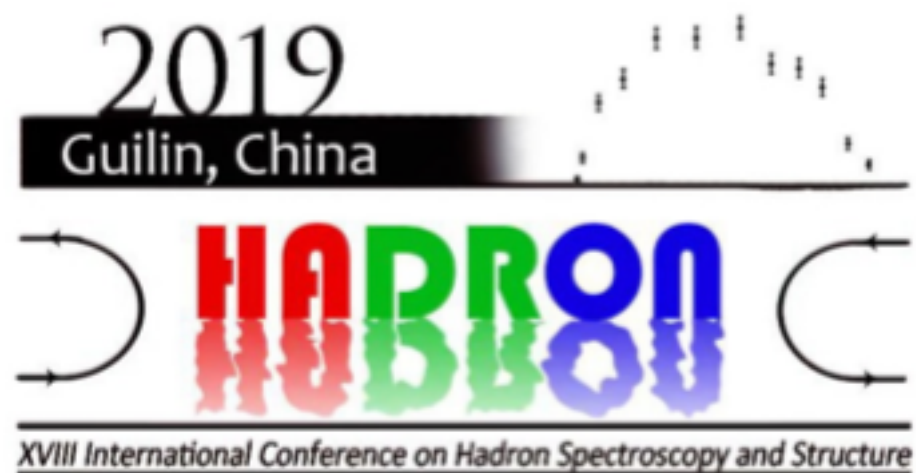


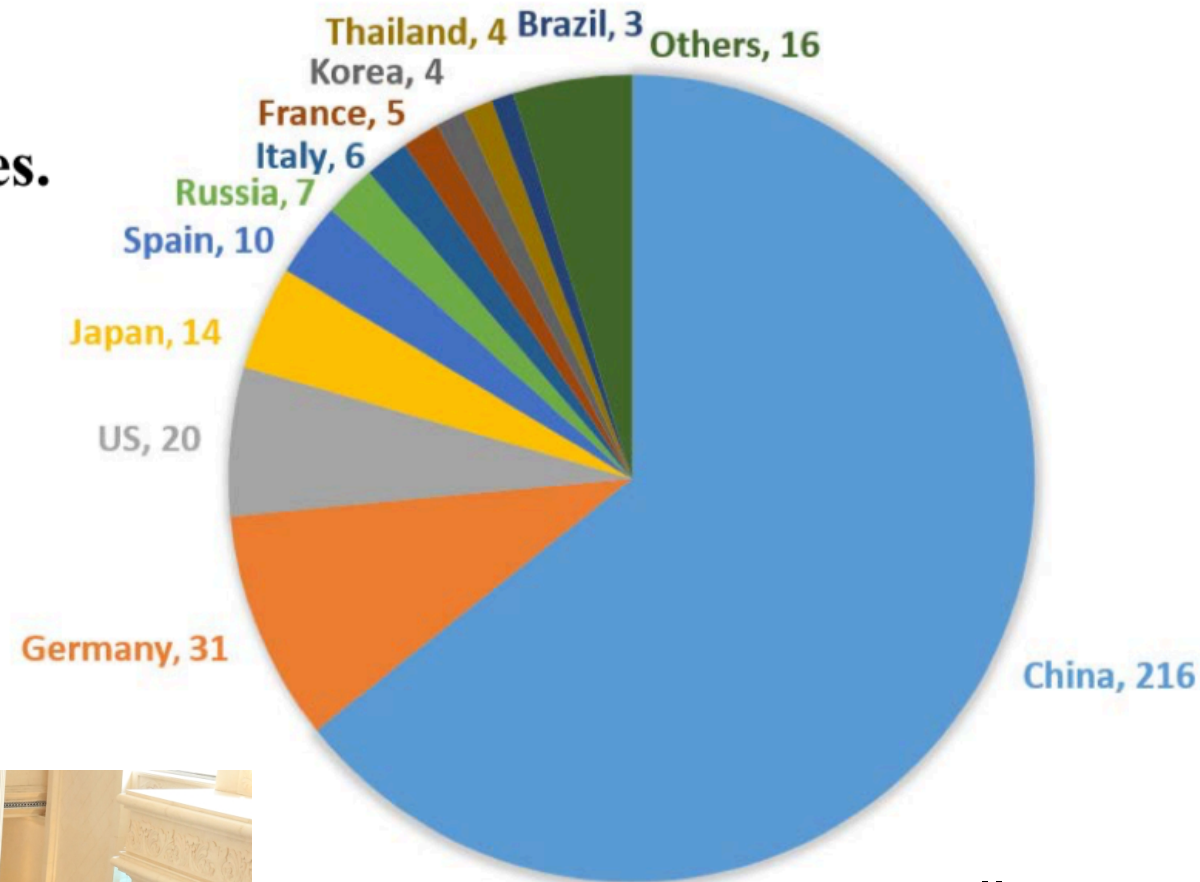


HADRON 2019



Distribution of participants

**336 participants,
around 150 institutions,
