
      - LPC, PRD101(2020)034020
      - BNL/MSU, arXiv:2005.12015
    - helicity
      - ETMC, PRL121(2018)112001
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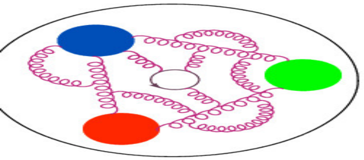
      - LP3, PRL121(2018)242003
      - BNL/MSU, arXiv:2005.12015
    - transversity
      - ETMC, PRD98(2018)091503(Rapid)
      - ETMC, PRD99(2019)114504
      - LP3, arXiv:1810.05043
  - Nucleon twist-3 PDFs
    - ETMC/Temple arXiv:2004.04130
  - Nucleon GPDs
    - ETMC arXiv:1910.13229 (paper soon!)
  - Δ PDFs
    - ETMC/Beijing arXiv:2002.12044 (accepted in PRD)
  - Pion/Kaon DA
    - LP3, PRD95(2017)094514
    - LP3, NPB939(2019)429
  - Pion PDF
    - LP3, PRD100(2019)034505
    - BNL, PRD100(2019)034516
    - BNL, arXiv:2007.06590
  - Pion GPD
    - Chen, NPB952(2020)114940
- **Pseudo-distributions**
  - Nucleon PDFs
    - JLab, PRD96(2017)094503
    - JLab, JHEP12(2019)081
    - JLab, arXiv:2004.01687
    - ETMC, arXiv:2005.02102
  - Pion PDF
    - JLab, PRD100(2019)114512
- **Good LCSs**
  - Pion PDF
    - JLab, PRD99(2019)074507
    - JLab, arXiv:2001.04960
- **Auxiliary light quark**
  - Pion DA
    - Regensburg, EPJC78(2018)217
    - Regensburg, PRD98(2018)094507
- **Auxiliary heavy quark**
  - Pion DA
    - Taiwan/MIT, arXiv:1810.12194
- **OPE without OPE**
  - Nucleon F_1
    - QCDSF, PRL118(2017)242001
    - QCDSF, arXiv:2007.01523
- **Hadronic tensor**
  - Nucleon W_{11}
    - χQCD, PRD101(2020)114503



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- Pion GPD: Chen, NPB952(2020)114940

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- Pion PDF: JLab, PRD100(2019)114512

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- Pion PDF: JLab, PRD99(2019)074507; JLab, arXiv:2001.04960

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Review Article

A Guide to Light-Cone PDFs from Lattice QCD: An Overview of Approaches, Techniques, and Results

Krzysztof Cichy[1] **and Martha Constantinou**[2]

[1]*Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland*
[2]*Department of Physics, Temple University, Philadelphia, PA 19122 - 1801, USA*

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arXiv:1811.07248
Special issue *Transverse Momentum Dependent Observables from Low to High Energy: Factorization, Evolution, and Global Analyses*

discusses in detail quasi-distributions
reviews also other approaches

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

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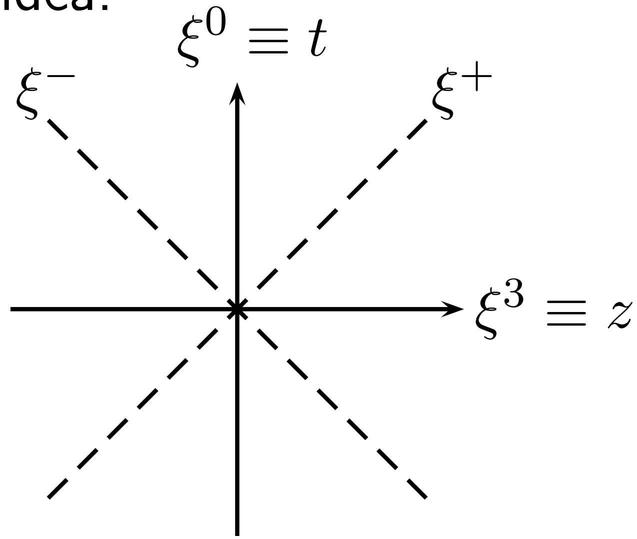

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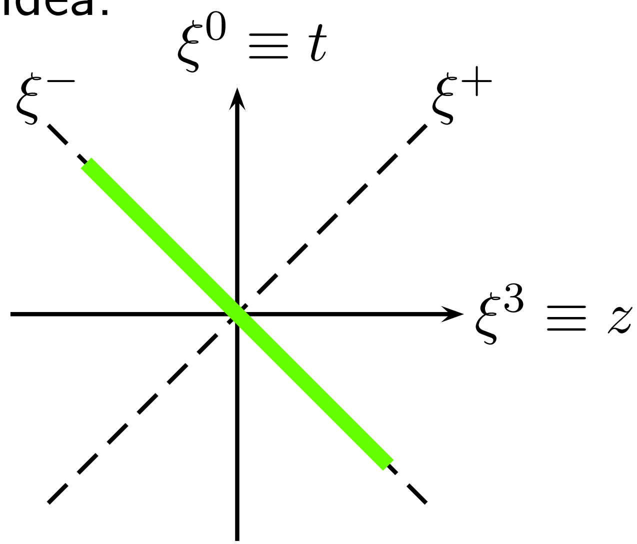

Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

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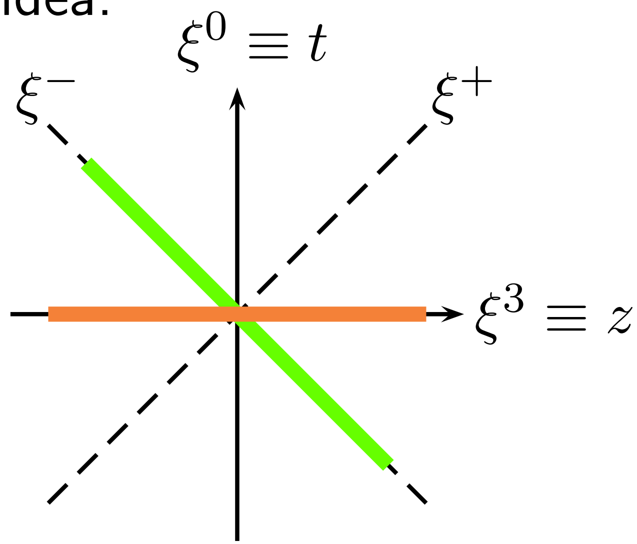

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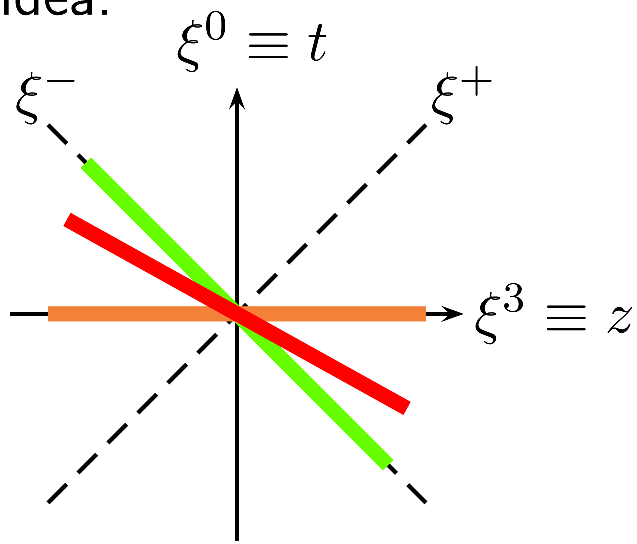

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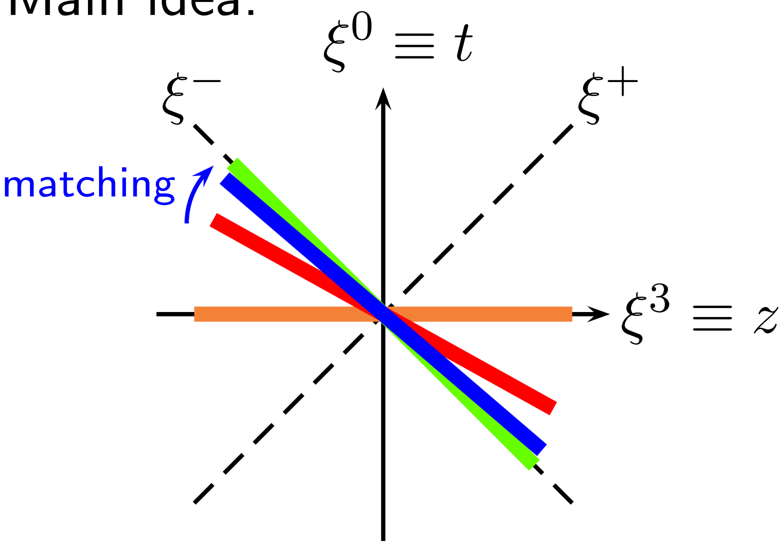

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Matching (Large Momentum Effective Theory (LaMET)

X. Ji, *Parton Physics from Large-Momentum Effective Field Theory*, Sci.China Phys.Mech.Astron. 57 (2014) 1407

→ brings quasi-distribution to the light-cone distribution, up to power-suppressed effects:

$$\tilde{q}(x,\mu,P_3) = \int_{-1}^{1} \frac{dy}{|y|}\, C\left(\frac{x}{y}, \frac{\mu}{P_3}\right) q(y,\mu) + \mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, M_N^2/P_3^2\right)$$

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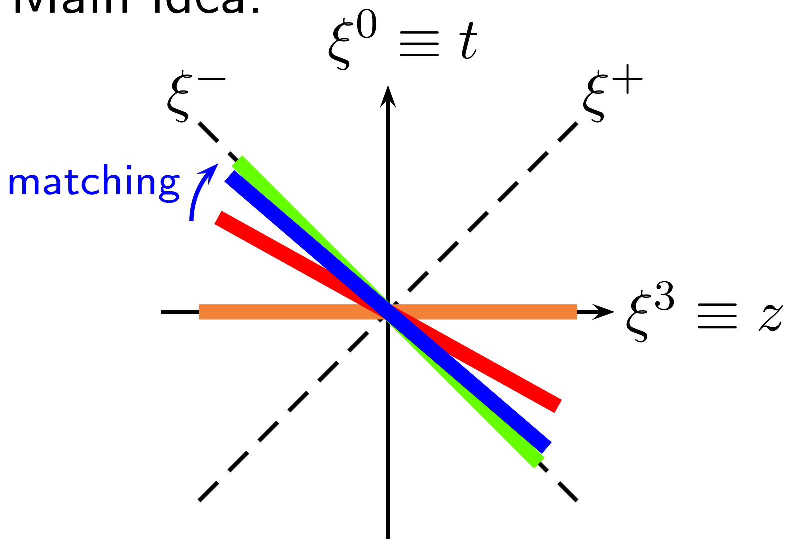

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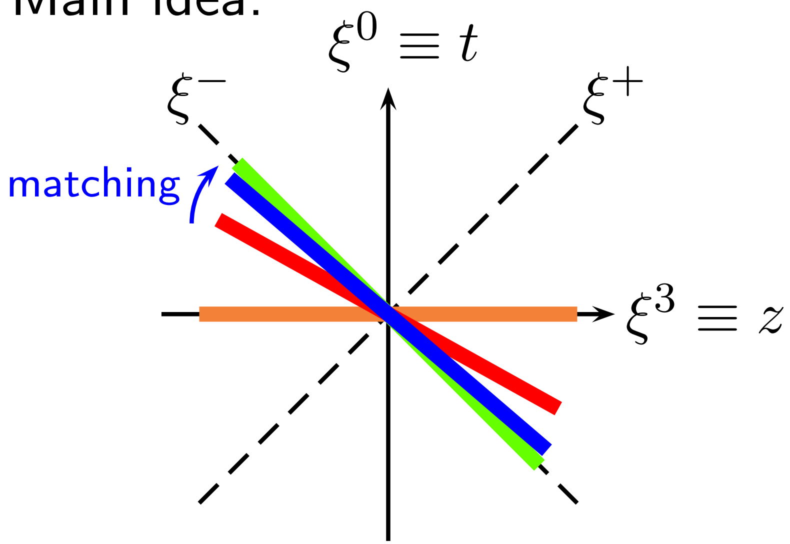

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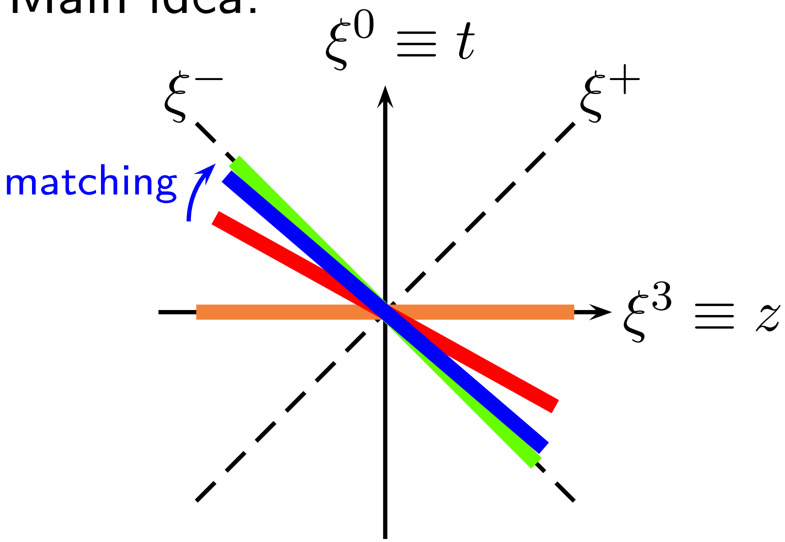

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quasi-PDF   pert.kernel   PDF

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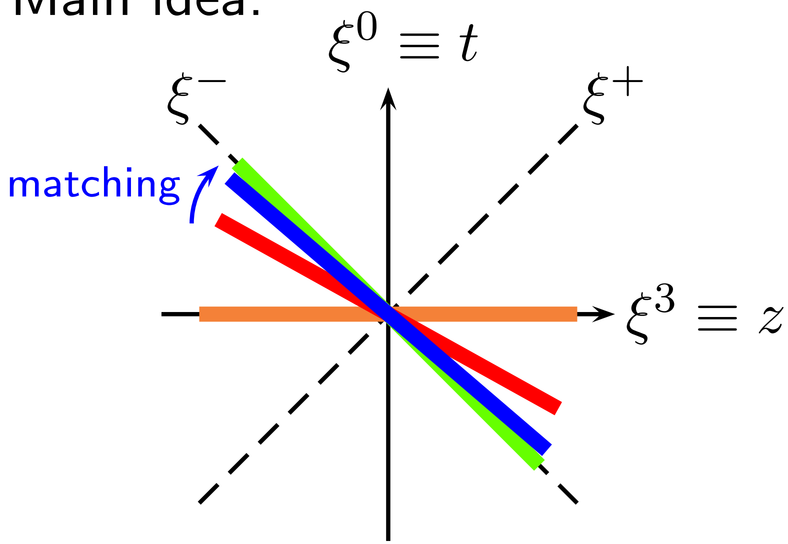

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quasi-PDF — pert.kernel — PDF — higher-twist effects

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- Originated from Radyushkin's pioneering studies of relations between quasi-distributions, virtuality distribution functions (VDFs) and "primordial" TMDs.
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Central object: **"Ioffe-time distribution" (ITD)** – $Q(\nu,\mu^2)$
Fourier-conjugate to PDF: $Q(\nu,\mu^2)=\int_{-1}^{1}dx\,e^{i\nu x}q(x,\mu^2)$
$\nu\equiv zP_3$ – "Ioffe time"

- Later, the approach has been broadly investigated theoretically...
  A. Radyushkin, Phys. Lett. B781 (2018) 433
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  A. Radyushkin, Phys. Rev. D100 (2019) 116011 (pseudo-GPDs)
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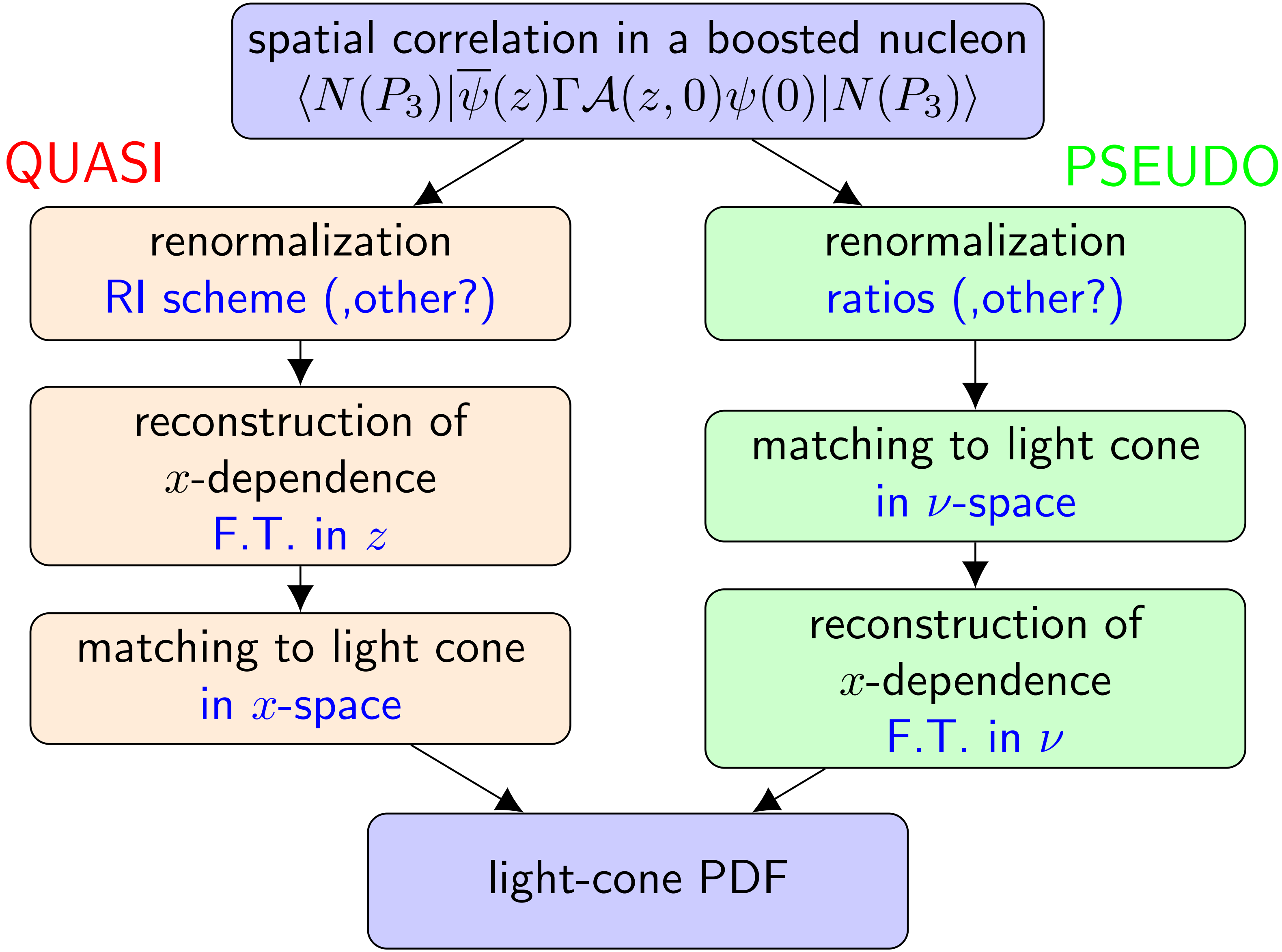
- Excellent review:
  A. Radyushkin, "Theory and applications of parton pseudodistributions", Int. J. Mod. Phys. A35 (2020) 2030002

Quasi-PDFs vs. pseudo-PDFs



Renormalization from a double ratio

The matrix element $\langle N(P_3)|\overline{\psi}(z)\gamma_0\mathcal{A}(z,0)\psi(0)|N(P_3)\rangle$ exhibits two kinds of divergences:

- standard logarithmic divergence,
- power divergence related to the Wilson line.

Shown to be multiplicatively renormalizable to all orders in PT

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Both divergences can be canceled by forming a double ratio with zero-momentum and local $(z=0)$ matrix elements:

(also removes part of HTE (generically $\mathcal{O}(z^2\Lambda_{\mathrm{QCD}}^2)$))

$$\mathfrak{M}(\nu, z^2) = \frac{\mathcal{M}(\nu, z^2)\,/\,\mathcal{M}(\nu, 0)}{\mathcal{M}(0, z^2)\,/\,\mathcal{M}(0, 0)}.$$

$\mathfrak{M}(\nu, z^2)$ – "reduced" matrix elements (or pseudo-ITDs).

The double ratio defines a renormalization scheme with renormalization scale proportional to $1/z$.

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The full 1-loop matching equation: A. Radyushkin, PLB781(2018)433, PRD98(2018)014019; J.-H. Zhang et al., PRD97(2018)074508; T. Izubuchi et al., PRD98(2018)056004

$$\mathfrak{M}(\nu, z^2) = Q(\nu, \mu^2) - \frac{\alpha_s C_F}{2\pi} \int_0^1 du \left[\ln\left(z^2 \mu^2 \frac{e^{2\gamma_E + 1}}{4} \right) B(u) + L(u) \right] Q(u\nu, \mu^2)$$

with:

$$B(u) = \left[\frac{1+u^2}{1-u} \right]_+, \qquad L(u) = \left[4 \frac{\ln(1-u)}{1-u} - 2(1-u) \right]_+,$$

$$\int_0^1 [f(u)]_+ \, Q(u\nu) = \int_0^1 f(u) \left(Q(u\nu) - Q(\nu) \right).$$

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  The evolved ITD $\mathfrak{M}'$ has 3 arguments:
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  Again 3 arguments and test of independence on the initial scale.

For the reconstruction of the final PDF
$\longrightarrow$ average the matched ITDs $Q(\nu, z^2, \mu^2)$ for cases where a given Ioffe time is achieved by different combinations of (P_3, z), denote such average by $Q(\nu, \mu^2)$.

Reconstruction of x-dependence

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Ways out used in our work:

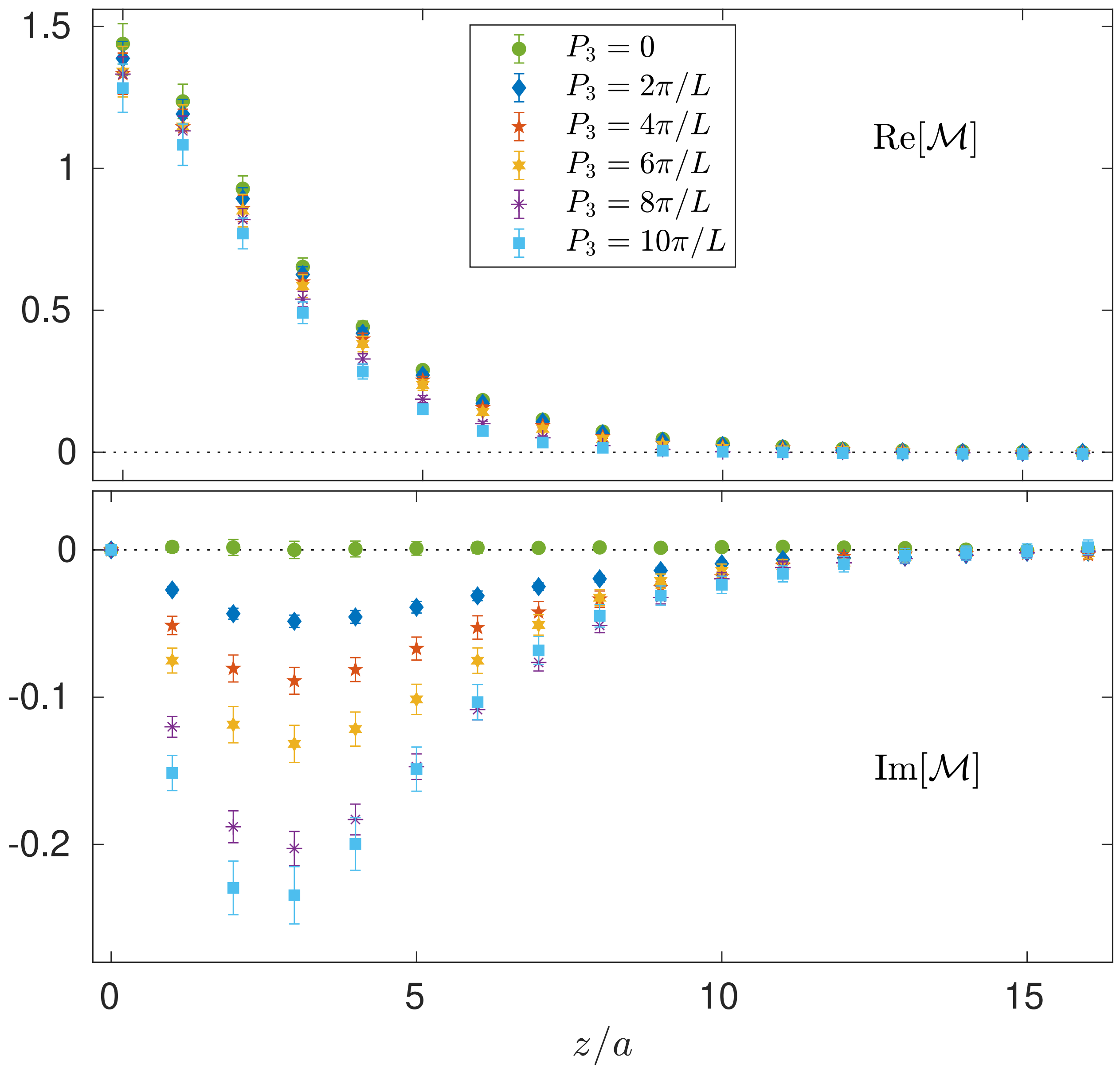
- Backus-Gilbert approach (with and without preconditioning),
- fitting ansatz reconstruction: $q(x) = N x^a (1-x)^b$.

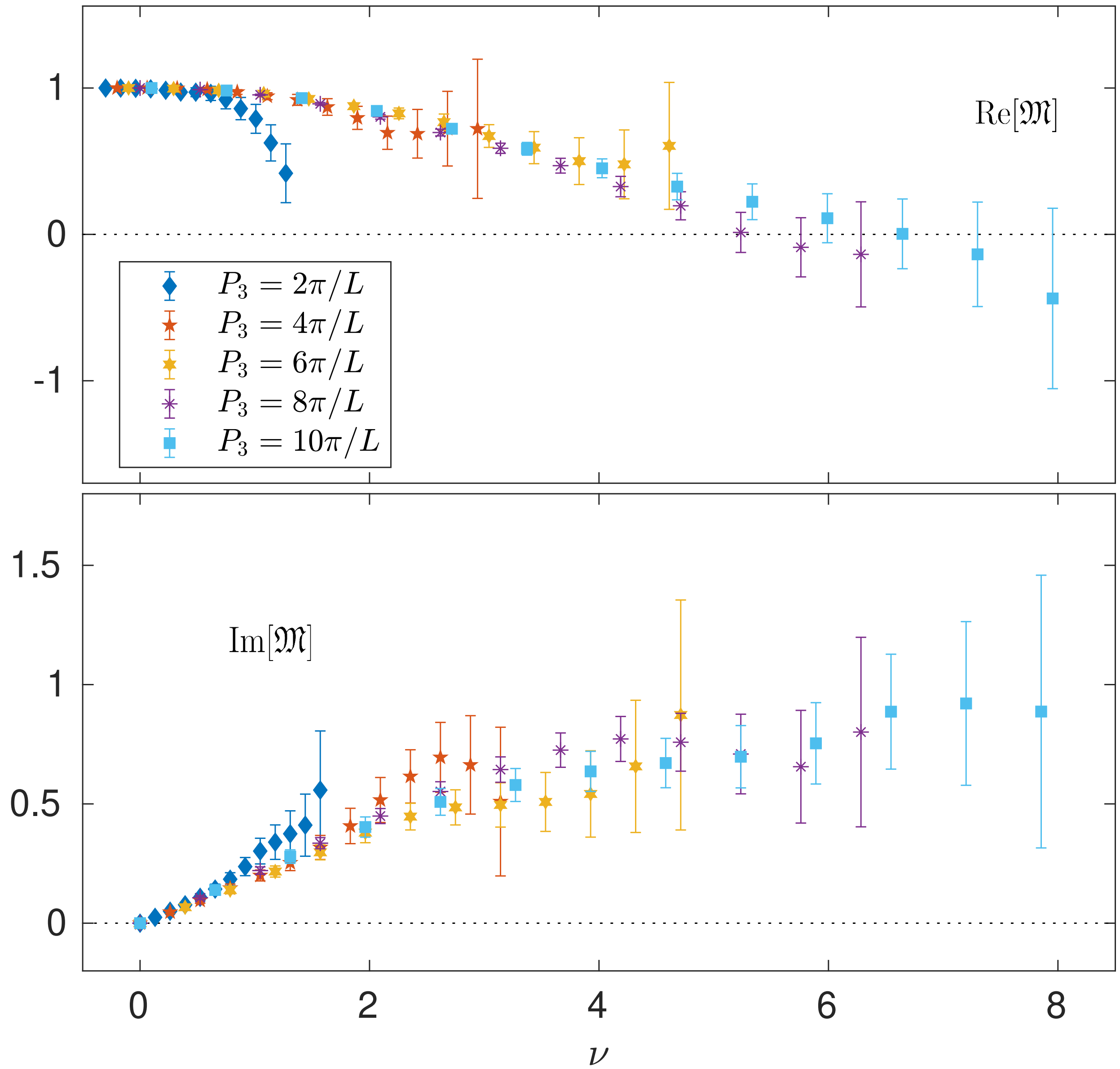
Lattice setup

- fermions: $N_f = 2$ twisted mass fermions + clover term
- gluons: Iwasaki gauge action, $\beta = 2.1$
- gauge field configurations generated by ETMC

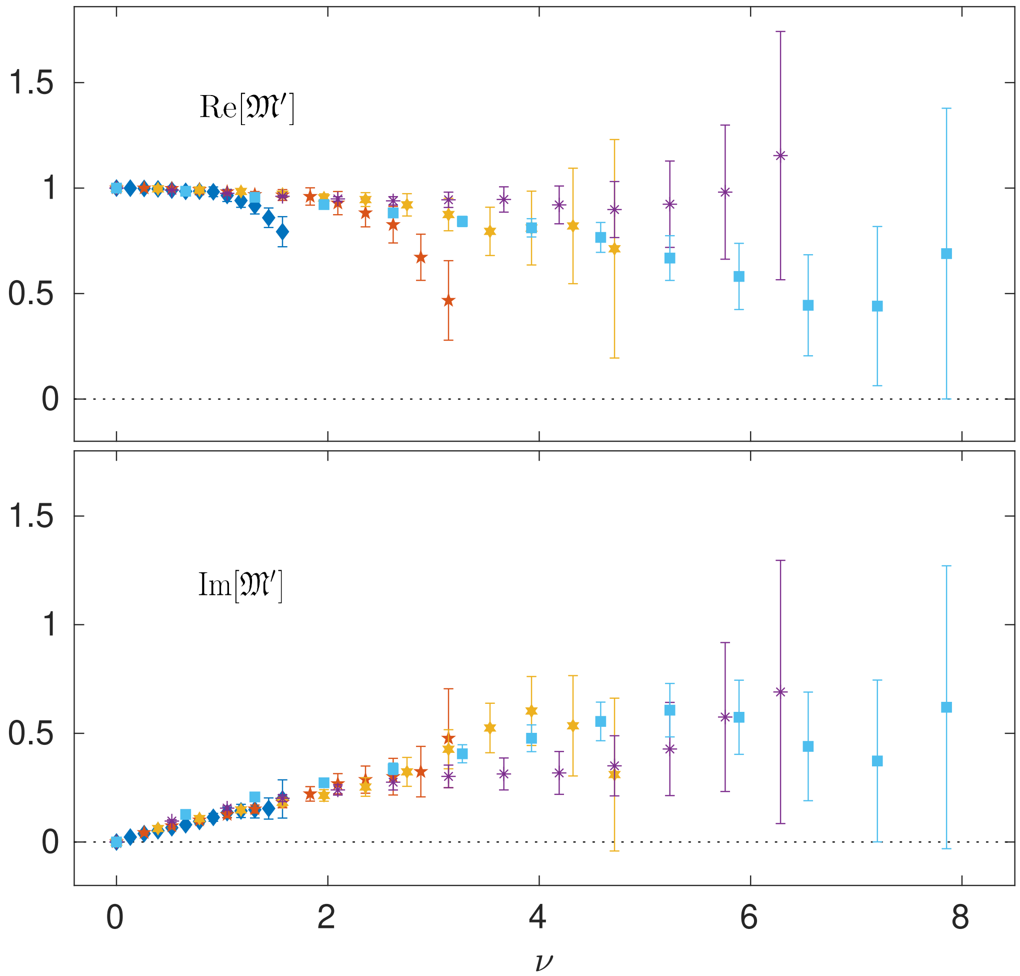
β=2.10,	$c_{\rm SW}$=1.57751,	a=0.0938(3)(2) fm
$48^3 \times 96$	$a\mu = 0.0009$	$m_N = 0.932(4)$ GeV
$L = 4.5$ fm	$m_\pi = 0.1304(4)$ GeV	$m_\pi L = 2.98(1)$

P_3	P_3 [GeV]	$N_{\rm confs}$	$N_{\rm meas}$
0	0	20	320
$2\pi/L$	0.28	19	1824
$4\pi/L$	0.55	18	1728
$6\pi/L$	0.83	50	4800
$8\pi/L$	1.11	425	38250
$10\pi/L$	1.38	811	72990


1.5
1
0.5
0
-0.1
-0.2
0
5
10
15
z/a
$P_3 = 0$
$P_3 = 2\pi/L$
$P_3 = 4\pi/L$
$P_3 = 6\pi/L$
$P_3 = 8\pi/L$
$P_3 = 10\pi/L$
Re[$\mathcal{M}$]
Im[$\mathcal{M}$]


Re[𝔐]
Im[𝔐]
$P_3 = 2\pi/L$
$P_3 = 4\pi/L$
$P_3 = 6\pi/L$
$P_3 = 8\pi/L$
$P_3 = 10\pi/L$
ν
1
0
-1
1.5
1
0.5
0
0
2
4
6
8

Evolved and $\overline{\mathrm{MS}}$-converted matrix elements



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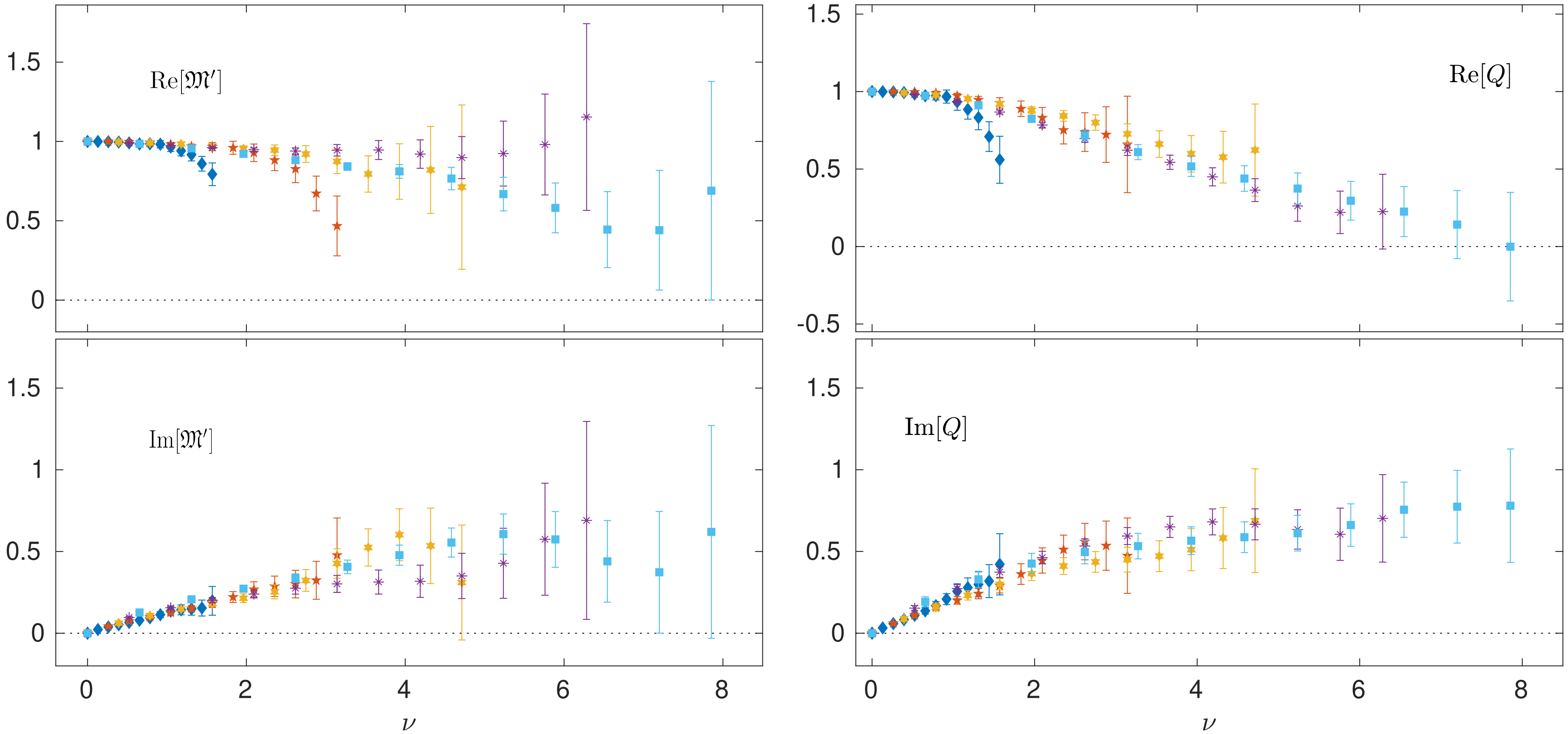
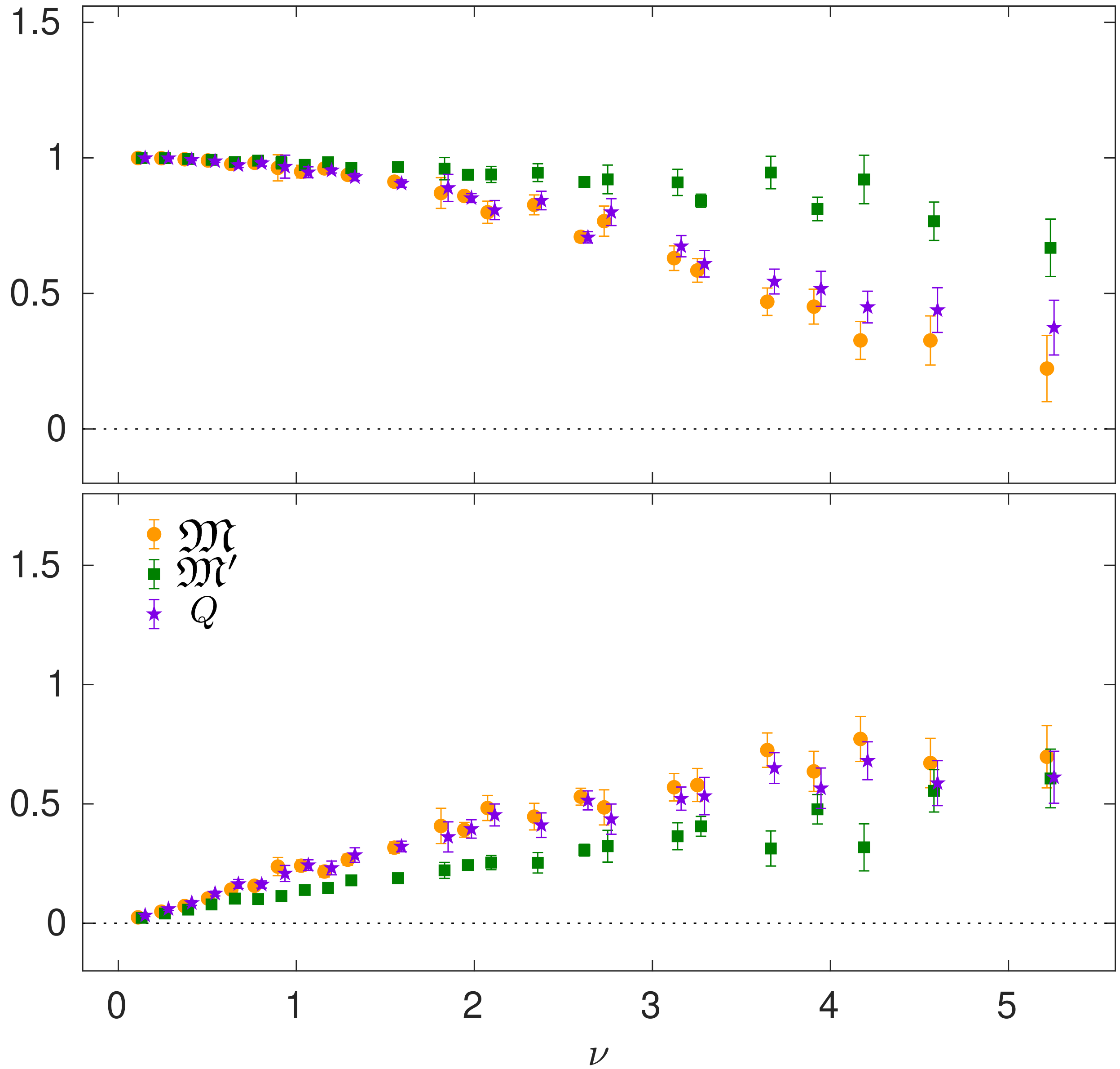
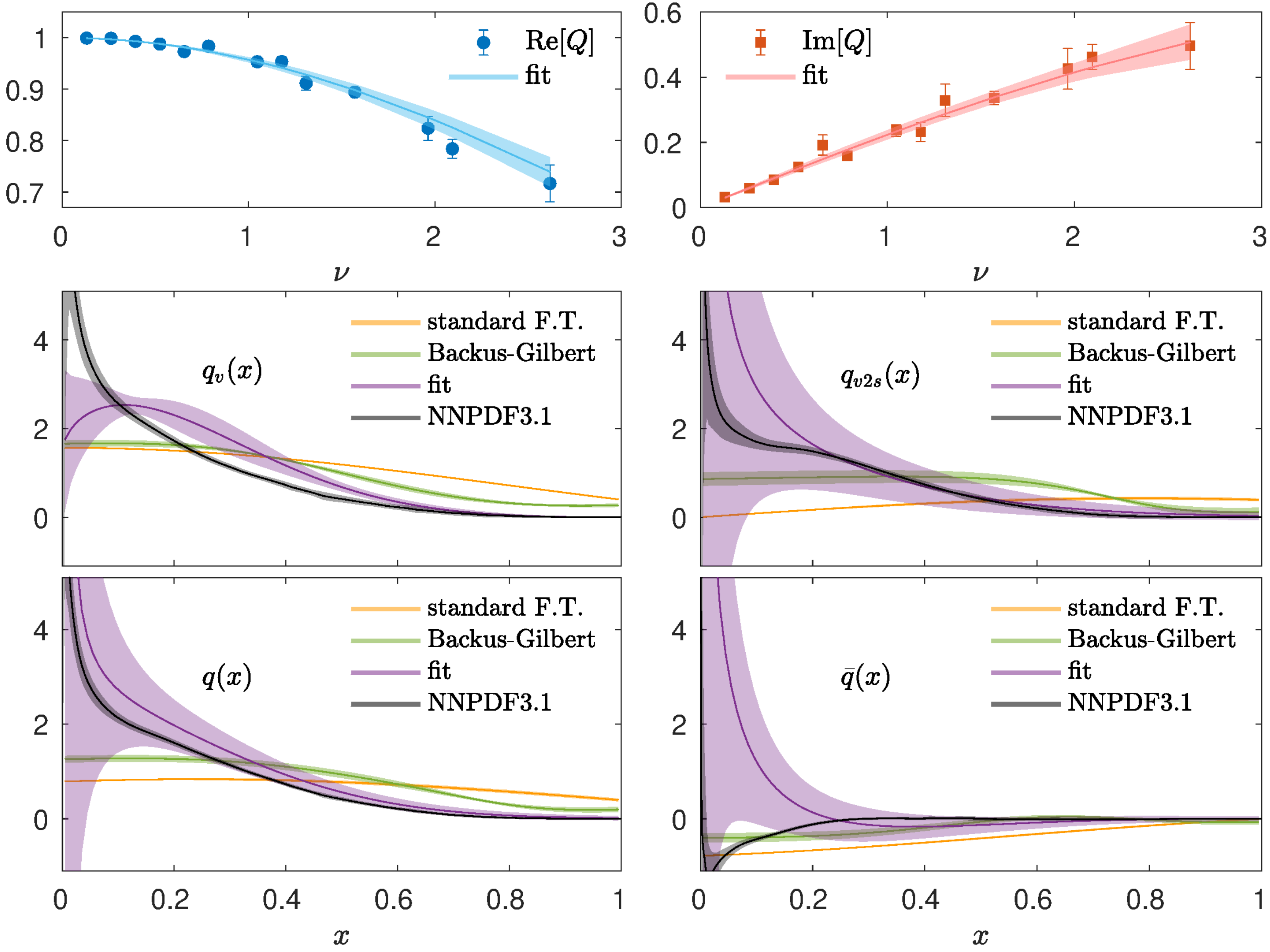