from more than 20 countries.**



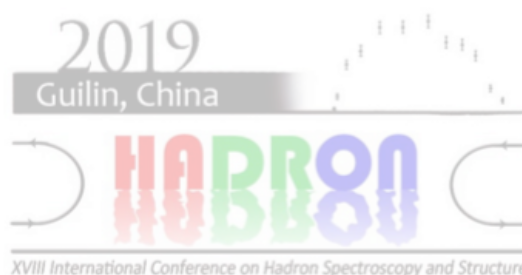
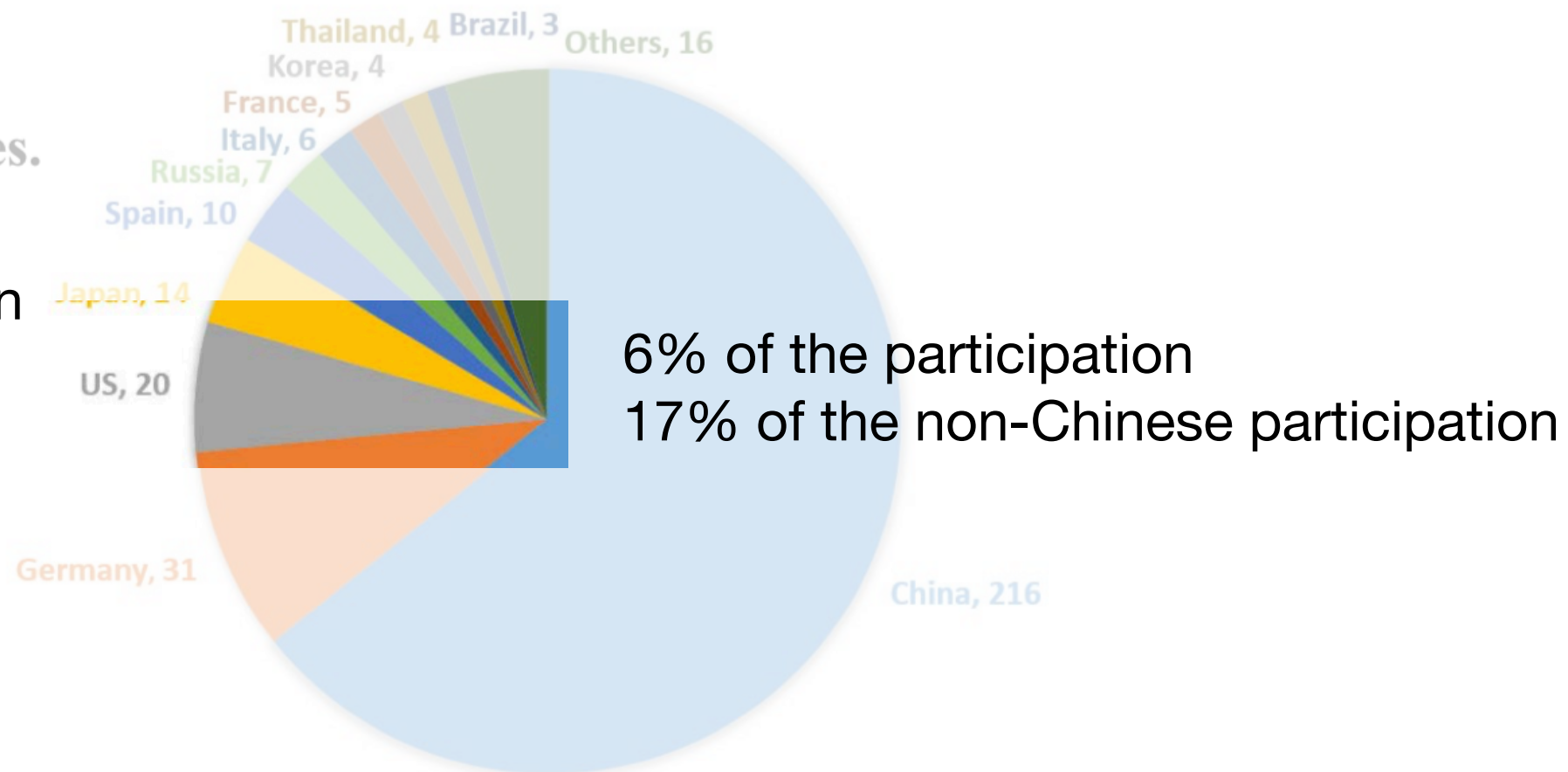
excellent opportunity to interact
with Chinese hadron community