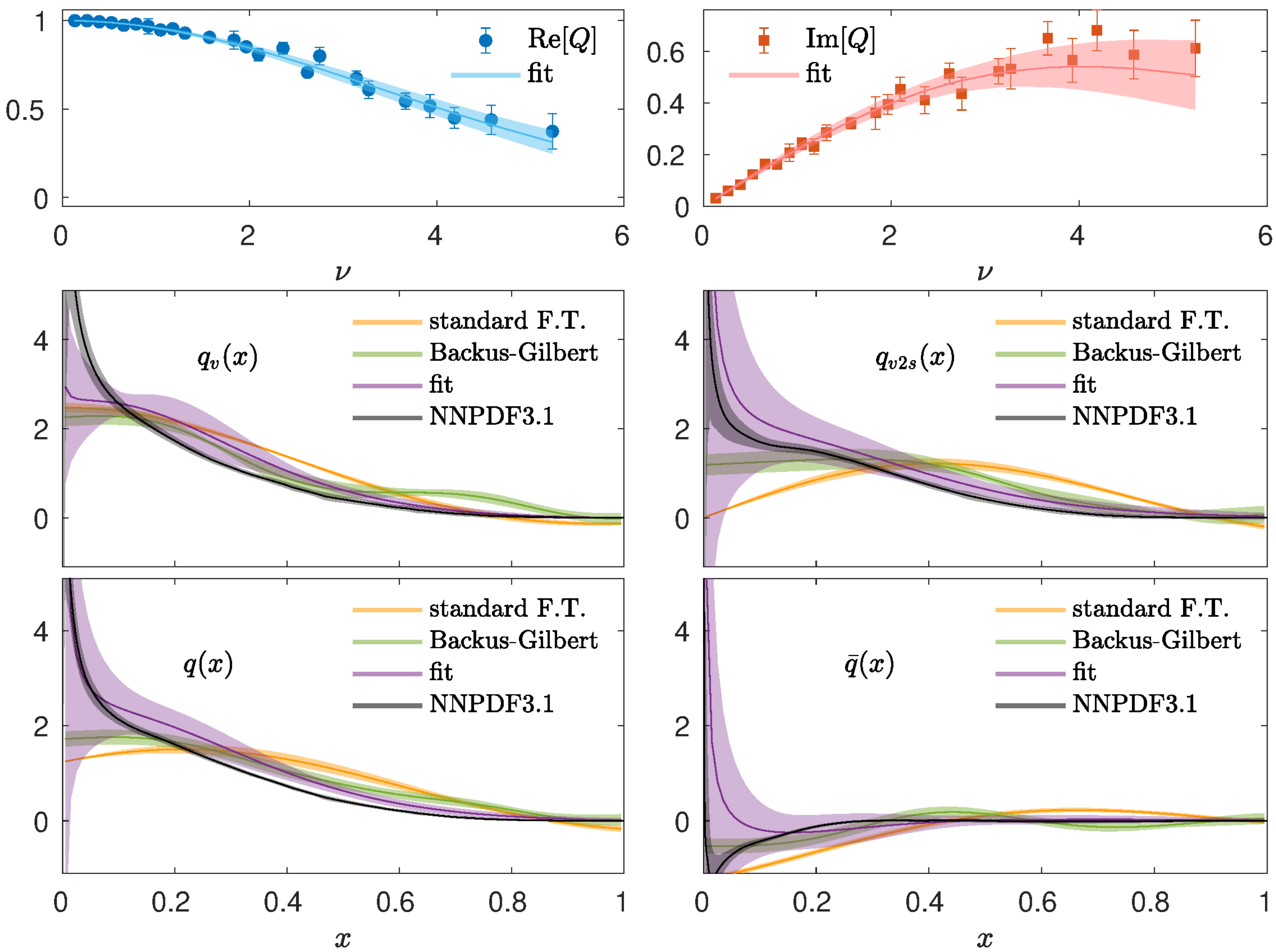
Averaged matrix elements



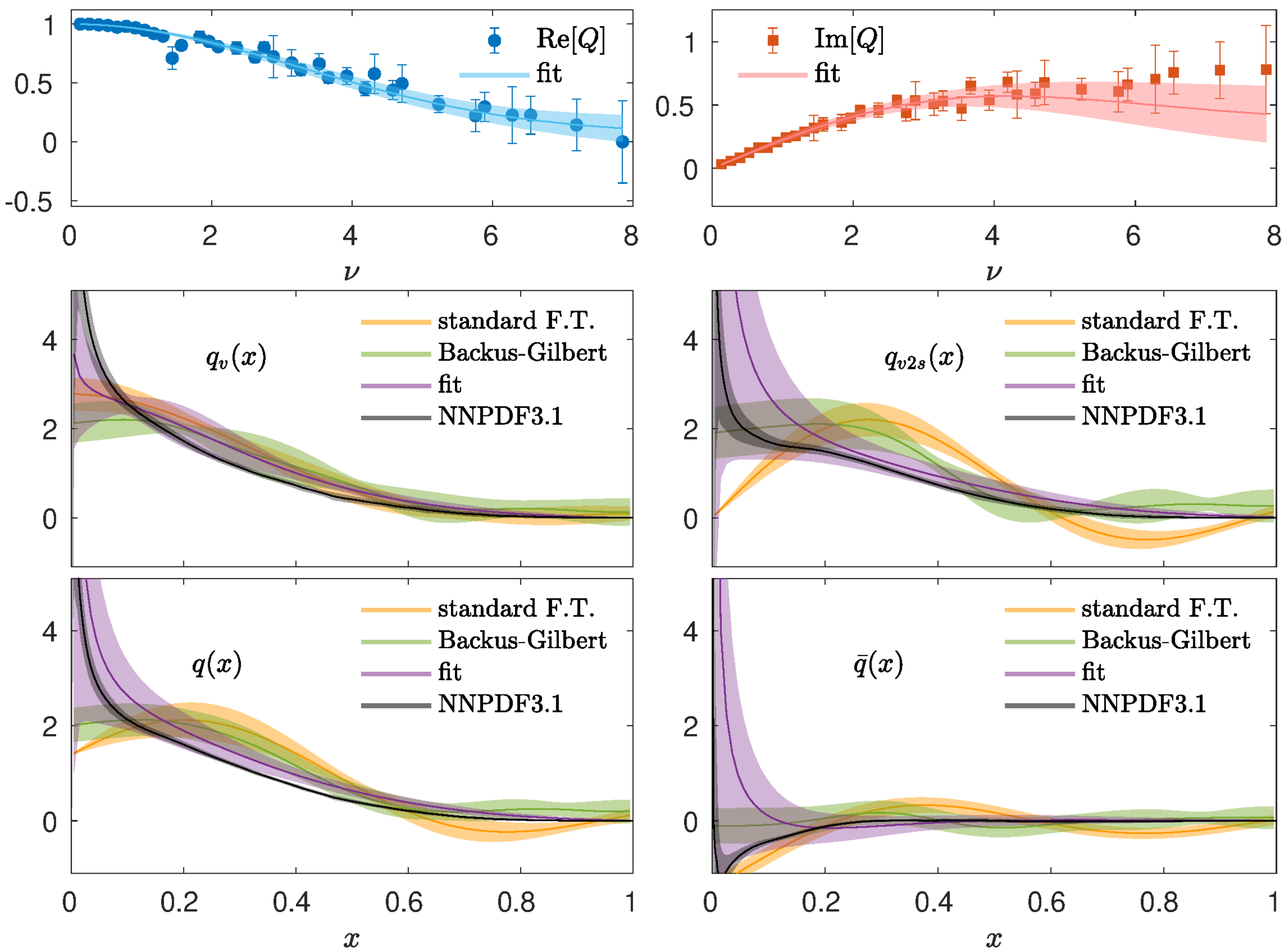
PDFs using ITDs with $z_{\max} = 4a$



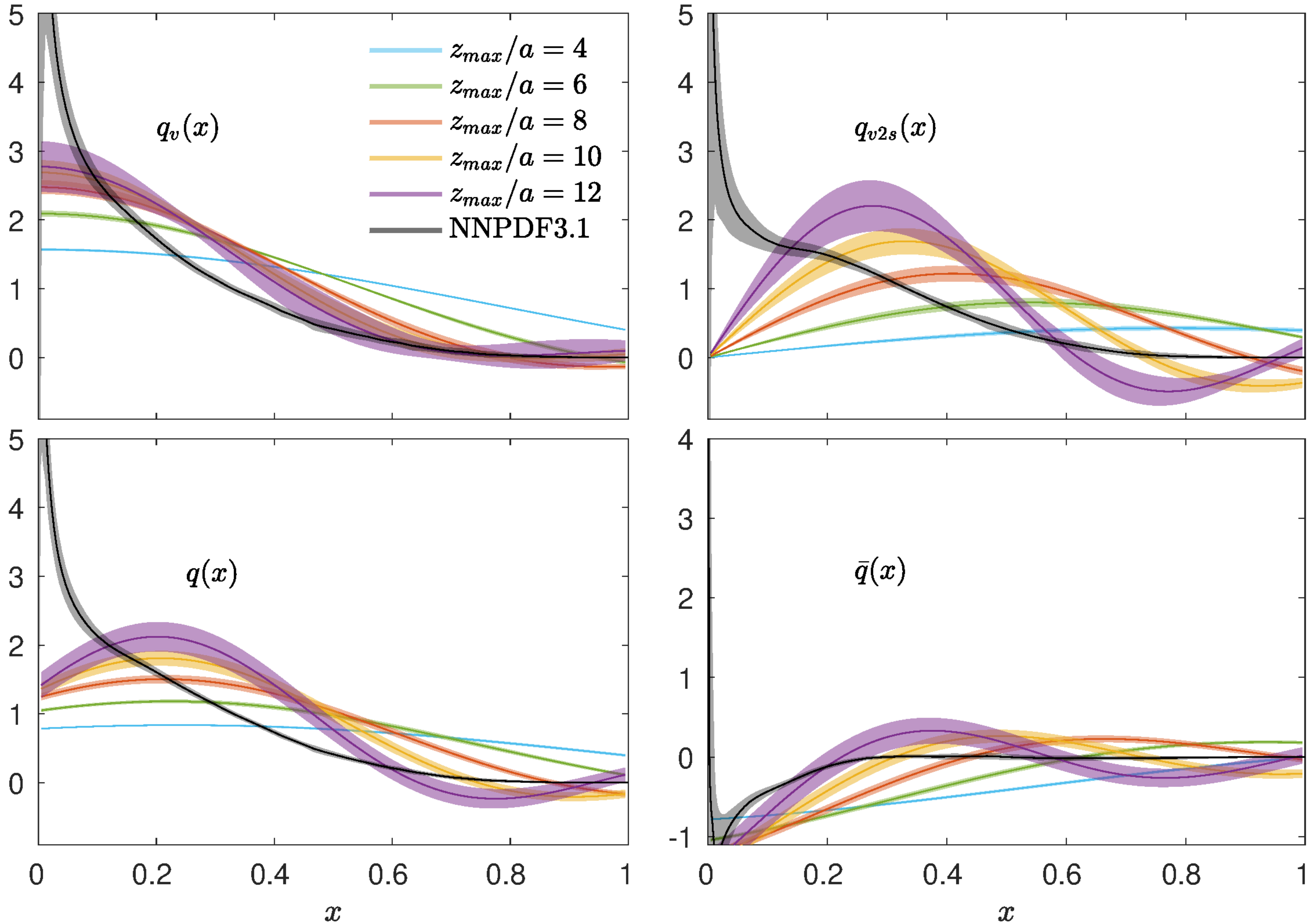
PDFs using ITDs with $z_{\max} = 8a$



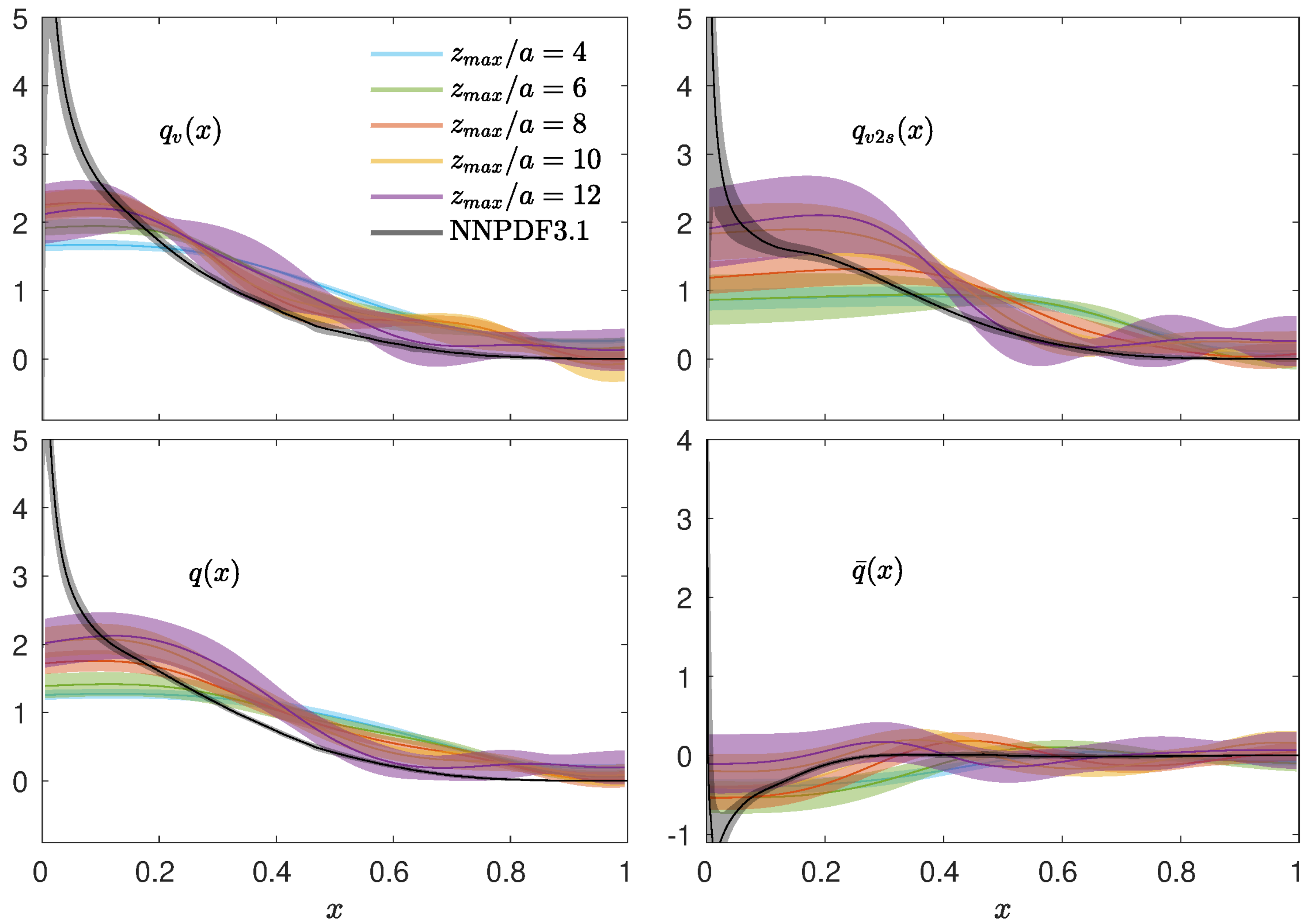
PDFs using ITDs with $z_{\max} = 12a$



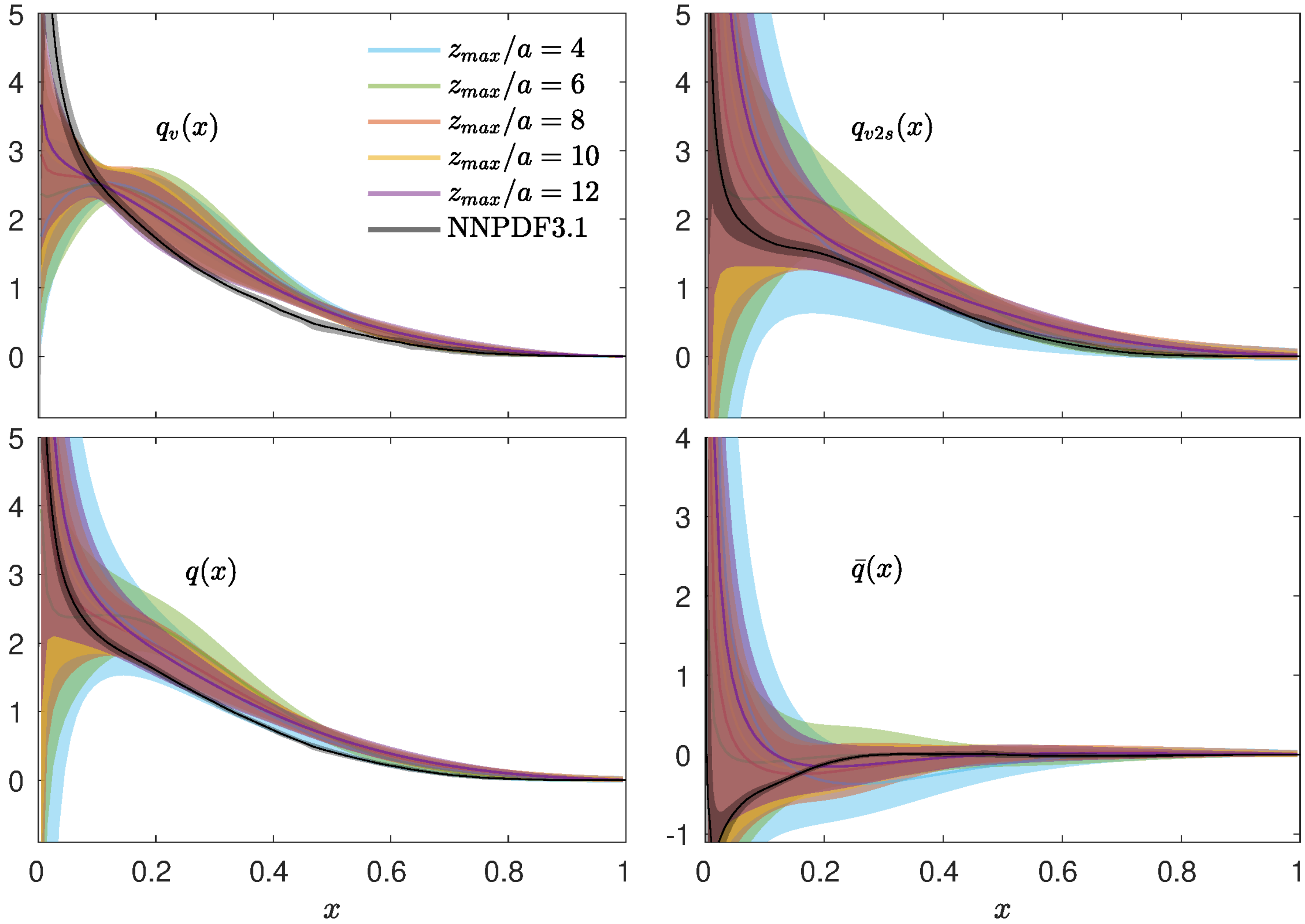
PDFs from naive FT – $z_{\max}$-dependence



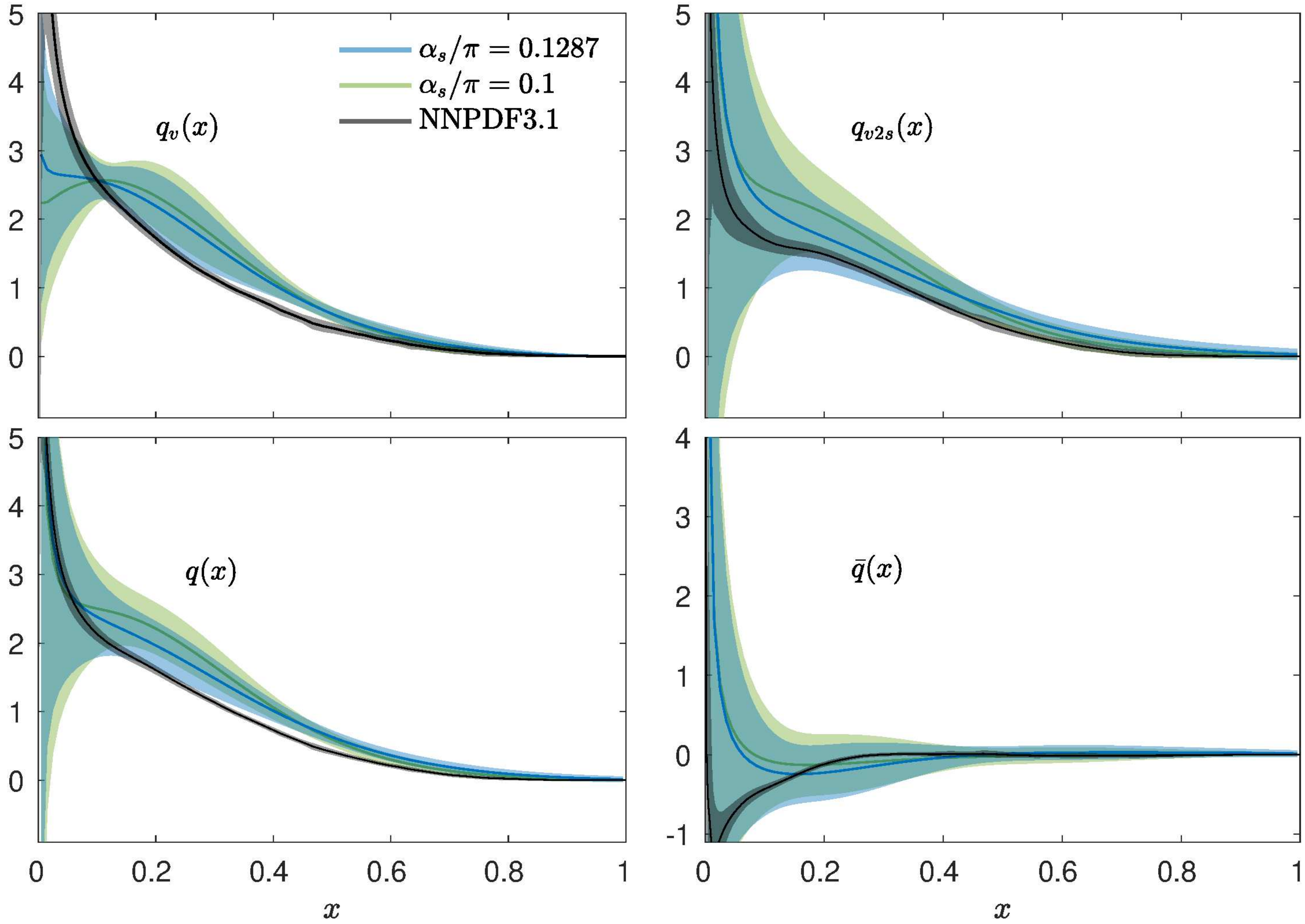
PDFs from BG – $z_{\max}$-dependence



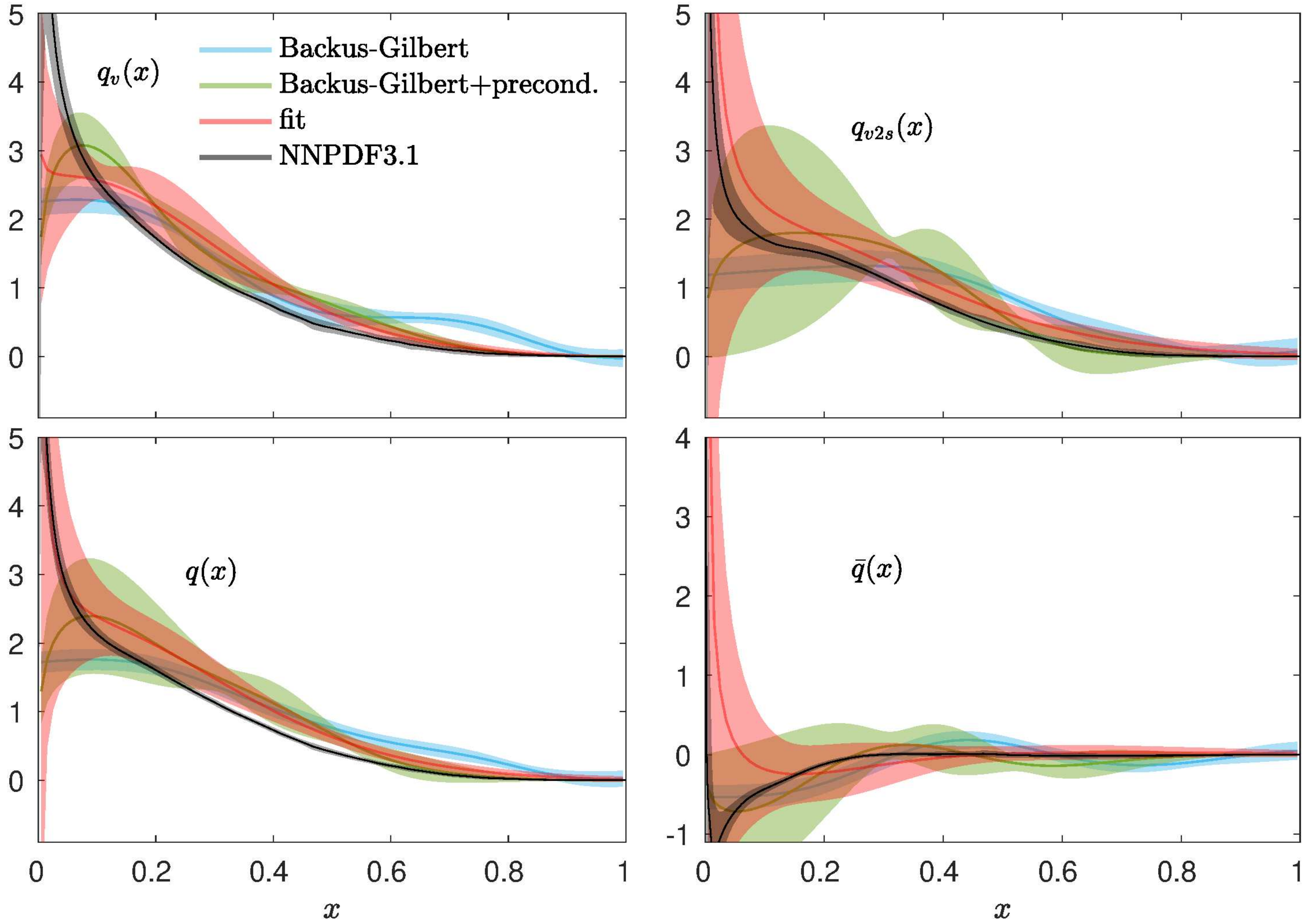
PDFs from fits – $z_{\max}$-dependence



PDFs from fits – α_s-dependence



BG with preconditioning vs. BG vs. fits



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Quantified systematics:

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- $z_{\max}$: $\Delta z_{\mathsf{max}}(x) = \frac{|q_{z_{\mathsf{max}}/a=12}(x) - q_{z_{\mathsf{max}}/a=4}(x)|}{2}$,
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  evidence: ETMC, Phys. Rev. D 99 (2019) 114504 – suppressed below stat. precision

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Estimated systematics:

- Discretization effects: assume 20%
  indirect support: no violation of continuum dispersion relation, $E^2 = P_3^2 + m_N^2$,
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- FVE: assume 5%
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  enhanced FVE? R. Briceño et al., Phys. Rev. D 98 (2018) 014511
  toy scalar model, relevant parameter for FVE: $m_N(L-z) \longrightarrow$ tiny,
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  also indirectly no indication for such effects in Z-factors for quasi-PDFs.
- Excited states: assume 10%
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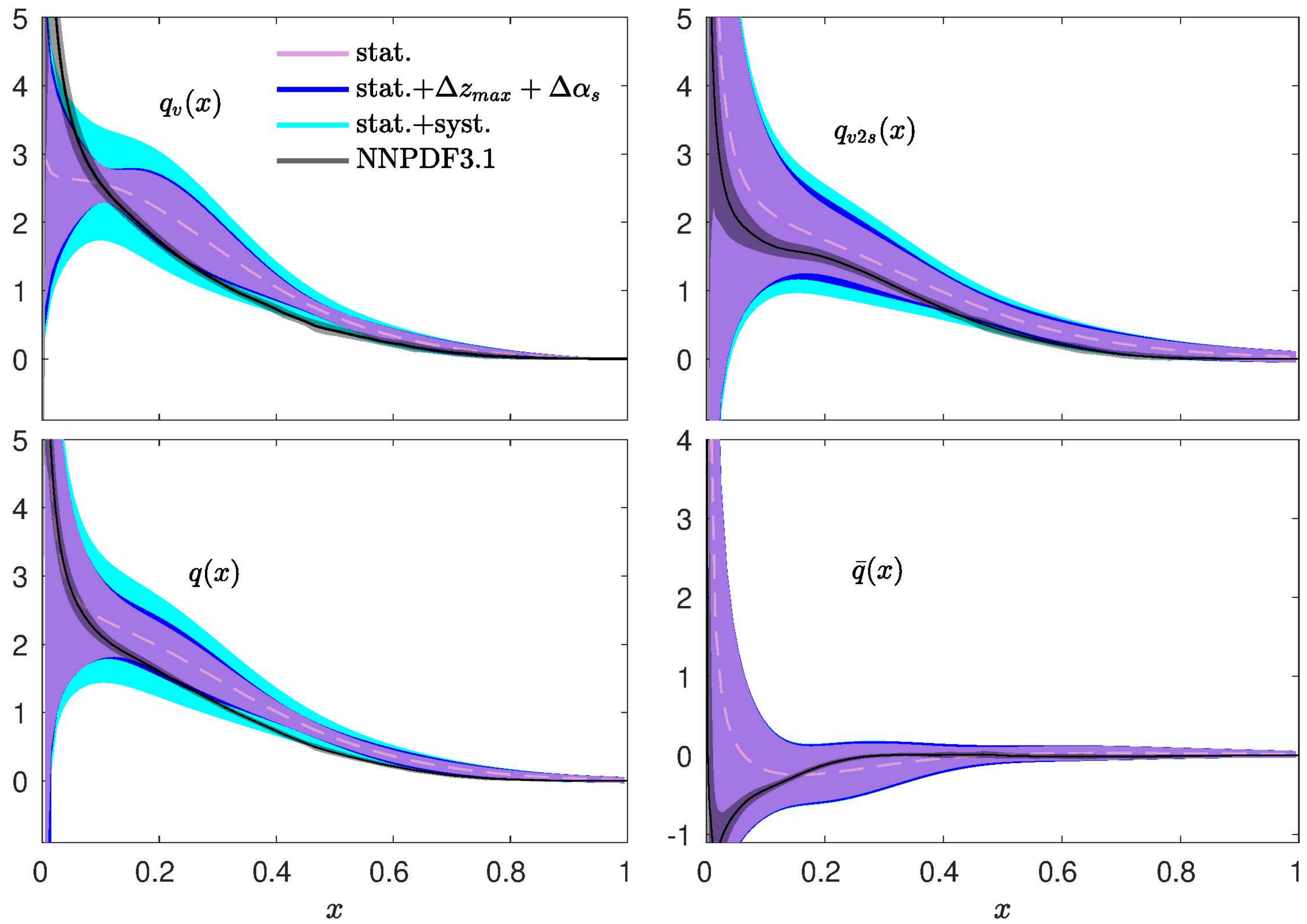
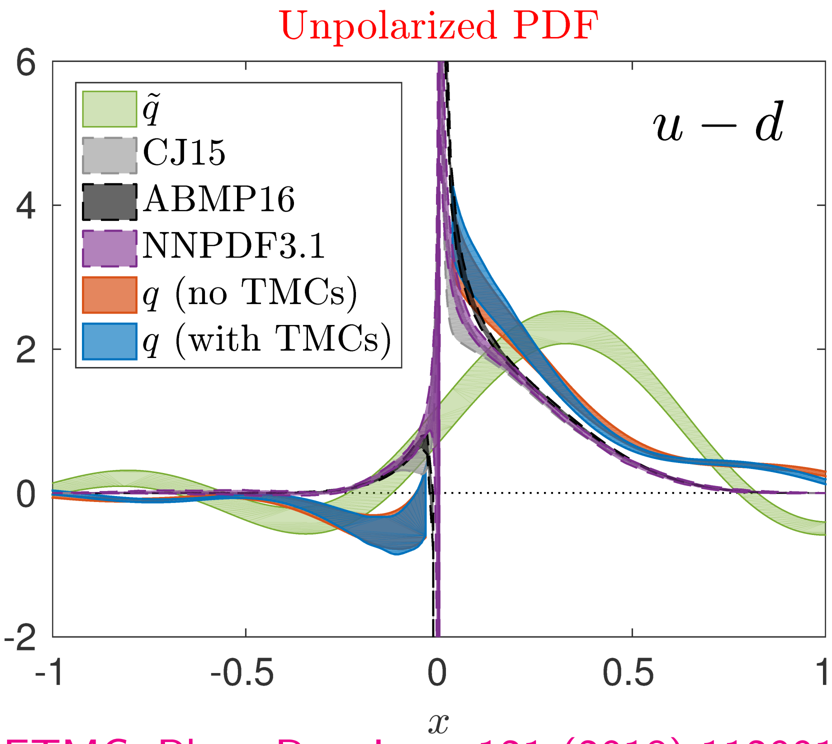
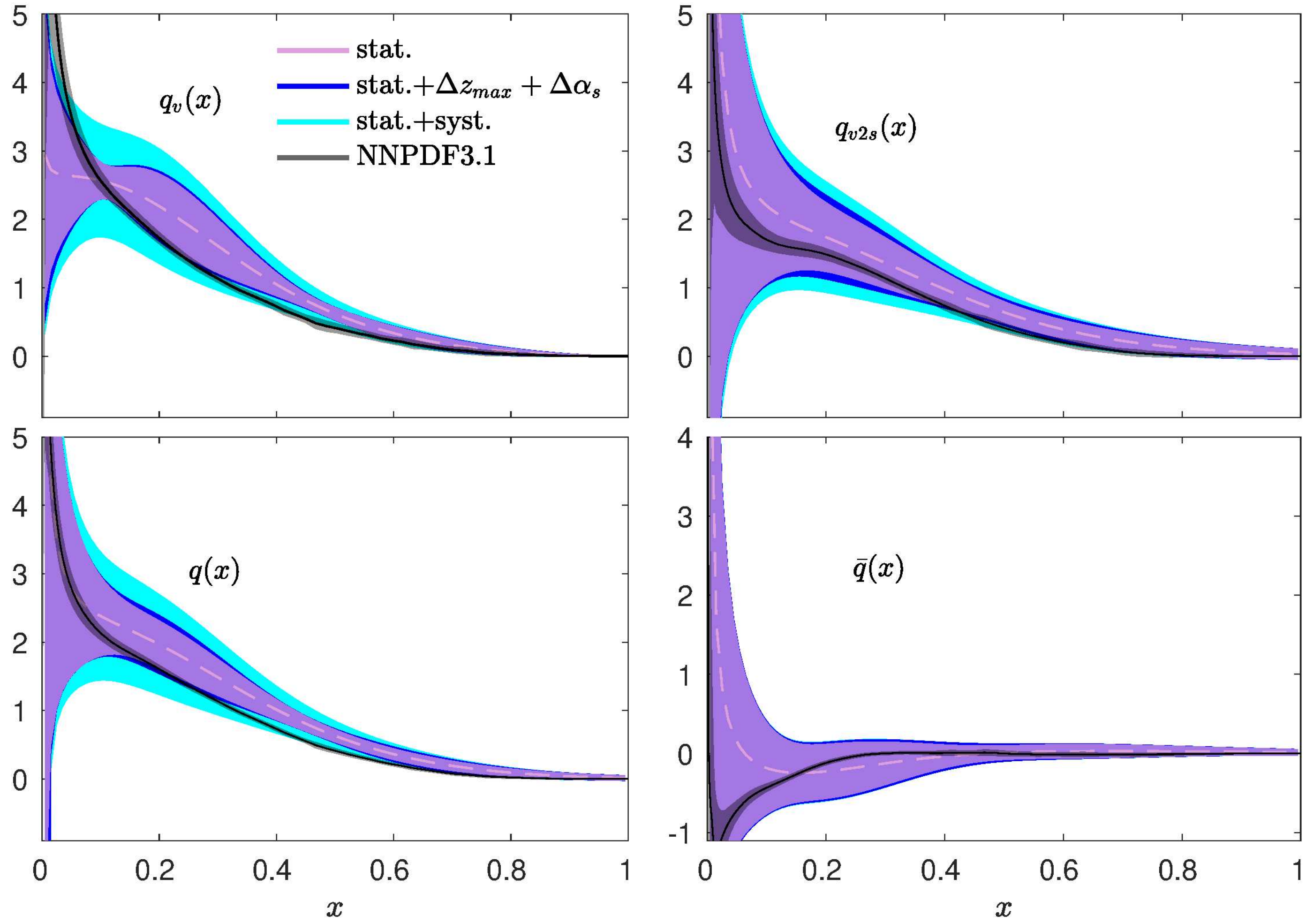
Final PDFs with systematics







ETMC, Phys. Rev. Lett. 121 (2018) 112001
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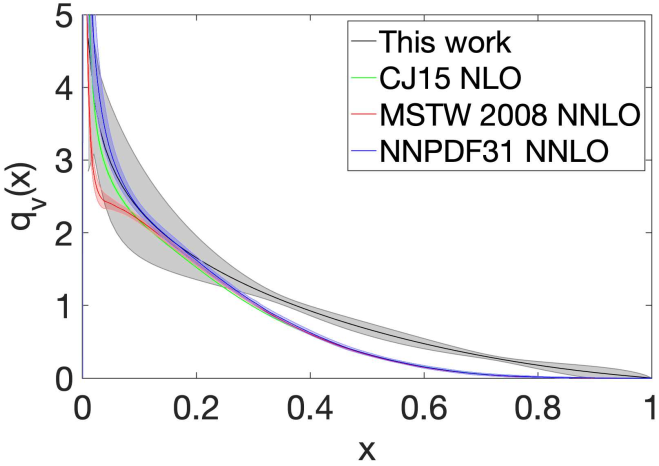
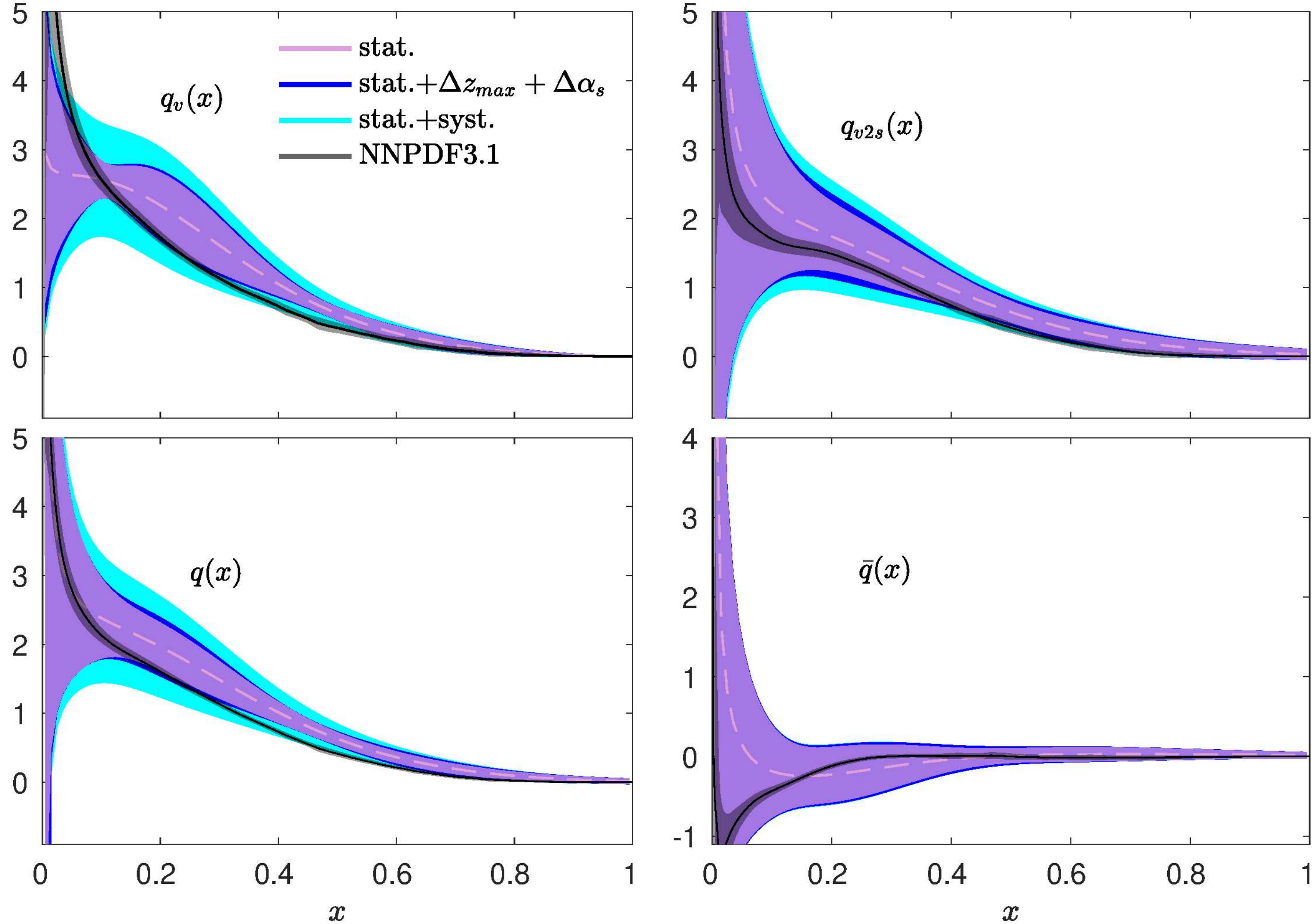
Comparison with JLab





B. Joó et al., arXiv:2004.01687

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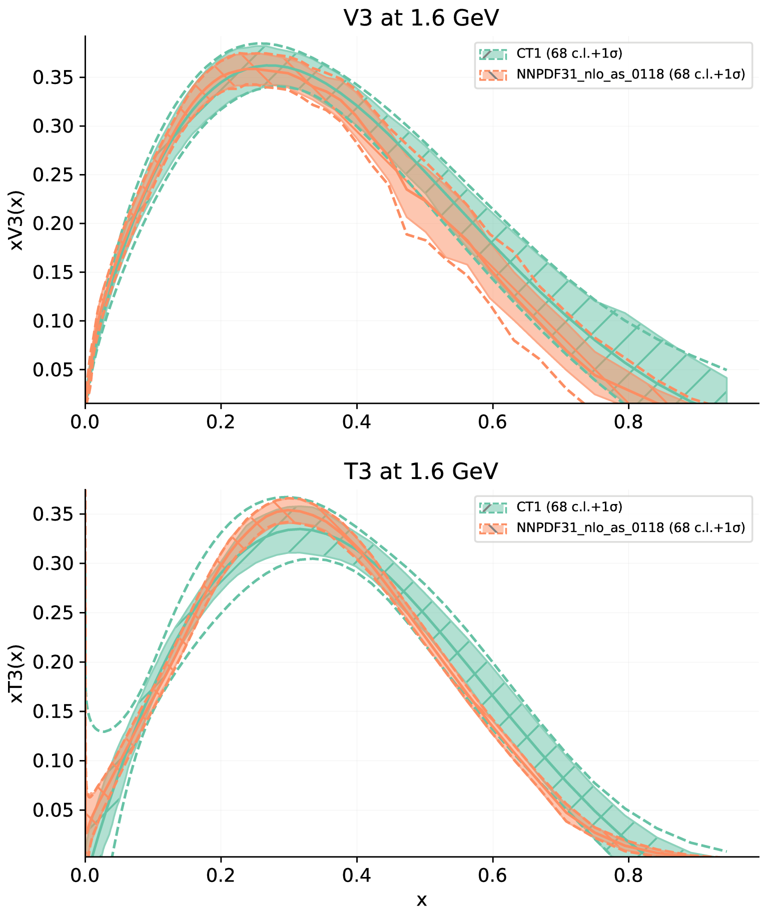

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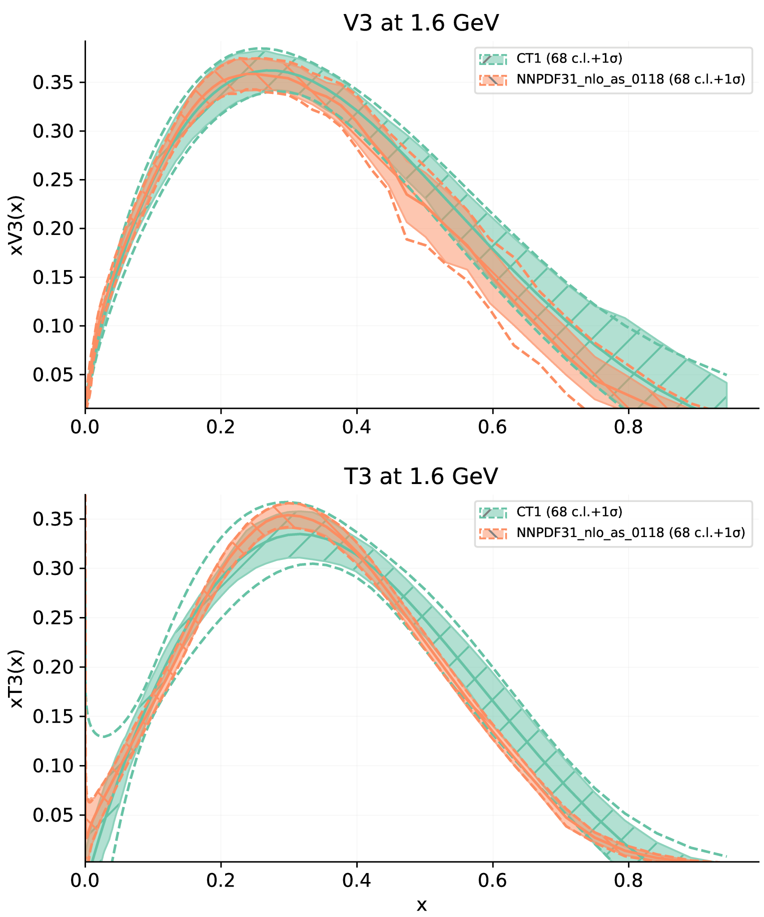

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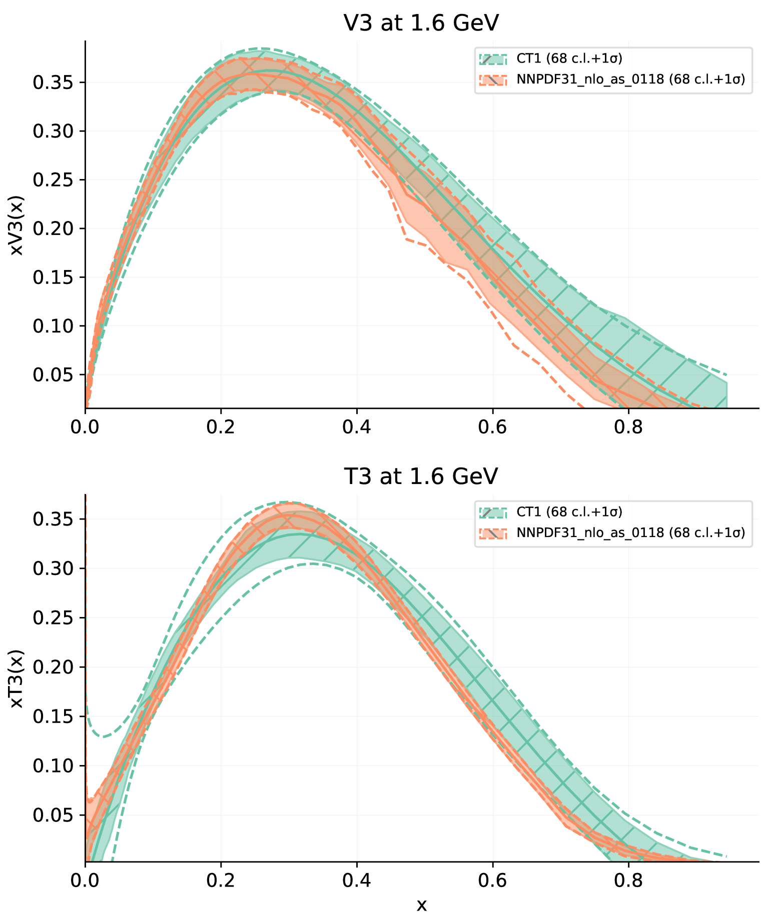

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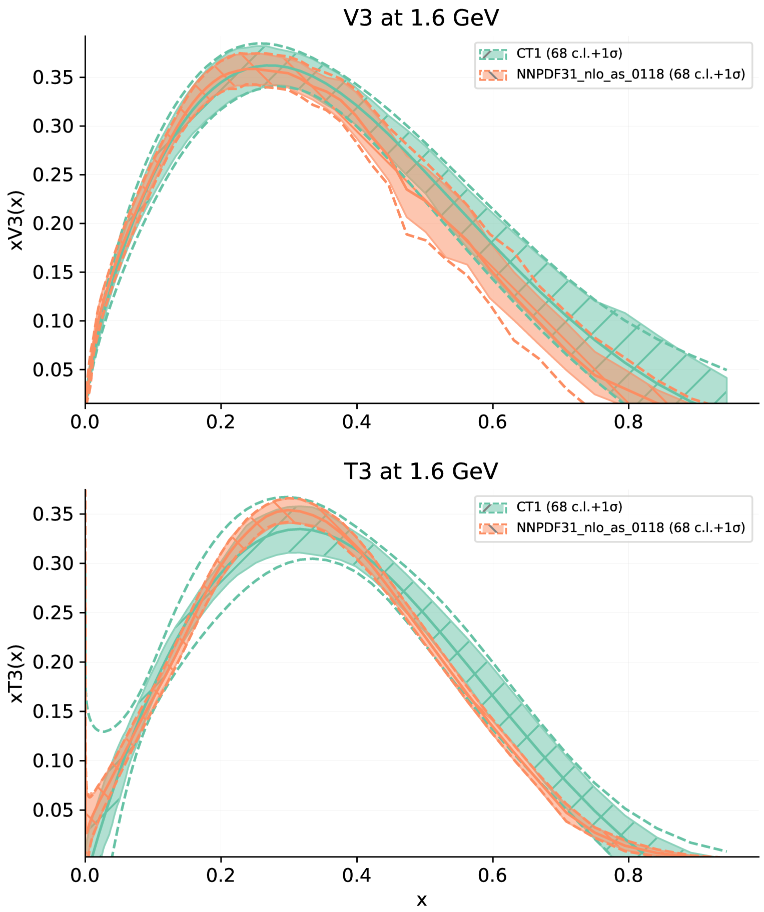

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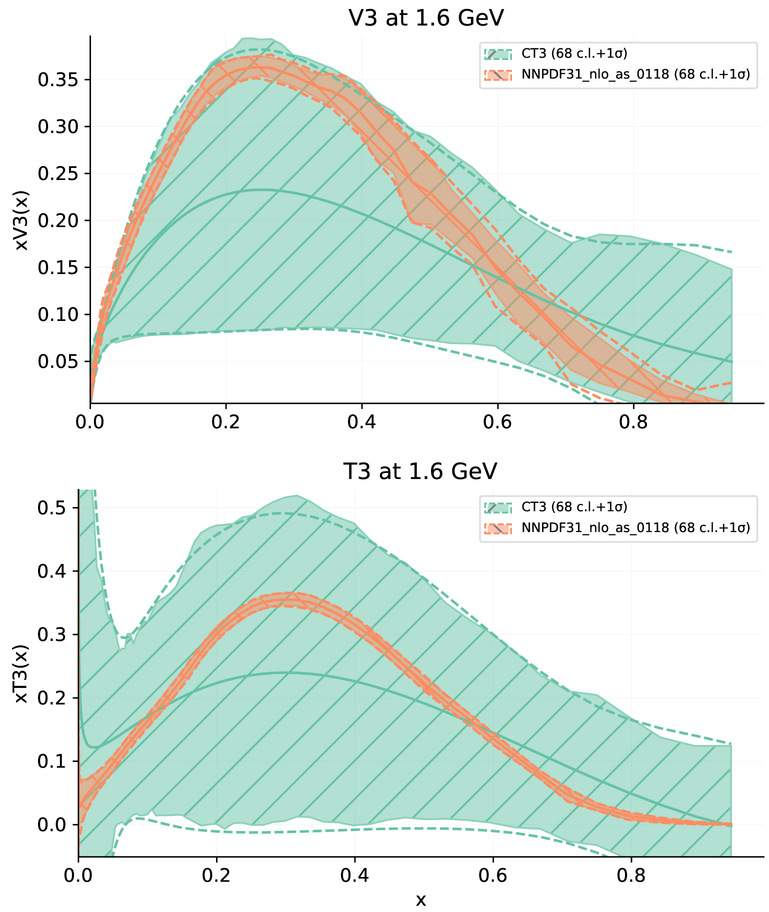

stat.error of ETMC lattice data + a scenario for systematics

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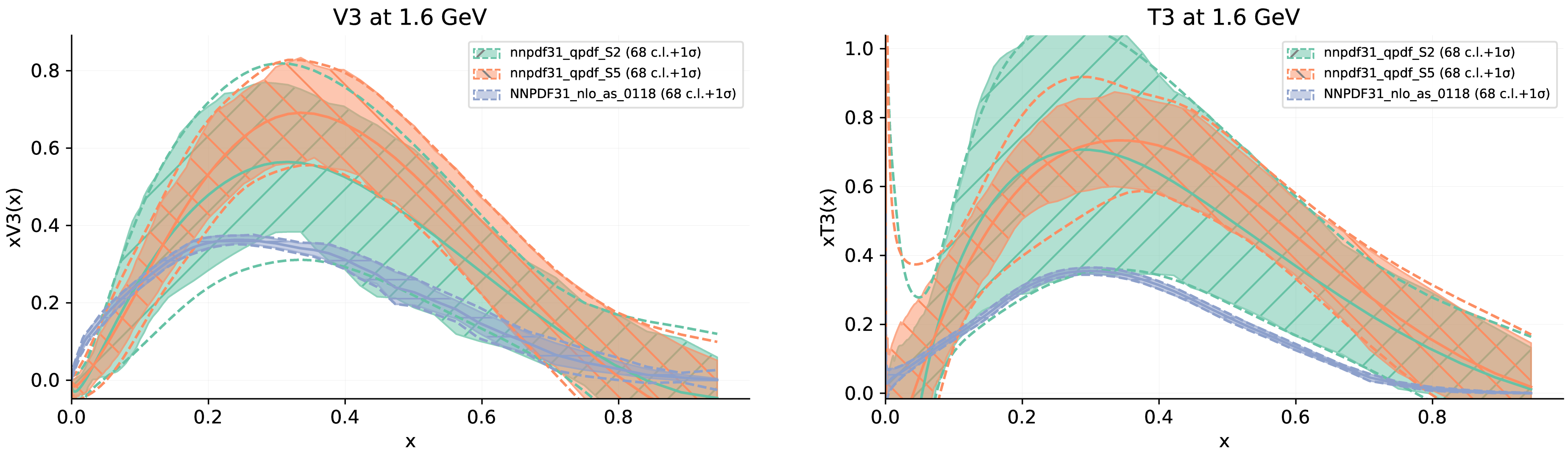

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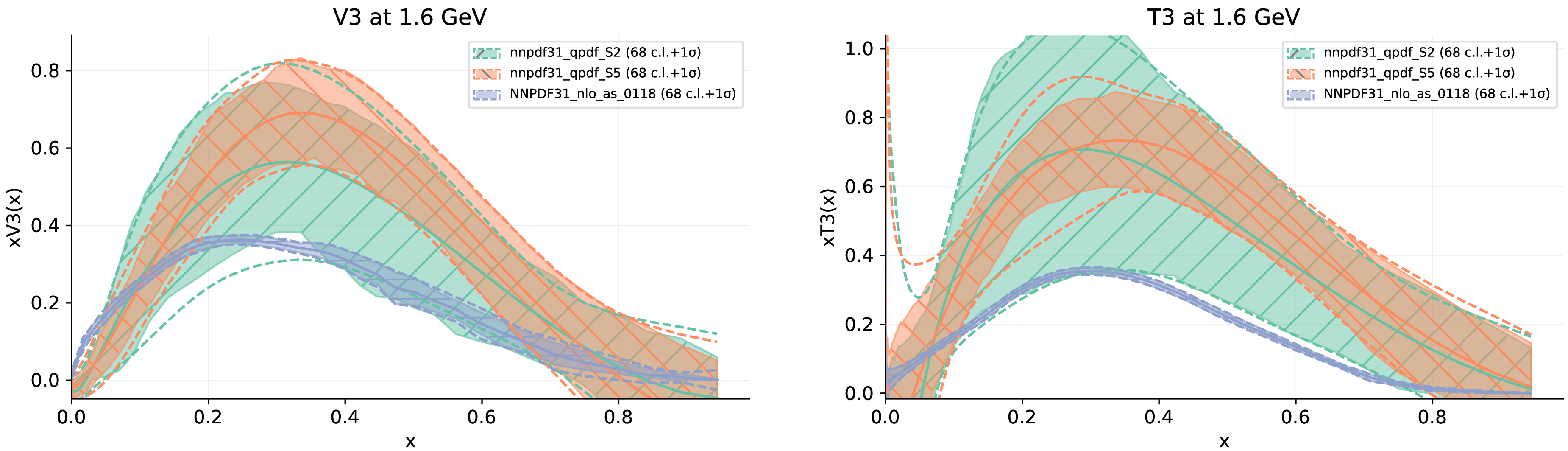

Reasonable agreement, but a lot of work for the lattice to reduce uncertainties!

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- however, measurements difficult due to their suppressed $\mathcal{O}(1/Q)$ kinematical behavior.

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- matching for twist-3 helicity $g_T(x)$
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  light-cone and quasi do not fully agree in the infrared
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Lattice matrix element:

$$\mathcal{M}_{g_T}(P,z) = \langle P|\,\overline{\psi}(0,z)\,\gamma^j\,\gamma^5\,W(z)\,\psi(0,0)\,|P\rangle\,.$$

$\gamma^j = \gamma^x$ or γ^y

Twist-3 PDFs

Lattice matrix element:

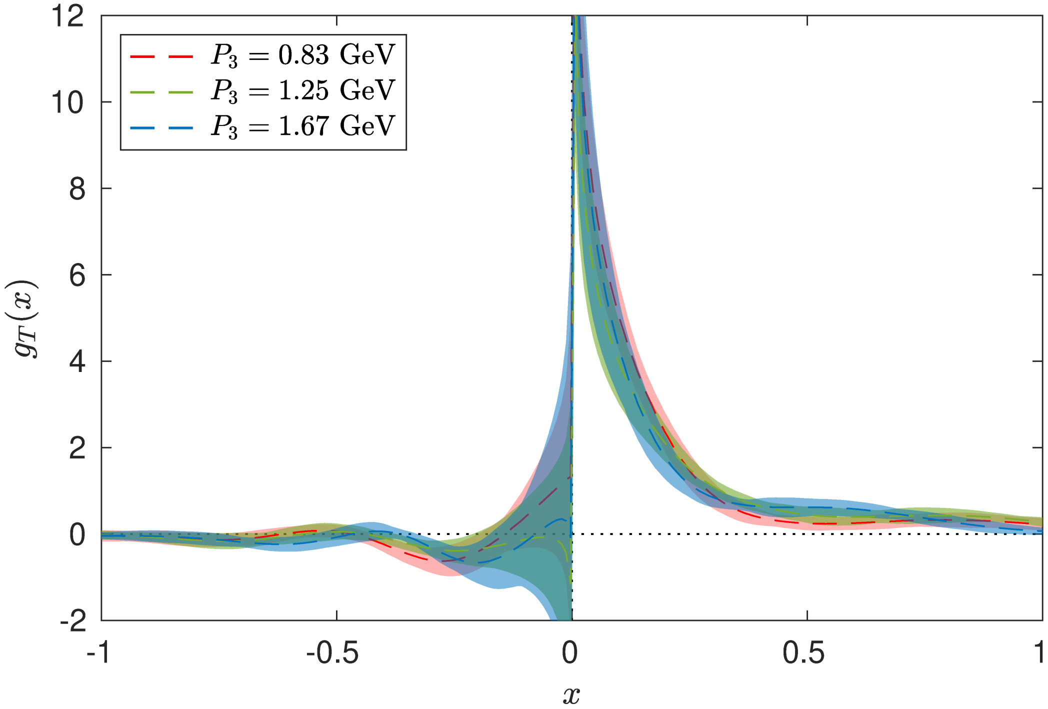
$$\mathcal{M}_{g_T}(P,z) = \langle P|\,\overline{\psi}(0,z)\,\gamma^j\,\gamma^5\,W(z)\,\psi(0,0)\,|P\rangle\,.$$

$\gamma^j = \gamma^x$ or γ^y

Lattice setup: S. Bhattacharya et al., arXiv:2004.04130

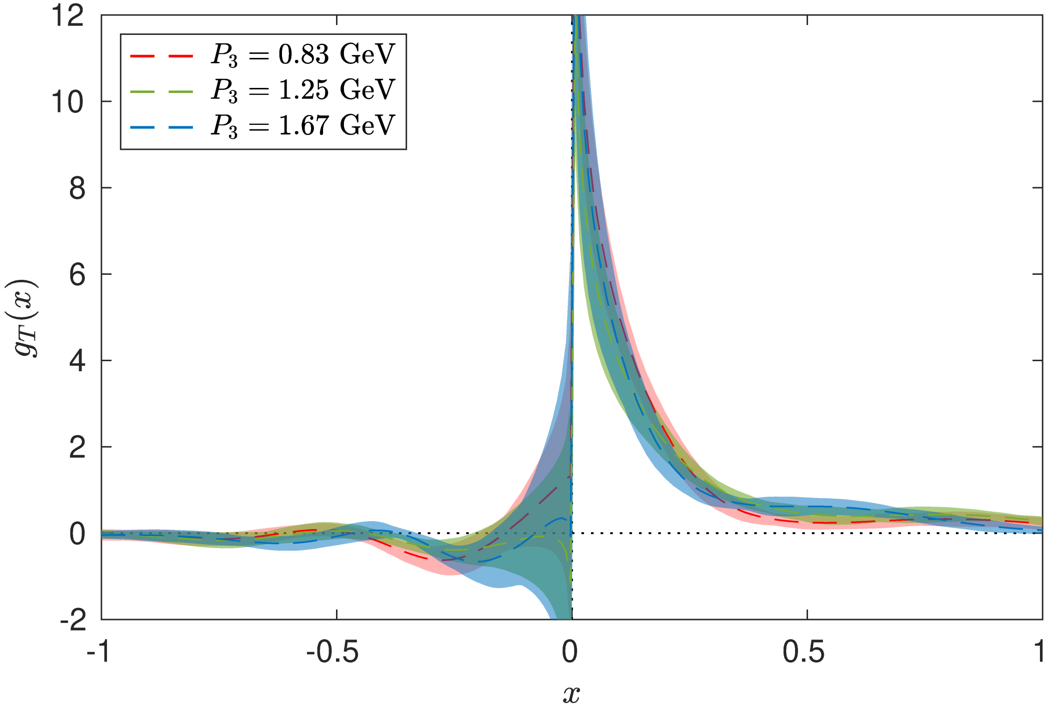
- fermions: $N_f = 2+1+1$ TM fermions + clover term,
- gluons: Iwasaki gauge action, $\beta = 1.778$,
- a=0.081 fm, $m_\pi \approx 270$ MeV.
- $32^3 \times 64$, $L = 3$ fm, $m_\pi L = 4$,

Nucleon boost dependence
(after matching)
(quasi-g_T reconstructed with BG)

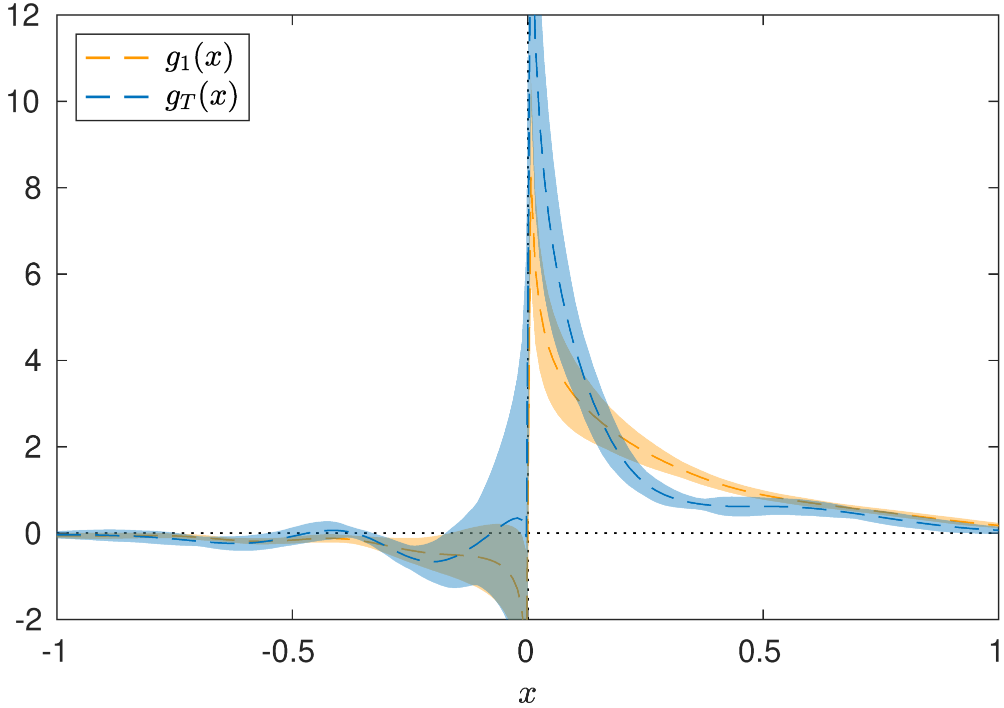


Results for g_T and g_1

Nucleon boost dependence
(after matching)
(quasi-g_T reconstructed with BG)



Twist-2 g_1 vs. twist-3 g_T
(at the largest boost)



WW approximation: twist-3 $g_T(x)$ fully determined by twist-2 $g_1(x)$:

$$g_T^{\rm WW}(x) = \int_x^1 \frac{dy}{y} g_1(y)$$

Wandzura-Wilczek approximation

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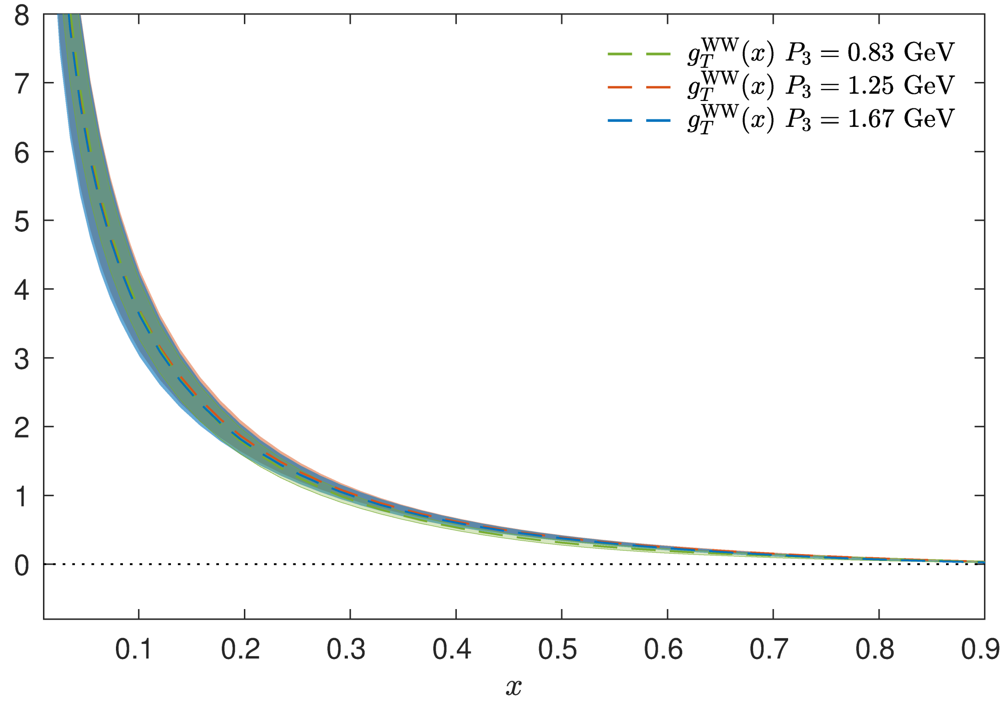

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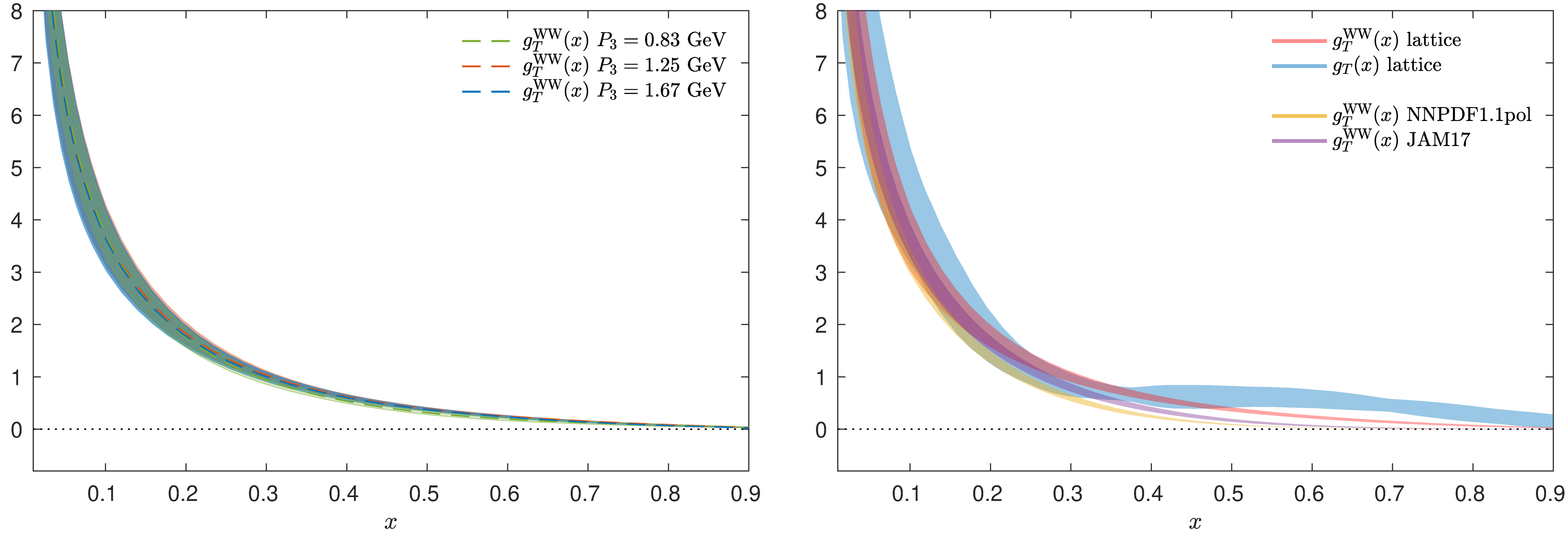

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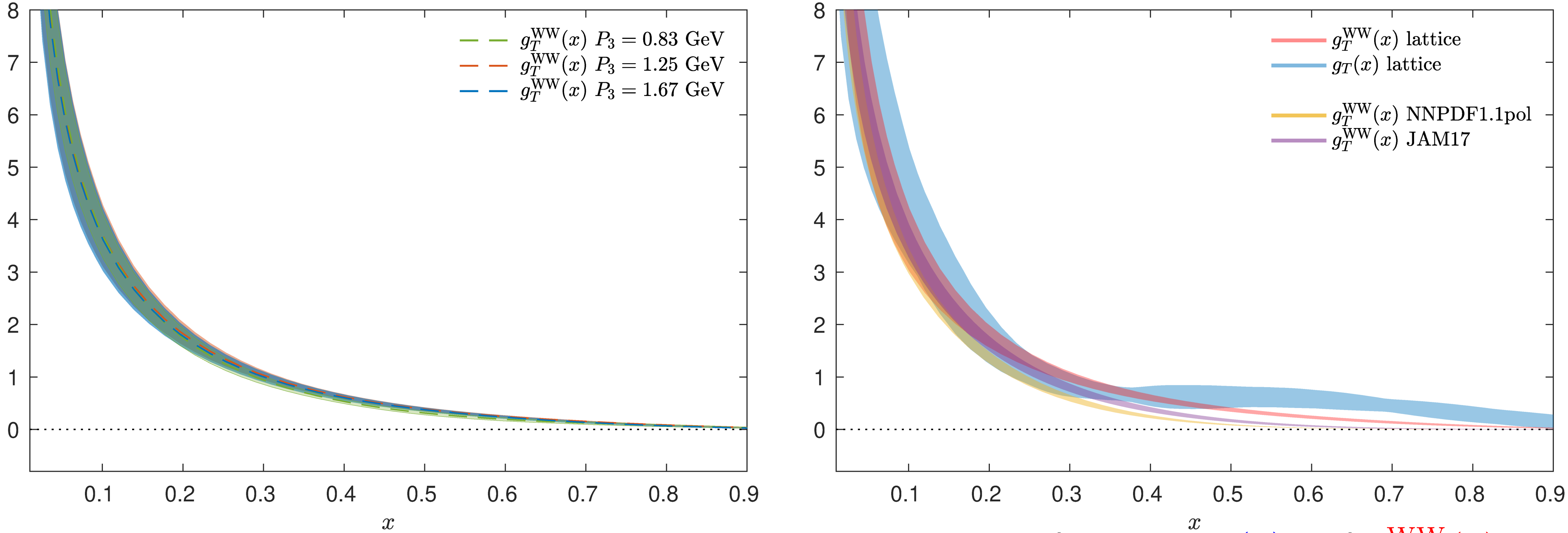

agreement between $g_T(x)$ and $g_T^{\mathrm{WW}}(x)$
for $x \lesssim 0.5$ within uncertainties