Distribution of participants

336 participants,
around 150 institutions,
from more than 20 countries.

is the US giving up on
leadership in hadron
physics ?



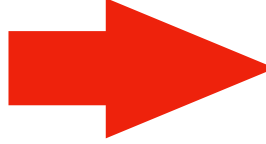
very low US national lab participation
— does this reflect how hard it is to get travel approval ?





not very many new experimental or theoretical results reported at this conference

in my opinion plenary talks overly biased to theory — blame the IAC

oops 

S. Bianco (INFN-Frascati)	N. Brambilla (Muenchen)
V. Burkert (JLab)	W.-C. Chang (Academia Sinica)
S.-U. Chung (Brookhaven)	V. Crede (Florida State)
J. Dudek (William&Mary/JLab)	S. Eidelman (Novosibirsk)
D. R. Entem (Salamanca)	P. Eugenio (Florida State)
A. Feliciello (INFN-Torino)	Y.-N. Gao (PKU)
T. Gershon (Warwick)	A. Hosaka (Osaka)
U.-G. Meißner (Bonn/Jülich)	C. A. Meyer (Carnegie Mellon)
S. Nagamiya (RIKEN)	M. Nakao (KEK)
E. Oset (Valencia)	S. Paul (Muenchen)
K. Peters (GSI)	A. Reis (CBPF-Rio de Janeiro)
C. D. Roberts (Argonne)	X.-Y. Shen (CAS-IHEP)
A. Szczepaniak (Indiana/JLab)	U. Thoma (Bonn)
A. Thomas (Adelaide)	U. Wiedner (Bochum)
H. Wittig (Mainz)	B.-S. Zou (CAS-ITP)

were parallel talks on hadron structure, but I didn't go to any of them ...

plenary talks on heavy hadron spectroscopy

Spectroscopy of Singly-Heavy* Hadrons

see

John Yelton

University of Florida

for a nice summary of (mostly)
heavy baryon experimental results

his conclusion: observations are very ‘quark-model-like’

see

Experimental Status of the XYZ Structures

Yuping Guo

(KPH, Mainz University)

and

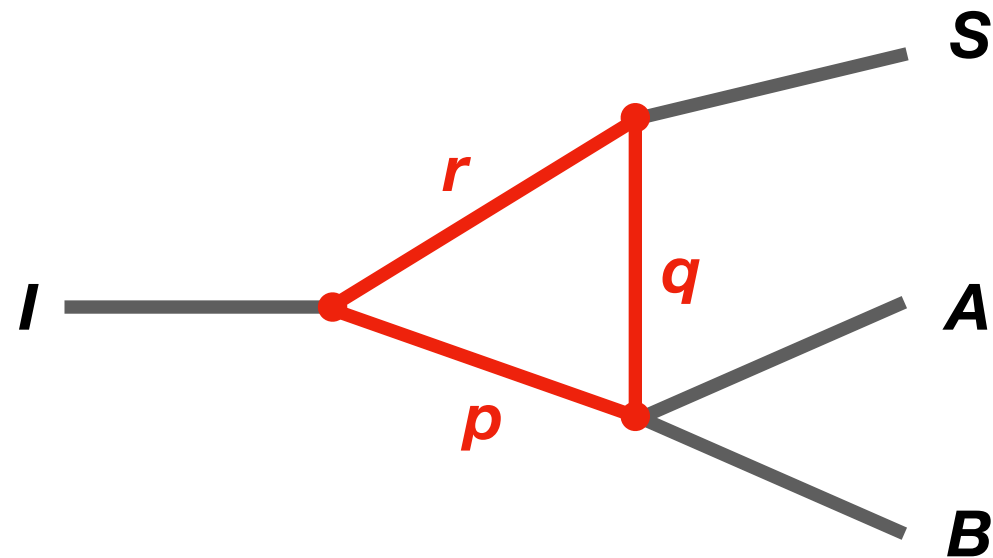
Theoretical aspects about XYZ and Pc
exotic candidates