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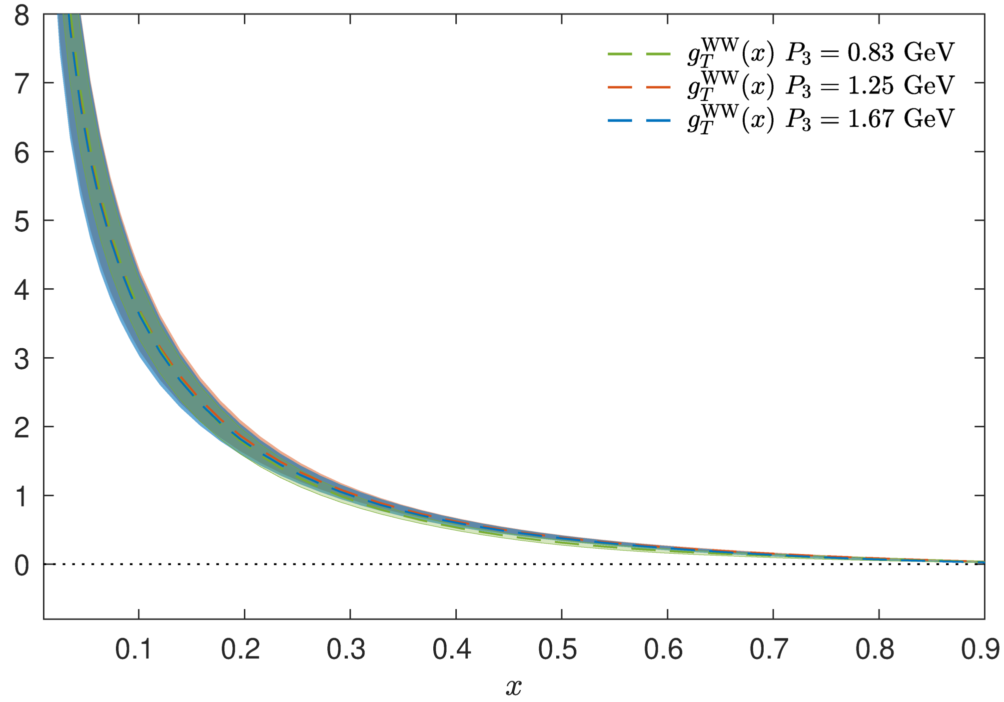

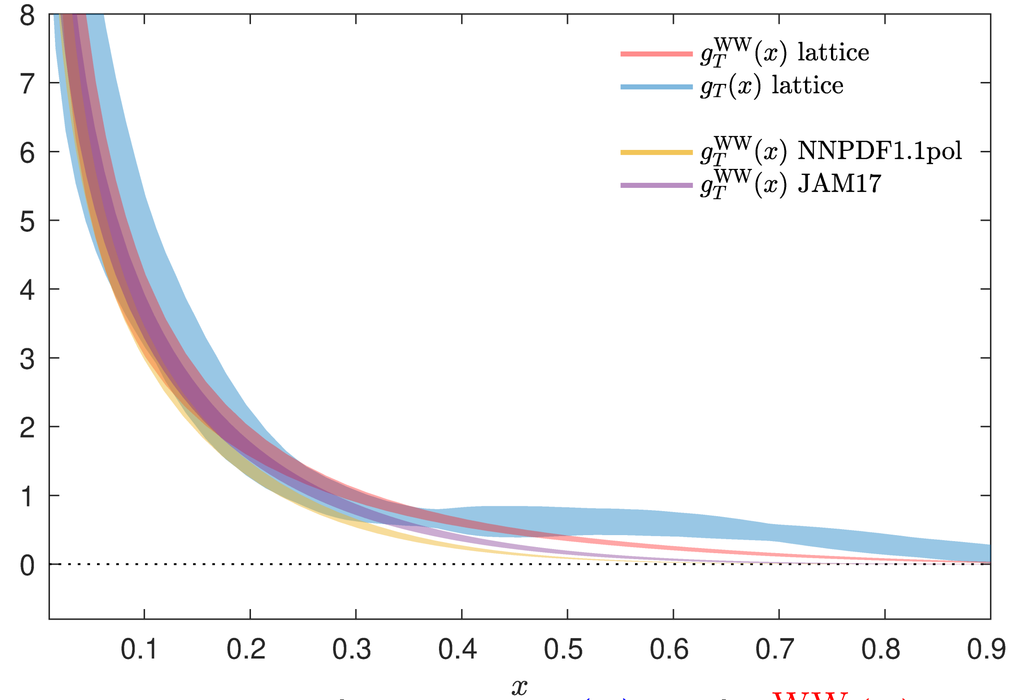

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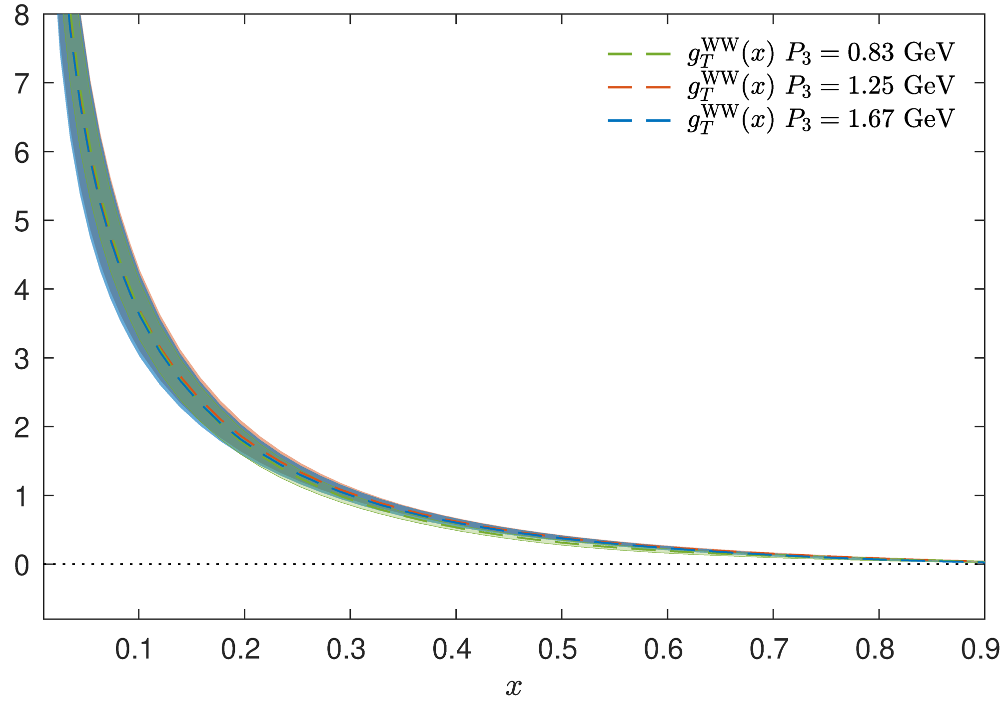

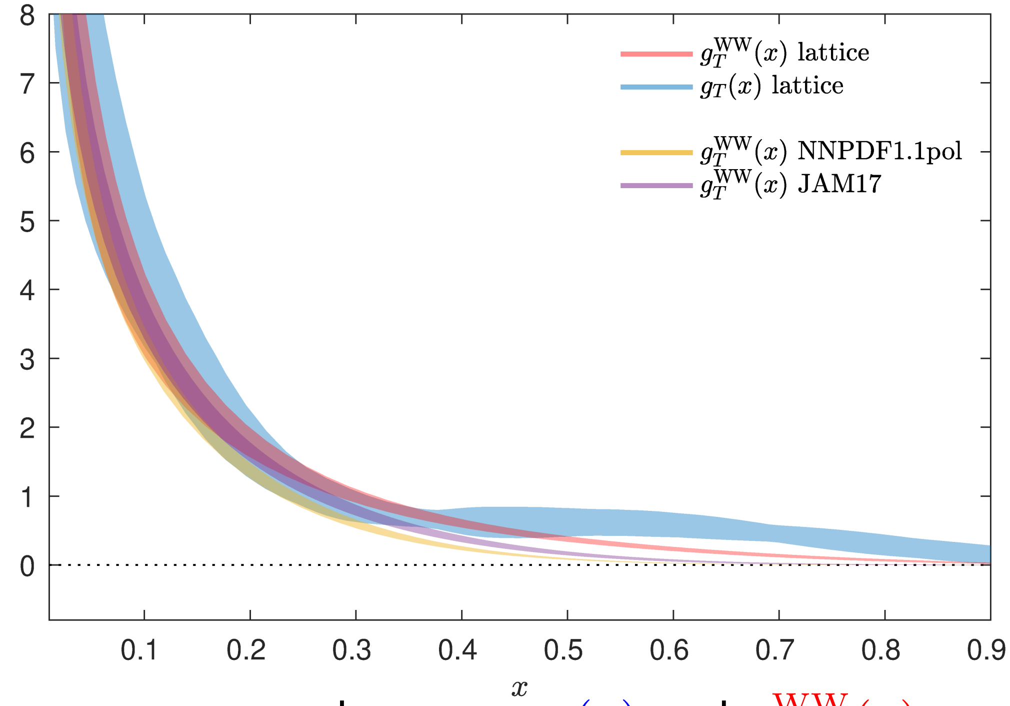

agreement between $g_T(x)$ and $g_T^{\mathrm{WW}}(x)$
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still: possible violation up to 30-40%
interestingly, similar possible violation (15-40%)
in experimental data analysis by JLab:

A. Accardi, A. Bacchetta, W. Melnitchouk, M. Schlegel, JHEP 11 (2009) 093

GPDs – can be accessed with the same type of matrix elements as PDFs:

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$$\mathcal{M}(z,t,\xi;\Gamma,\overline{\Gamma}) = \langle P''|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P'\rangle,$$

Quasi-GPDs

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Γ – Dirac structure of the insertion,
$\overline{\Gamma}$ – Dirac structure of the projector,

average momentum: $P = \frac{P'+P''}{2}$,

momentum transfer: $Q = P'' - P'$, $t = -Q^2$,

quasi-skewness: $\tilde{\xi} = -\frac{P_3''-P_3'}{P_3''+P_3'} = -\frac{Q_3}{2P_3}$, light-cone skewness: $\xi = \tilde{\xi} + \mathcal{O}\left(\frac{M^2}{P_3^2}\right)$.

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After renormalization, the above MEs can be decomposed into MEs of quasi-GPDs:

$$\mathcal{M}(z,t,\xi;\,\mu_R;\,\Gamma,\overline{\Gamma}) = \mathcal{K}_H(\Gamma,\overline{\Gamma})H(z,t,\xi;\mu_R) + \mathcal{K}_E(\Gamma,\overline{\Gamma})E(z,t,\xi;\mu_R).$$

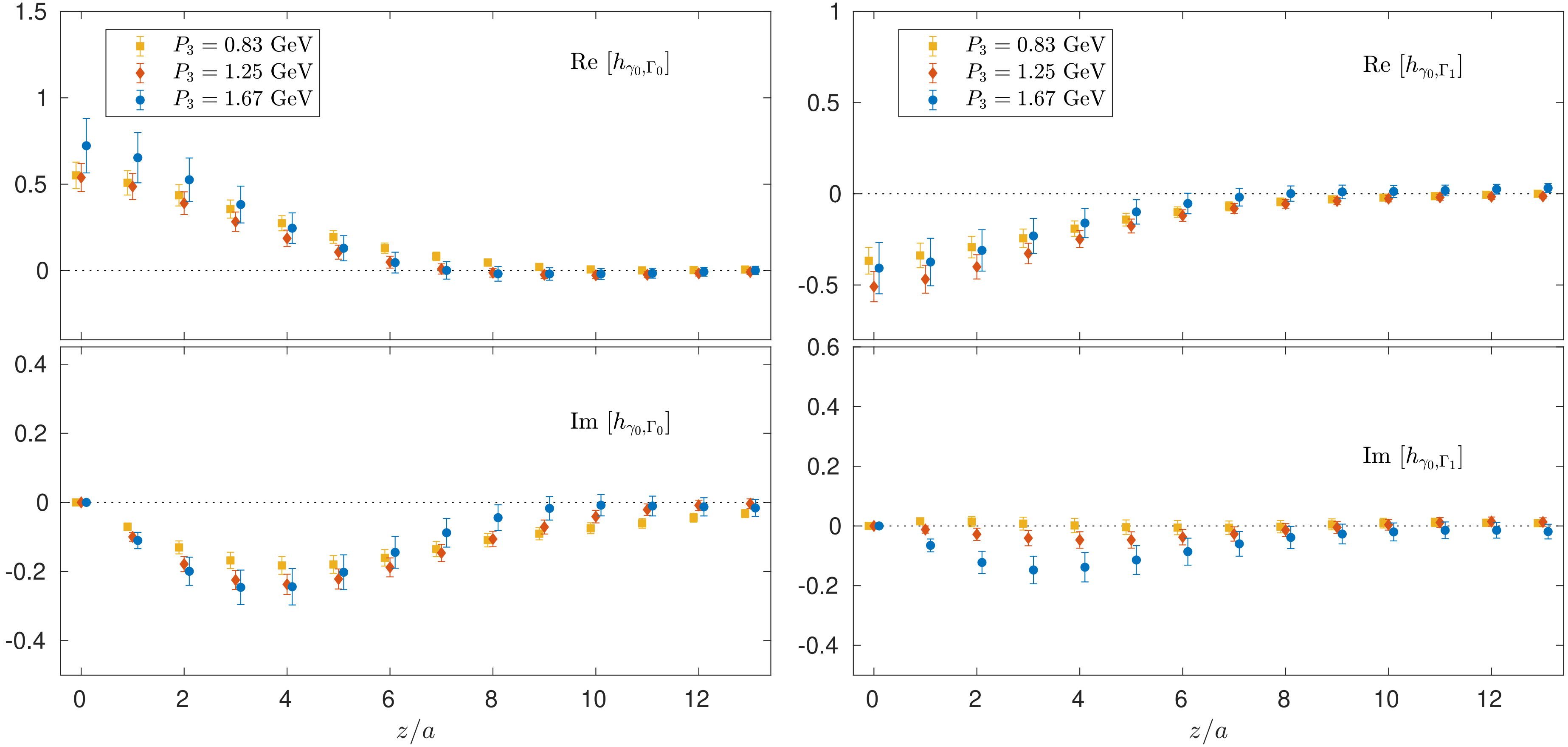
Bare matrix elements

Lattice setup: same as for twist-3

- fermions: $N_f = 2+1+1$ TM fermions + clover term,
- gluons: Iwasaki gauge action, $\beta = 1.778$,
- a=0.081 fm, $m_\pi \approx 270$ MeV.
- $32^3 \times 64$, $L = 3$ fm, $m_\pi L = 4$,

$P_3 = 0.83, 1.25, 1.67$ GeV
$Q^2 = 0.69$ GeV^2
$\xi = 0$

unpolarized projector | polarized projector



Disentangled renormalized matrix elements

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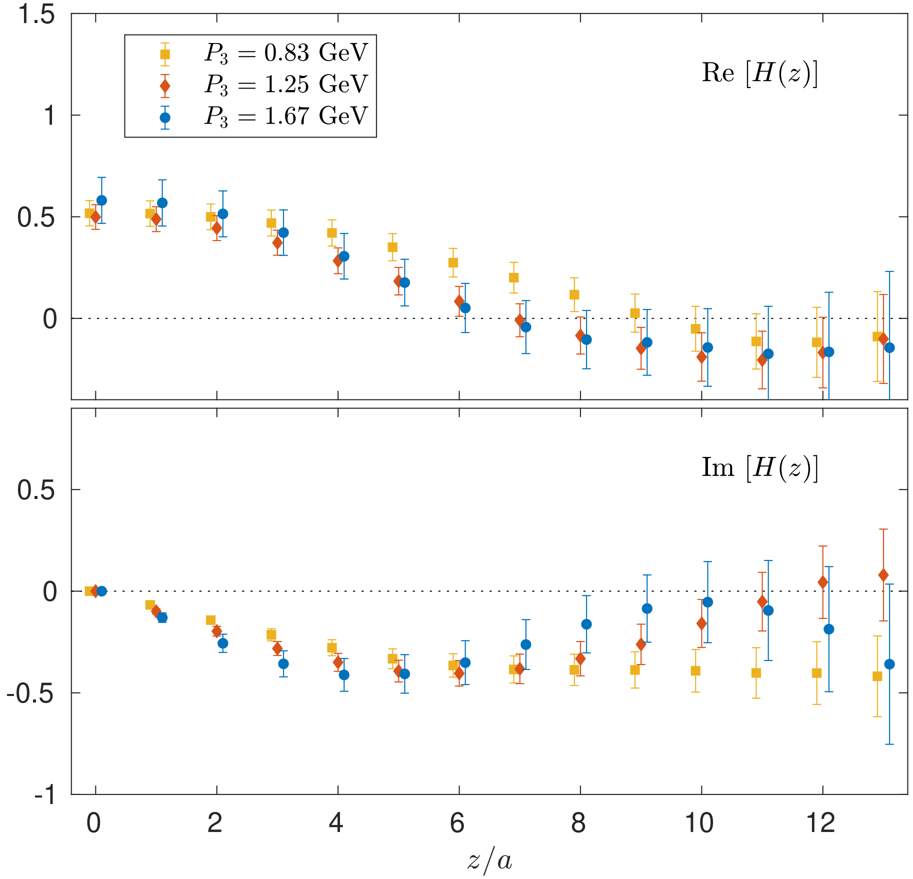
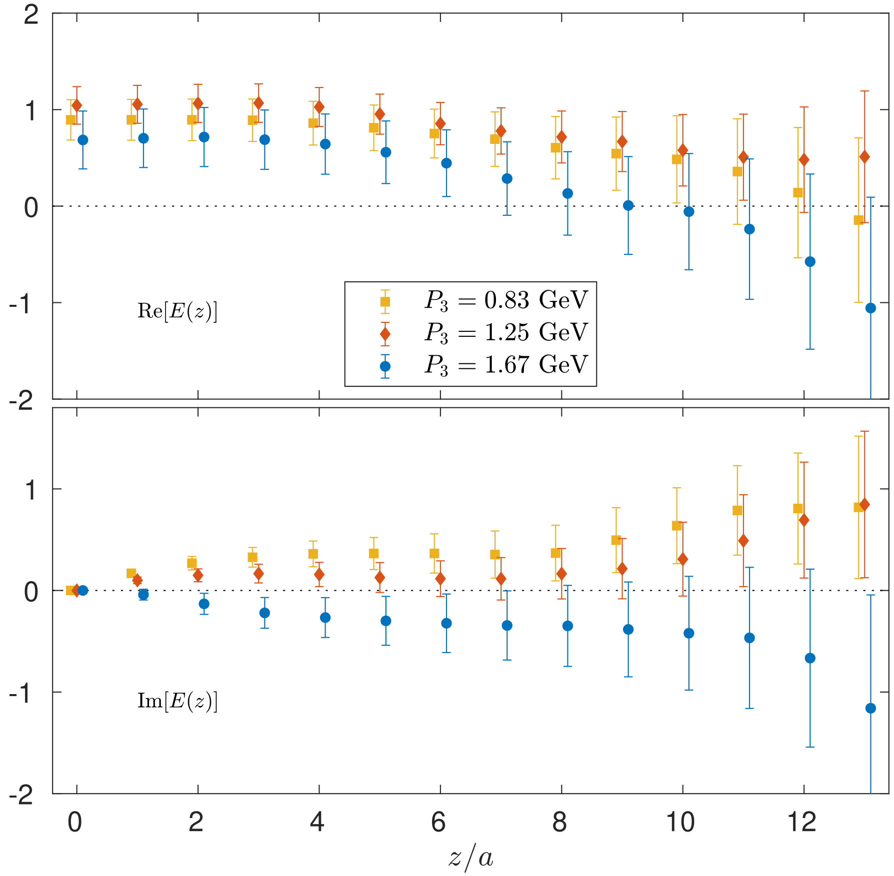
ME of H-function



ME of E-function



After matching: H and E functions

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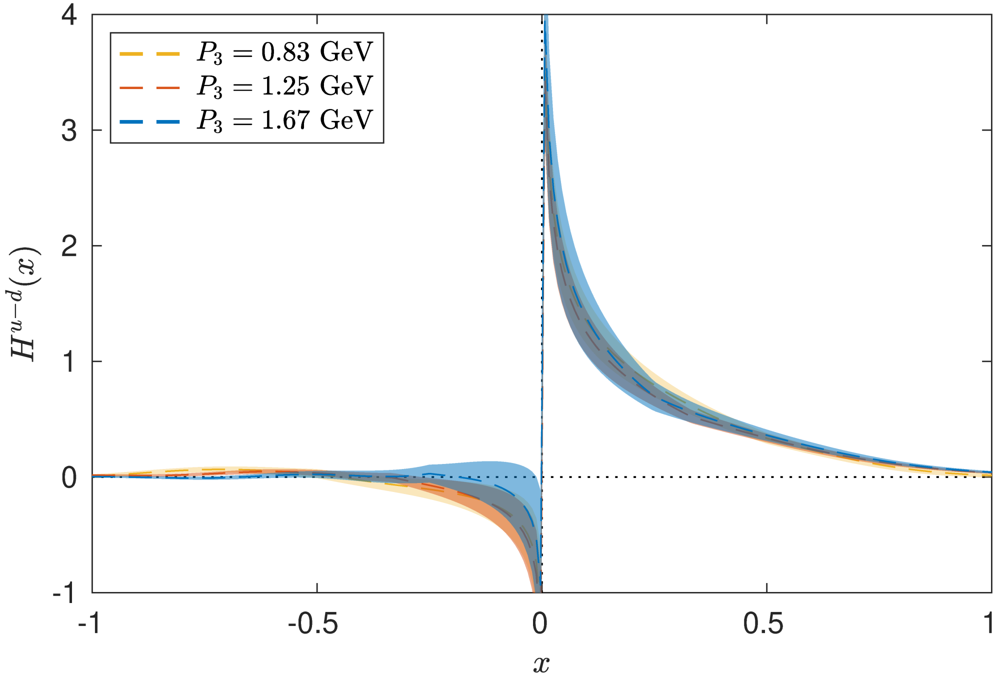
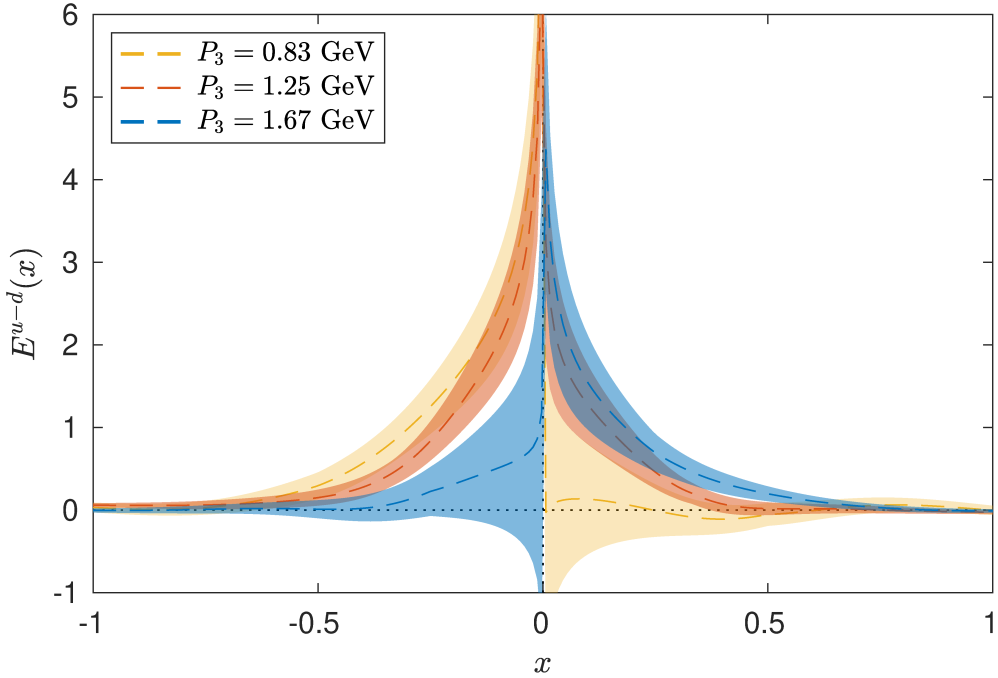
H-function



E-function



Comparison of PDFs and H-GPDs

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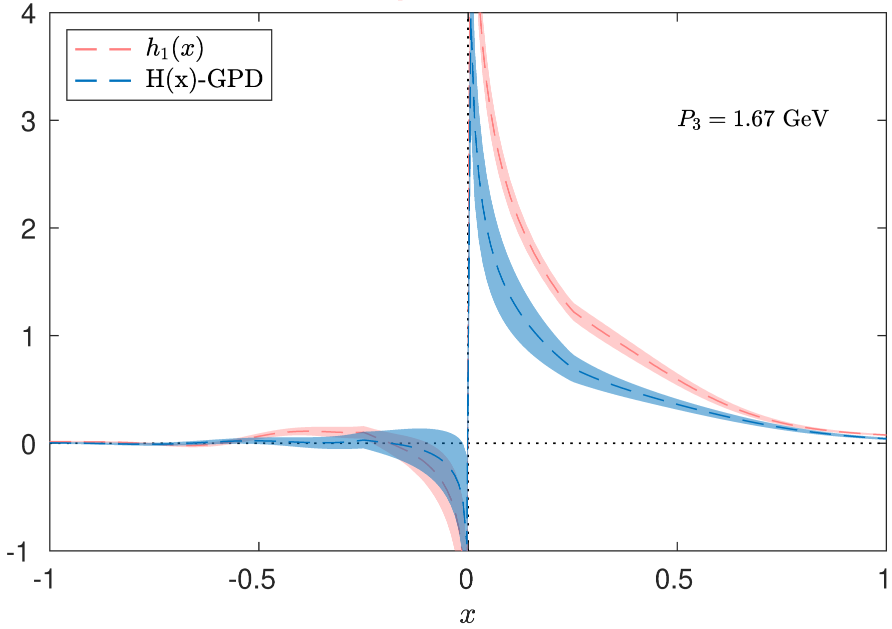
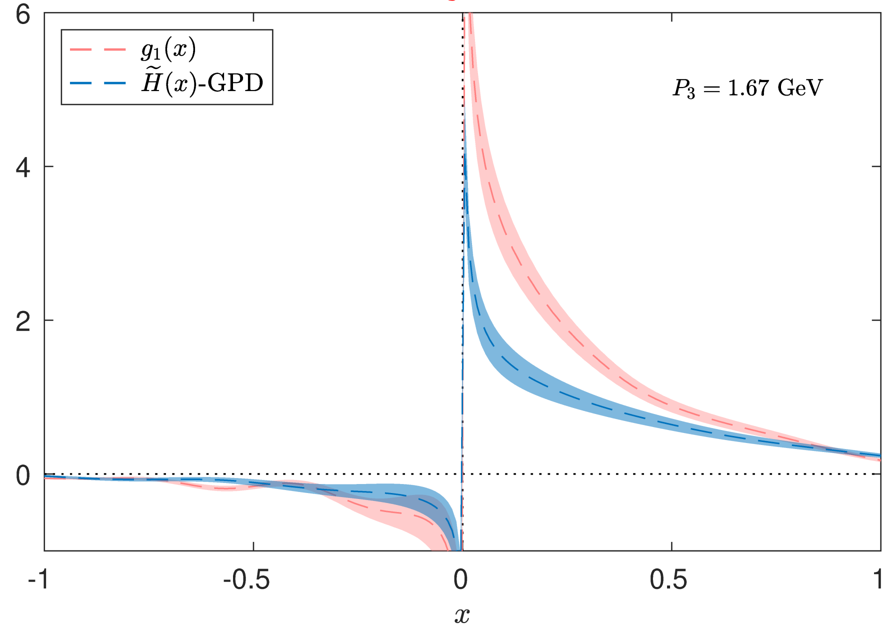
- $32^3 \times 64$, $L = 3$ fm, $m_\pi L = 4$,

$P_3 = 1.67$ GeV
$Q^2 = 0$ or 0.69 GeV^2
$\xi = 0$

unpolarized



helicity



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Conclusions and prospects

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Thank you for your attention!