Qiang Zhao

Institute of High Energy Physics, CAS

in short this is not getting any simpler ...

the shape of the conference — **triangles**



peaks in **$m(AB)$** if very particular kinematics hold

many of the XYZ states are seen as **AB** bumps in three-body decays

one school of thought — some/all of these are just triangles

data not precise enough to resolve line-shape

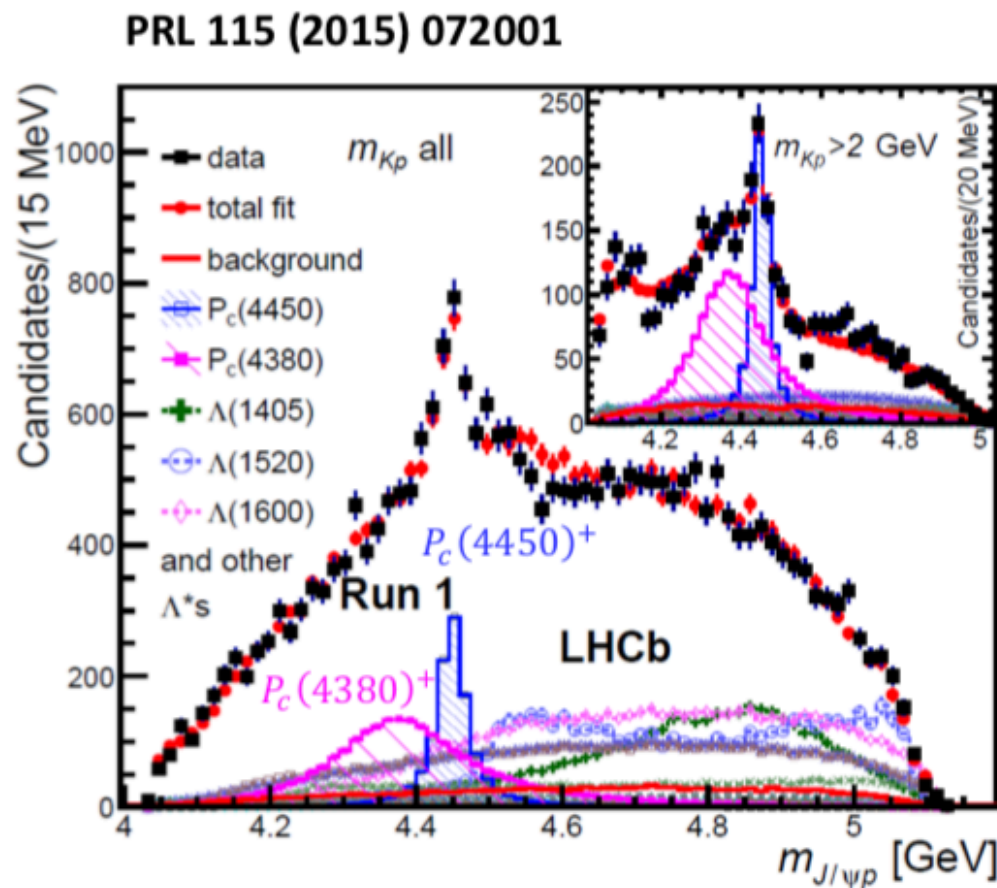
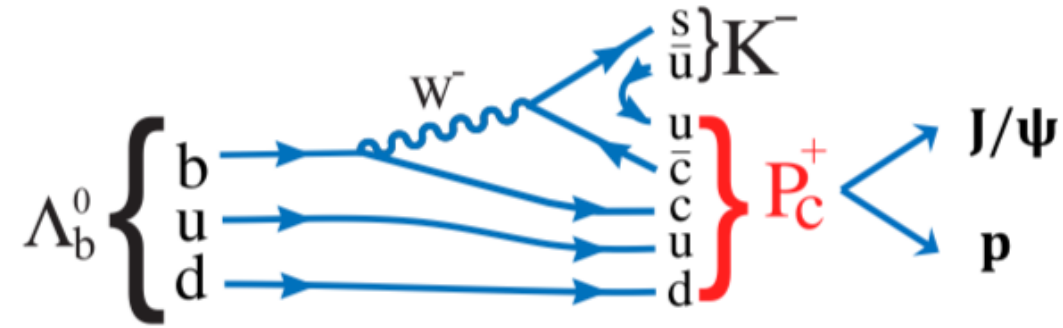
particular to this production process — observe in others ...