Backup slides

Quasi-PDFs procedure

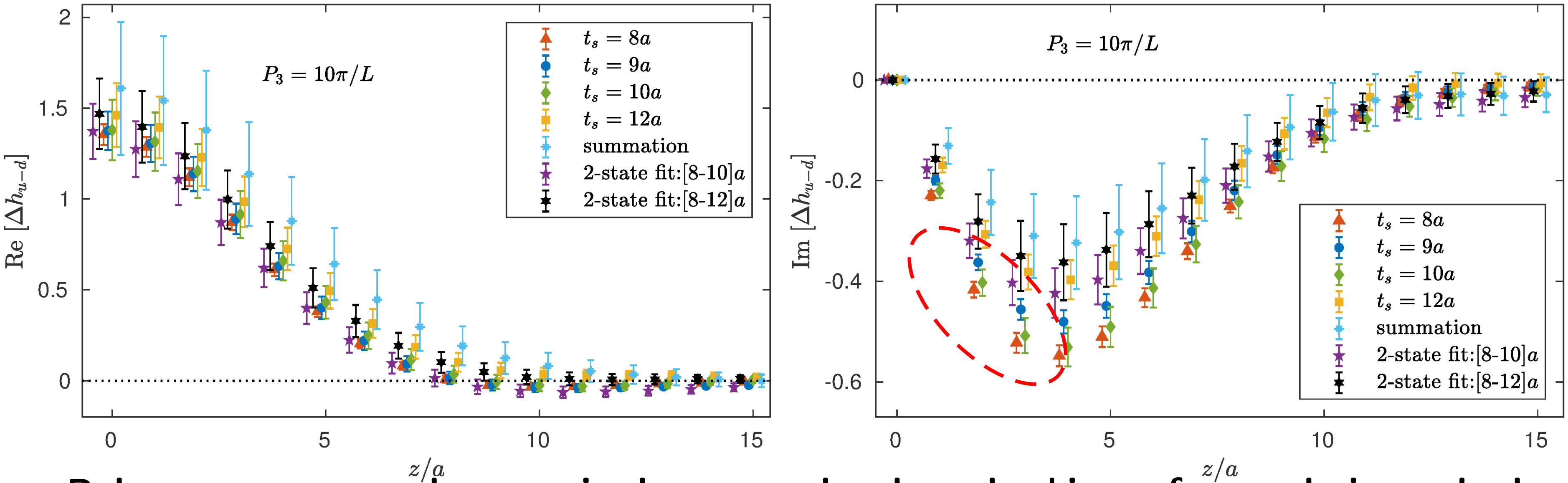
The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$.
2. Compute renormalization functions in an intermediate lattice scheme (here: RI'-MOM): $Z^{\mathrm{RI}'}(z,\mu)$.
3. Perturbatively convert the renormalization functions to the scheme needed for matching (here $\mathrm{M\overline{MS}}$) and evolve to a reference scale: $Z^{\mathrm{RI}'}(z,\mu) \to Z^{\mathrm{M\overline{MS}}}(z,\bar{\mu})$.
4. Apply the renormalization functions to the bare matrix elements, obtaining renormalized matrix elements in the $\mathrm{M\overline{MS}}$ scheme.
5. Calculate the Fourier transform, obtaining quasi-PDFs:
$$\tilde{q}^{\mathrm{M\overline{MS}}}(x,\bar{\mu},P_3) = \int \frac{dz}{4\pi}\, e^{ixP_3z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle^{\mathrm{M\overline{MS}}}.$$
6. Relate $\mathrm{M\overline{MS}}$ quasi-PDFs to $\overline{\mathrm{MS}}$ light-cone PDFs via a matching procedure: $\tilde{q}^{\mathrm{M\overline{MS}}}(x,\bar{\mu},P_3) \to q^{\overline{\mathrm{MS}}}(x,\bar{\mu})$.
7. Apply nucleon mass corr. to eliminate residual m_N^2/P_3^2 effects.

What momentum should be used to obtain reliable light-cone PDFs?

The answer is seemingly simple – **large** momentum, but:

- we have finite lattice spacing → UV cut-off of ≈ 2 GeV.
- large momentum means it is very difficult to isolate the ground state → excessive excited states contamination → one needs to go to large enough source-sink separation t_s ⇒ COSTLY!



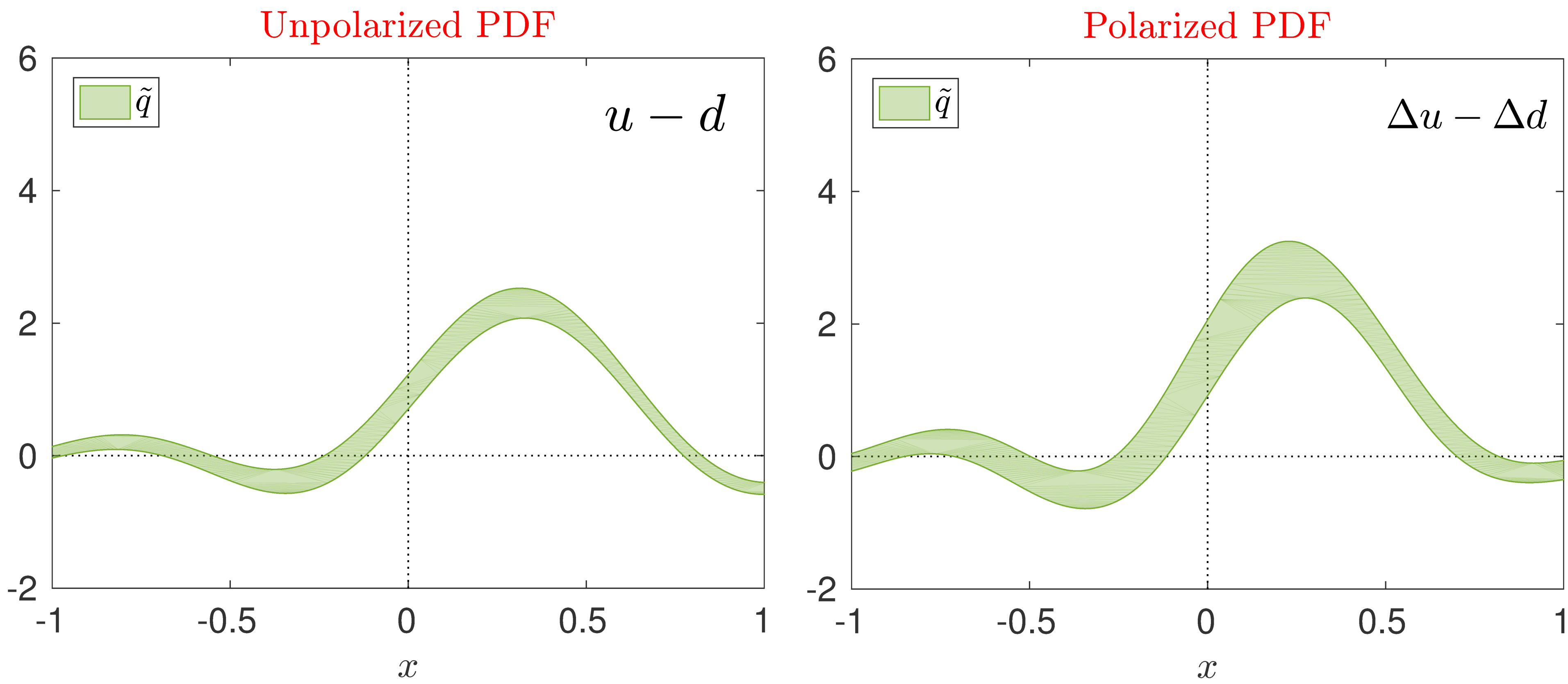
- **Robust statements about excited states only when checking a few analysis methods.**
  here: 2-state fit with $t_s/a = 8, 9, 10, 12$ shows full consistency with the 1-state fit at $t_s = 12a$.

Our largest momentum: ≈ 1.4 GeV

- safely below UV cut-off,
- excited states contamination shown to be smaller than statistical errors.

Fourier transform

Nucleon momentum $\frac{10\pi}{48}$, $Q^2 = 4$ GeV^2



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

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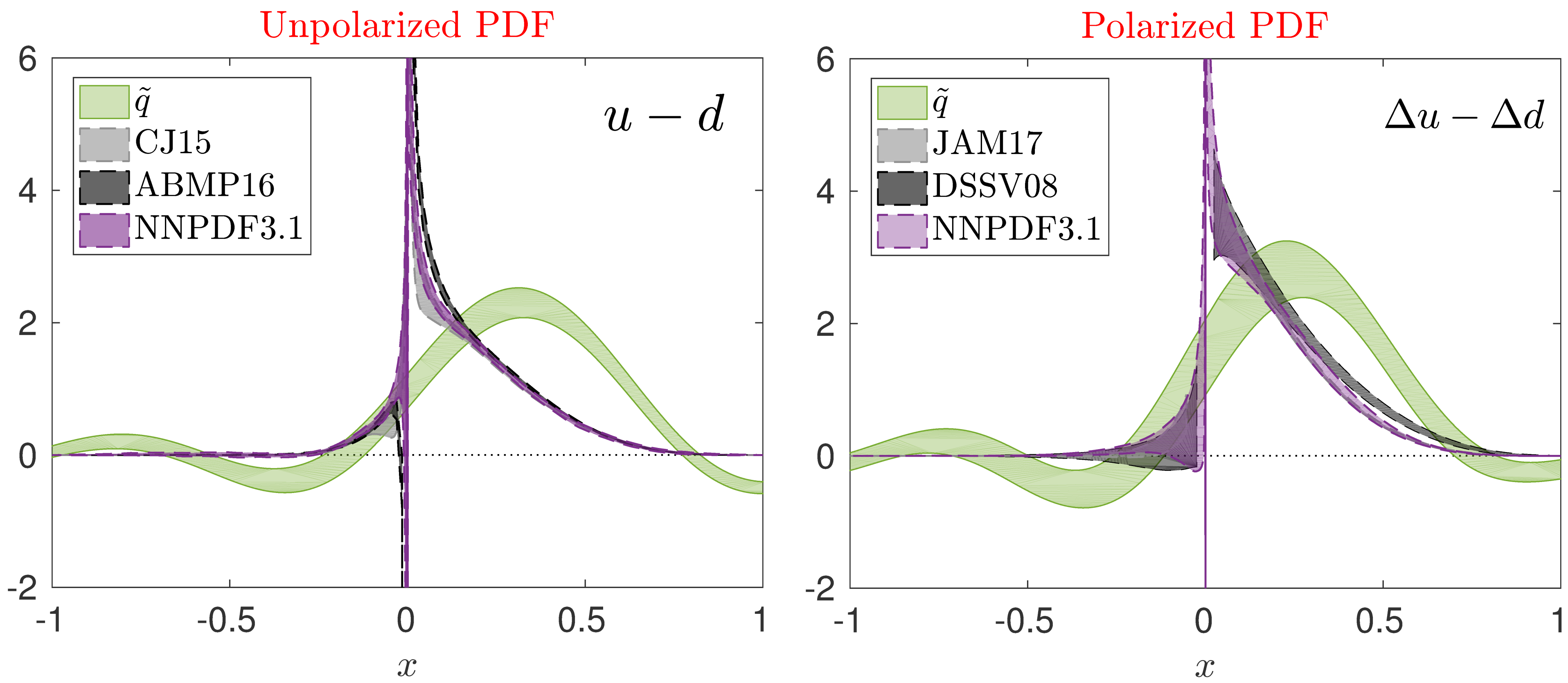


C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Matching to light-front PDFs

The matching formula can be expressed as:

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|} C\left(\xi, \frac{\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}, \mu, P_3\right)$$

C – matching kernel $\mathrm{M}\overline{\mathrm{MS}} \to \overline{\mathrm{MS}}$: [C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001]

$$C\left(\xi, \frac{\xi\mu}{xP_3}\right) = \delta(1-\xi) + \frac{\alpha_s}{2\pi} C_F \begin{cases} \left[\frac{1+\xi^2}{1-\xi} \ln \frac{\xi}{\xi-1} + 1 + \frac{3}{2\xi}\right]_+ & \xi > 1, \\ \left[\frac{1+\xi^2}{1-\xi} \ln \frac{x^2 P_3^2}{\xi^2\mu^2} (4\xi(1-\xi)) - \frac{\xi(1+\xi)}{1-\xi} + 2\iota(1-\xi)\right]_+ & 0 < \xi < 1, \\ \left[-\frac{1+\xi^2}{1-\xi} \ln \frac{\xi}{\xi-1} - 1 + \frac{3}{2(1-\xi)}\right]_+ & \xi < 0, \end{cases}$$

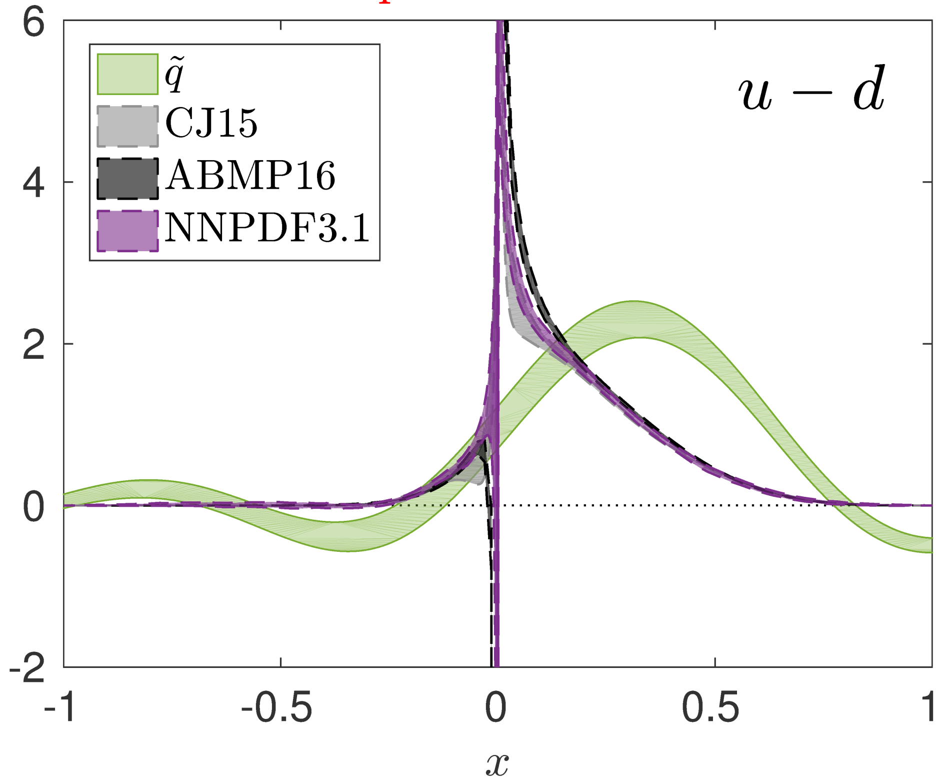
$\iota=0$ for γ_0 and $\iota=1$ for $\gamma_3/\gamma_5\gamma_3$.

- Additional subtractions with respect to $\overline{\mathrm{MS}}$ – made outside the physical region of the unintegrated vertex corrections.
- Thus, needs modified renormalization scheme for input quasi-PDF $\to$ $\mathrm{M}\overline{\mathrm{MS}}$ scheme.
- In this procedure, vector current is **conserved**.

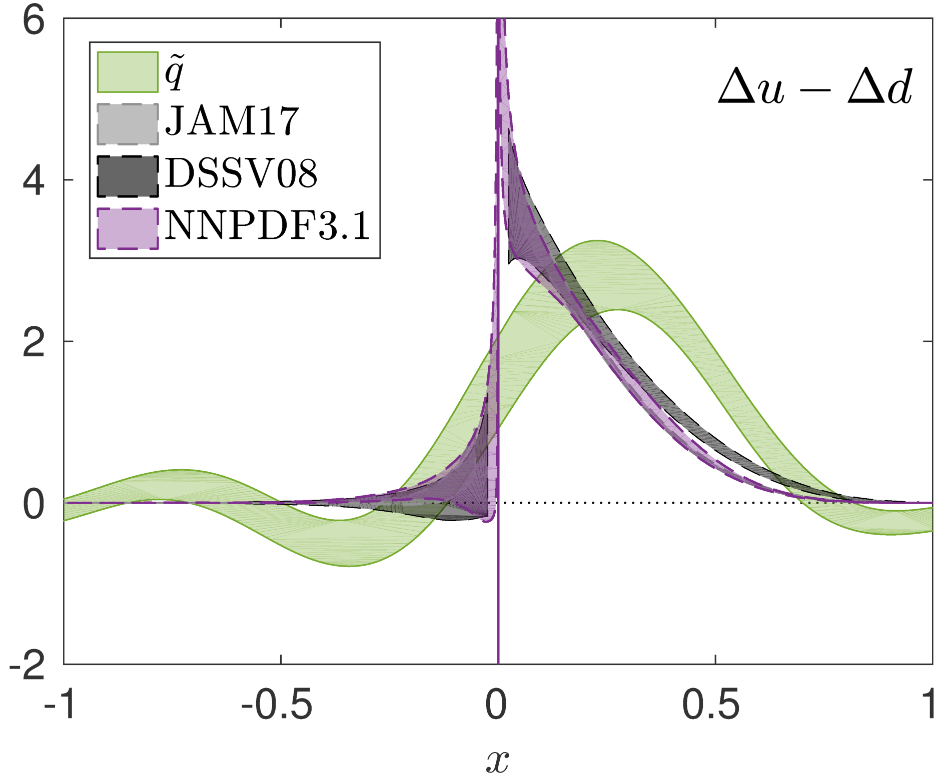
Matched PDFs

Nucleon momentum $\frac{10\pi}{48}$, $Q^2 = 4$ GeV^2

Unpolarized PDF



Polarized PDF



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

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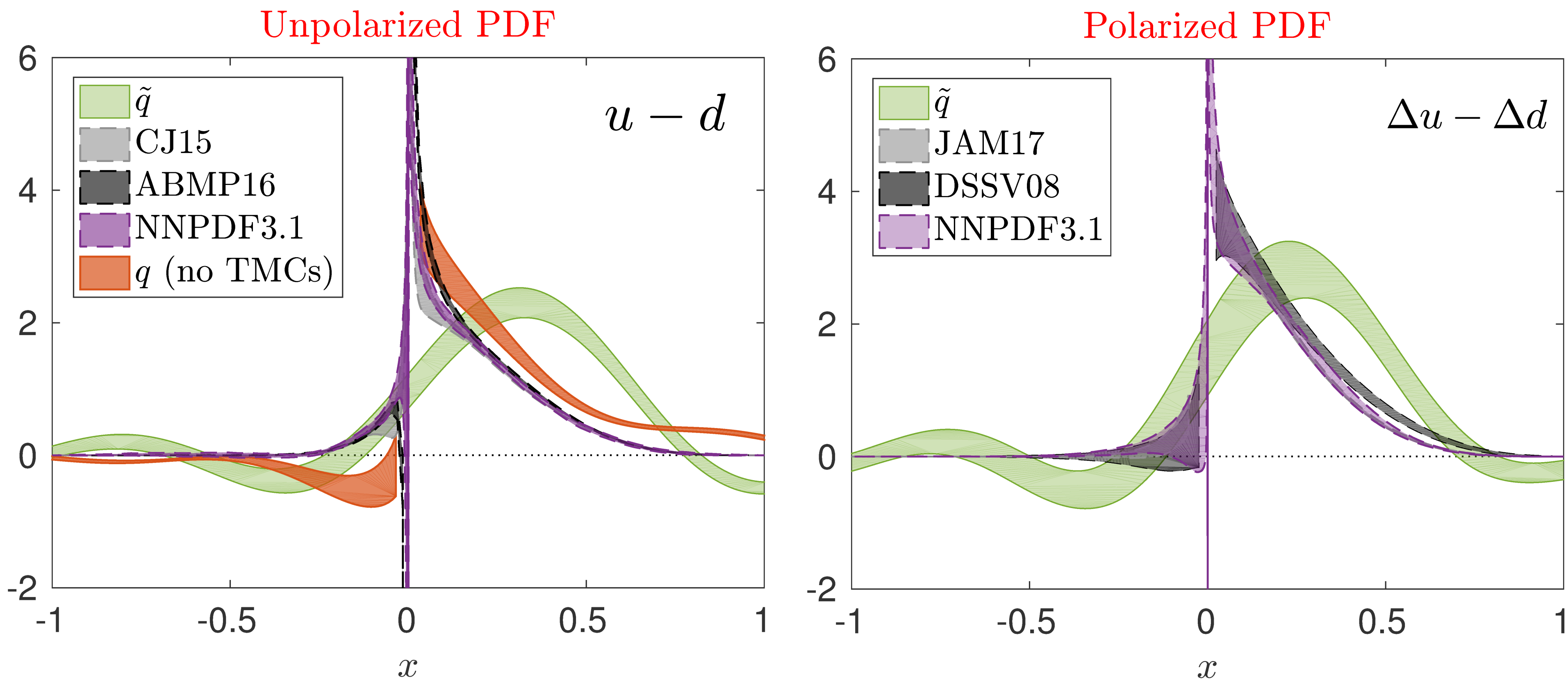

C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

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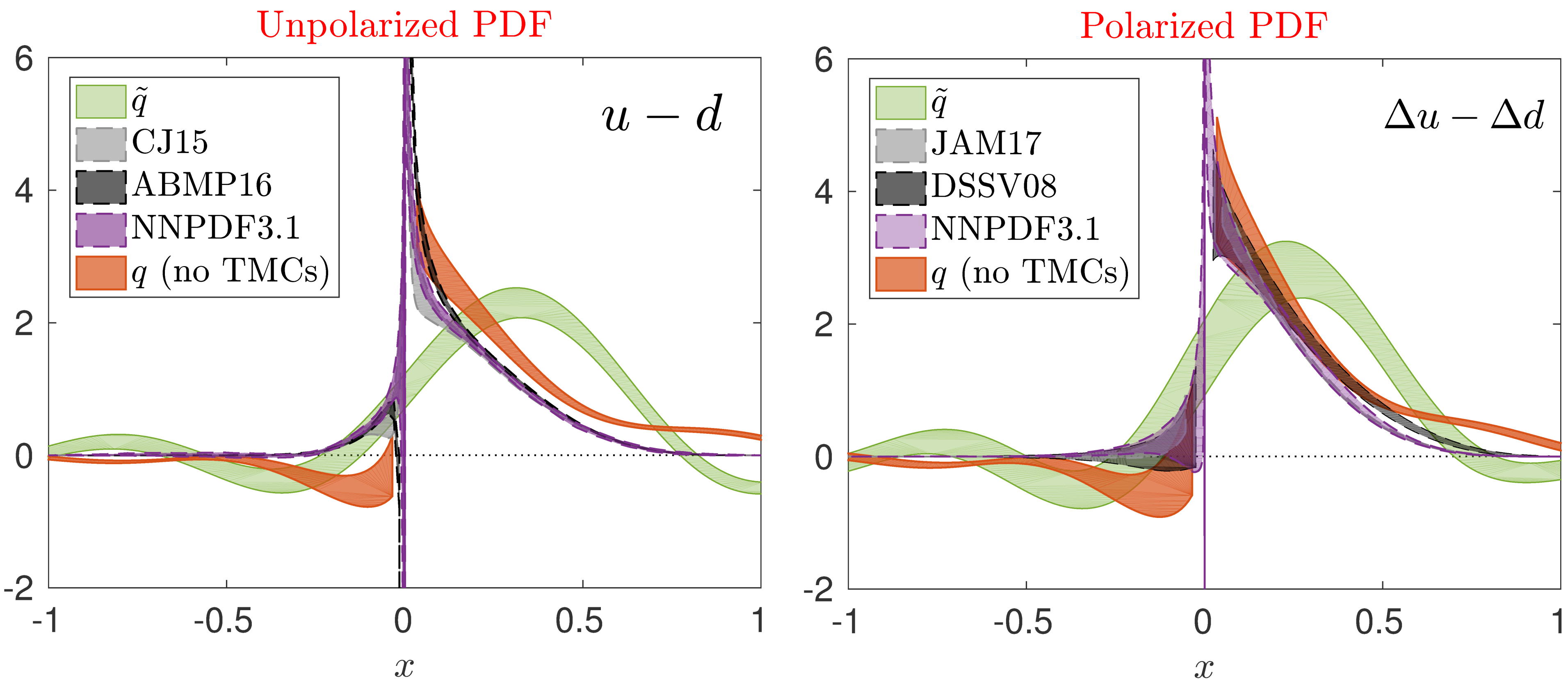
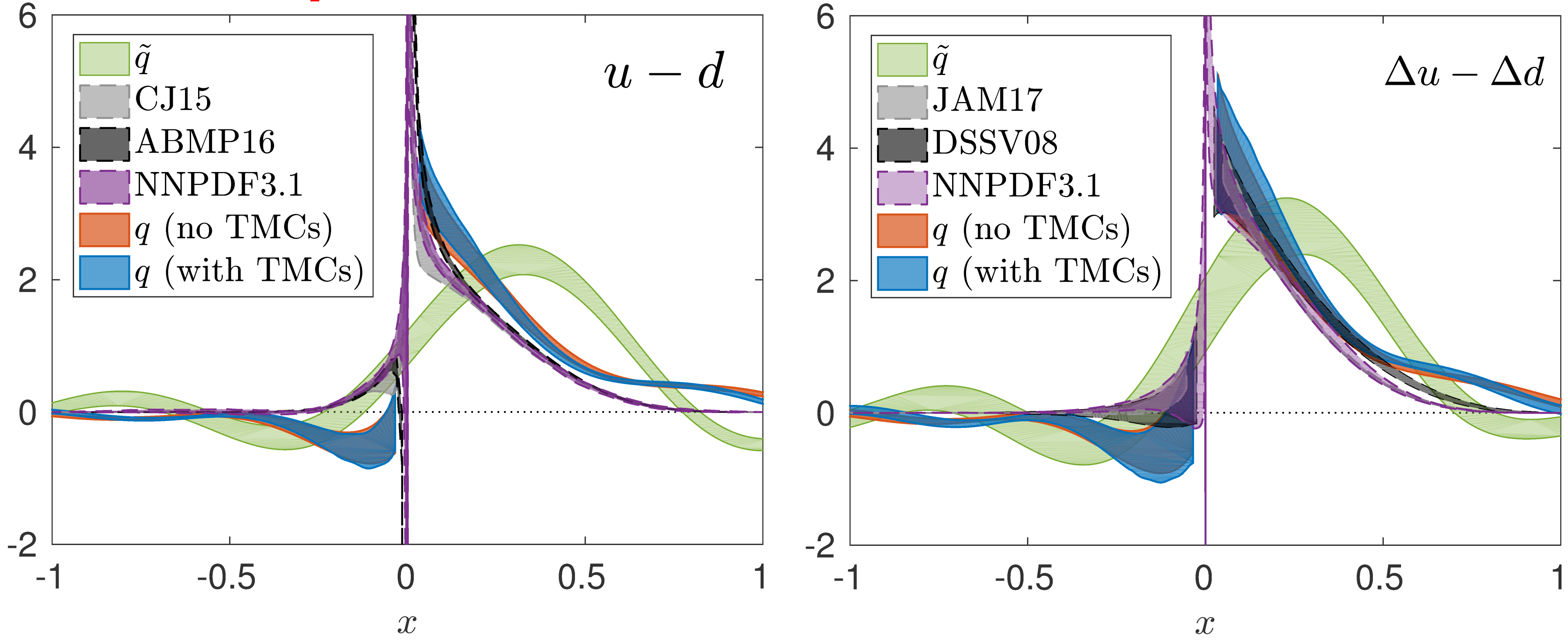


C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Matched PDF + TMCs

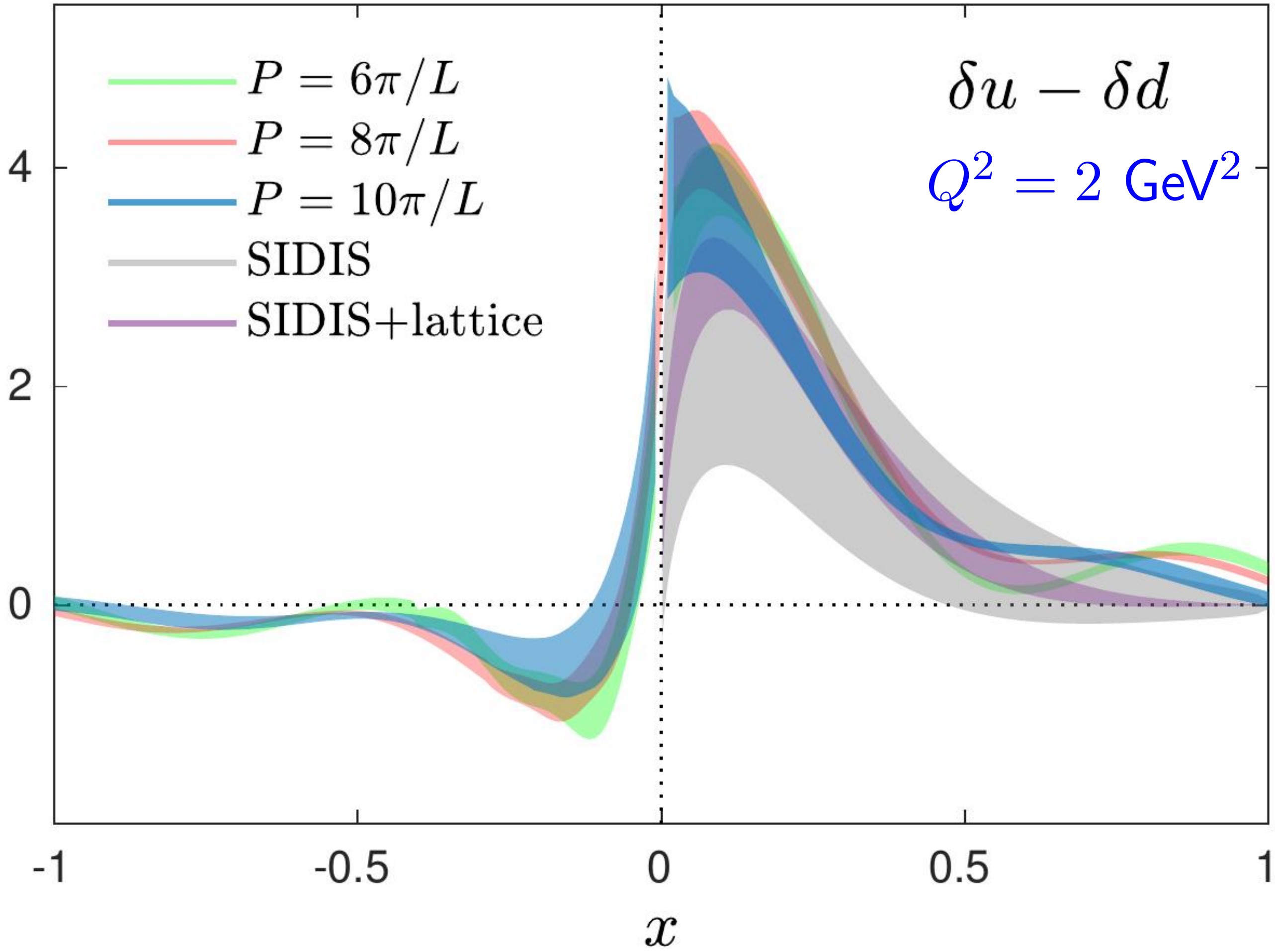
Nucleon momentum $\frac{10\pi}{48}$, $Q^2 = 4$ GeV^2

Unpolarized PDF | Polarized PDF



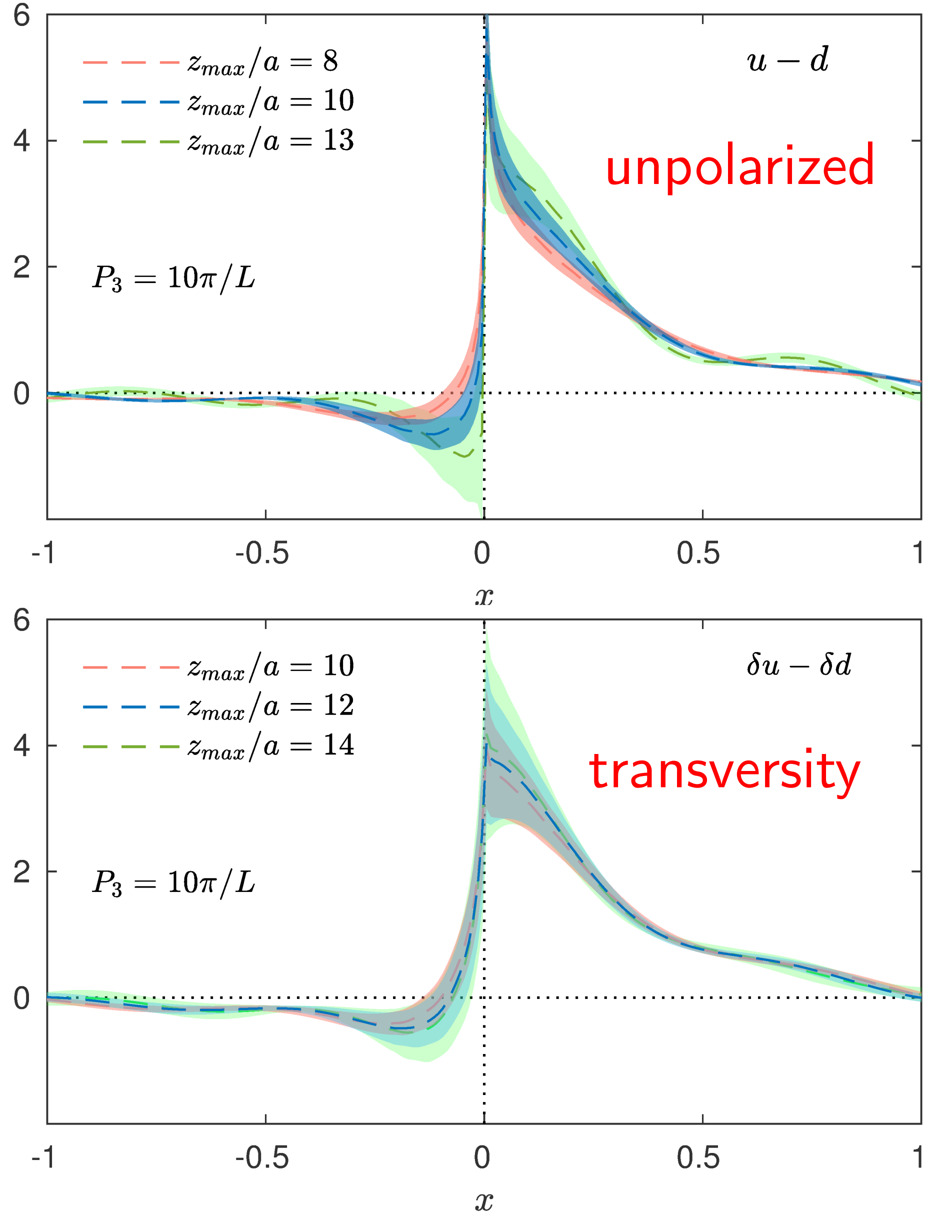
C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

C. Alexandrou et al., Phys. Rev. D98 (2018) 091503 (Rapid Communications)



Statistical precision already much better than the precision of phenomenological fits from SIDIS: JAM Collaboration, Phys. Rev. Lett. 120 (2018) 152502

Truncation of Fourier transform



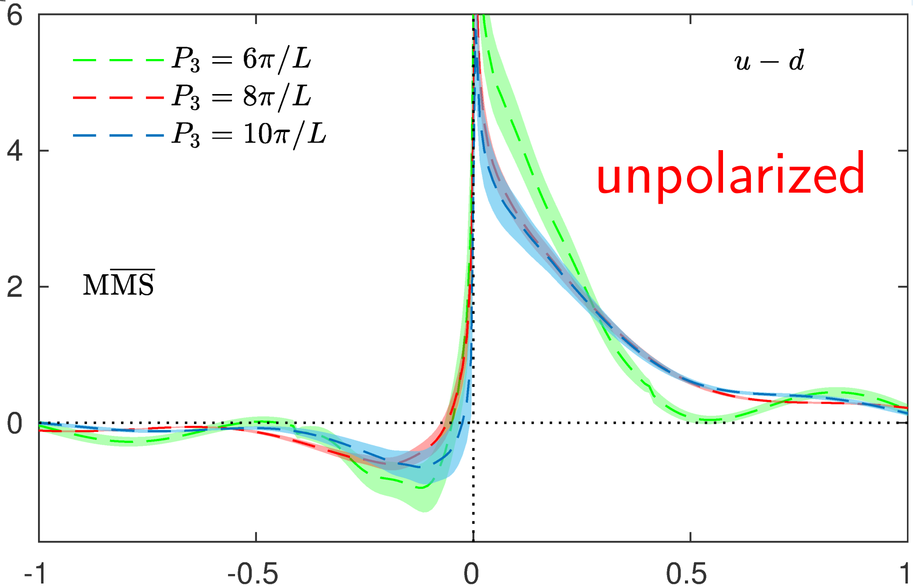
Nucleon momentum $\frac{10\pi}{48}$

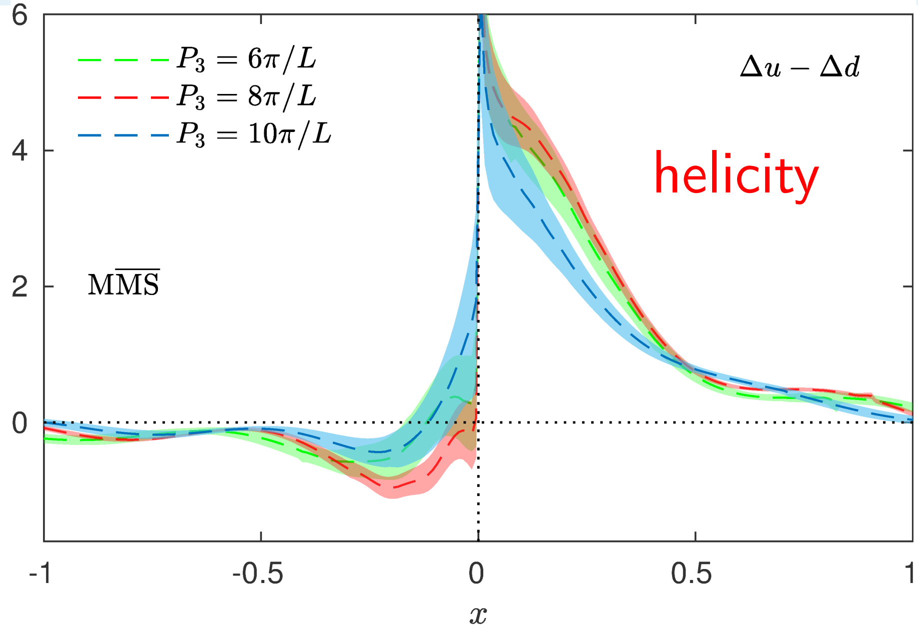
Needs the use of advanced reconstruction techniques

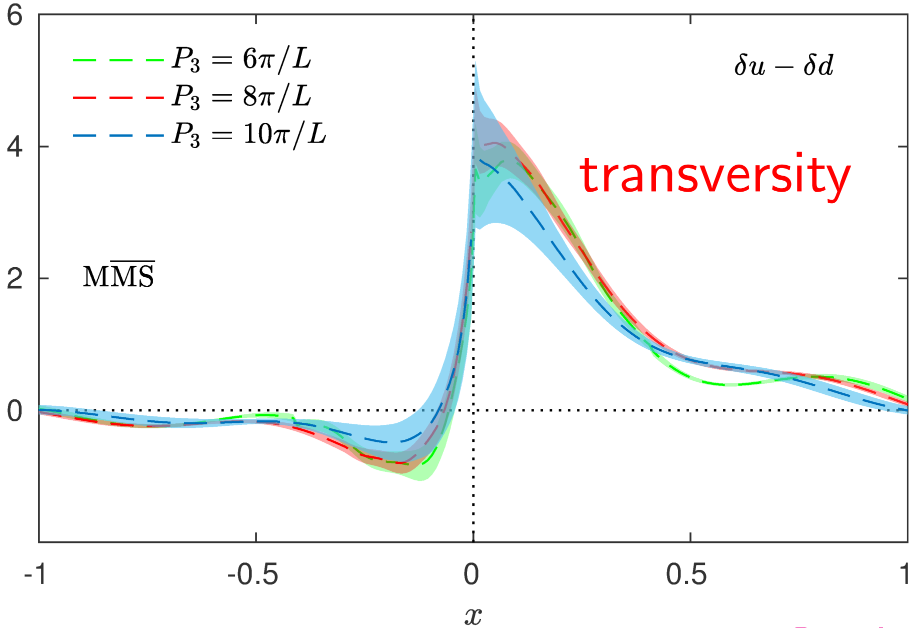
J. Karpie et al., JHEP 1904 (2019) 057

C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

Momentum dependence of final PDFs







Nucleon momenta $\frac{6\pi}{48}$, $\frac{8\pi}{48}$, $\frac{10\pi}{48}$

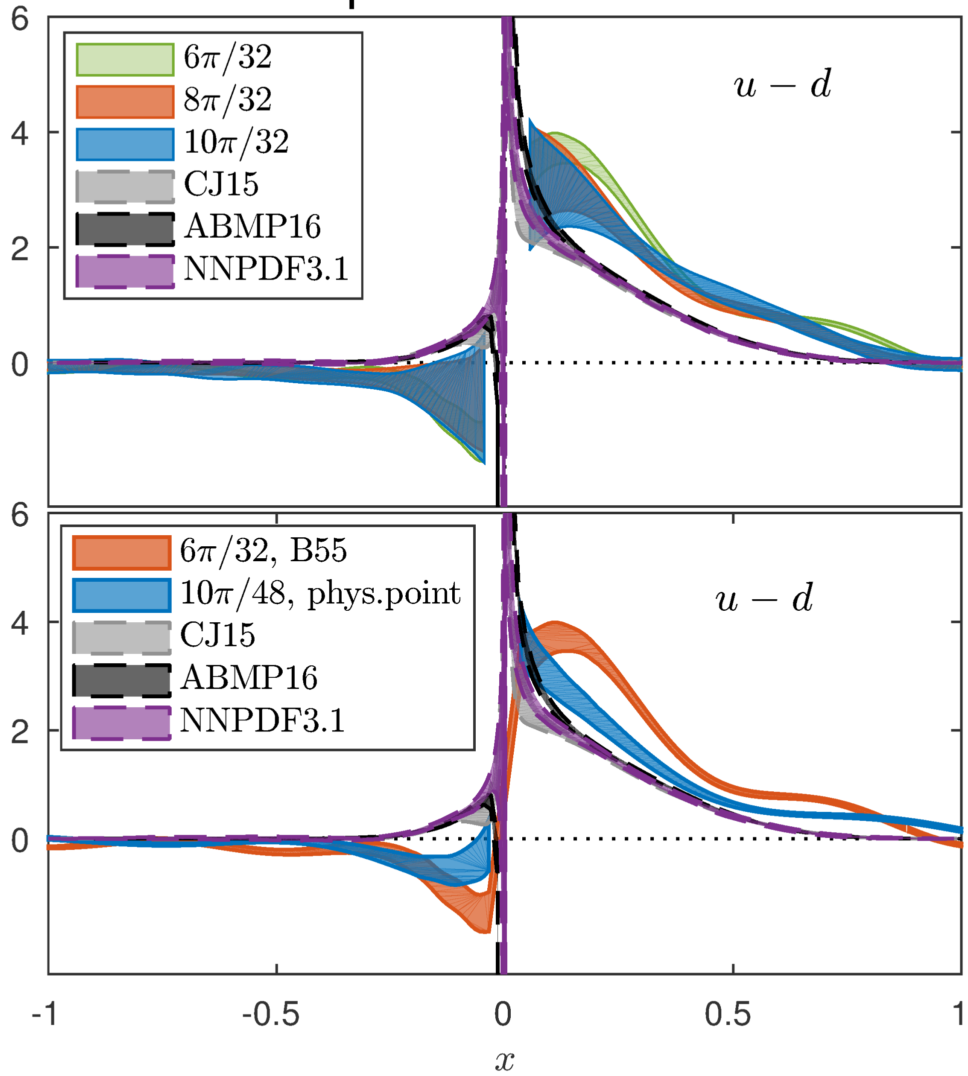
Results seem to indicate convergence in nucleon boost

Expected HTE:
$\mathcal{O}(\Lambda^2_{\mathrm{QCD}}/P_3^2) \approx 5\%$ at $P_3 = 1.4$ GeV

C. Alexandrou et al., Phys. Rev. D99 (2019) 114504

Comparison with non-physical pion mass

Physical vs. non-physical pion mass – 135 vs. 375 MeV
unpolarized PDF



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