situation largely unclear to me, will be improved by

- expt data in other production processes
- lattice QCD two-body scattering calcs

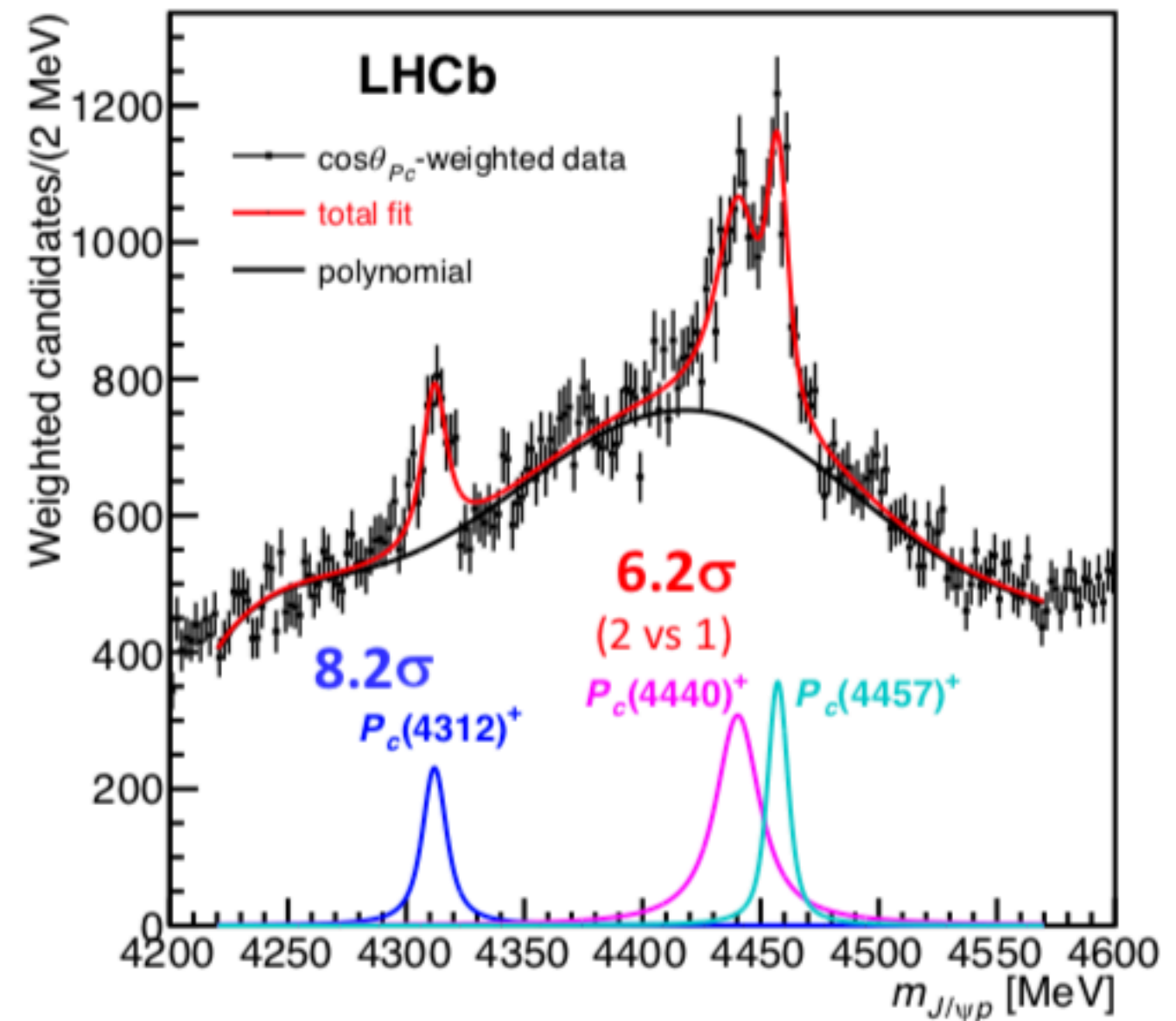
Experimental review of baryons with two heavy quarks (including P_c)

Liming Zhang (Tsinghua University)



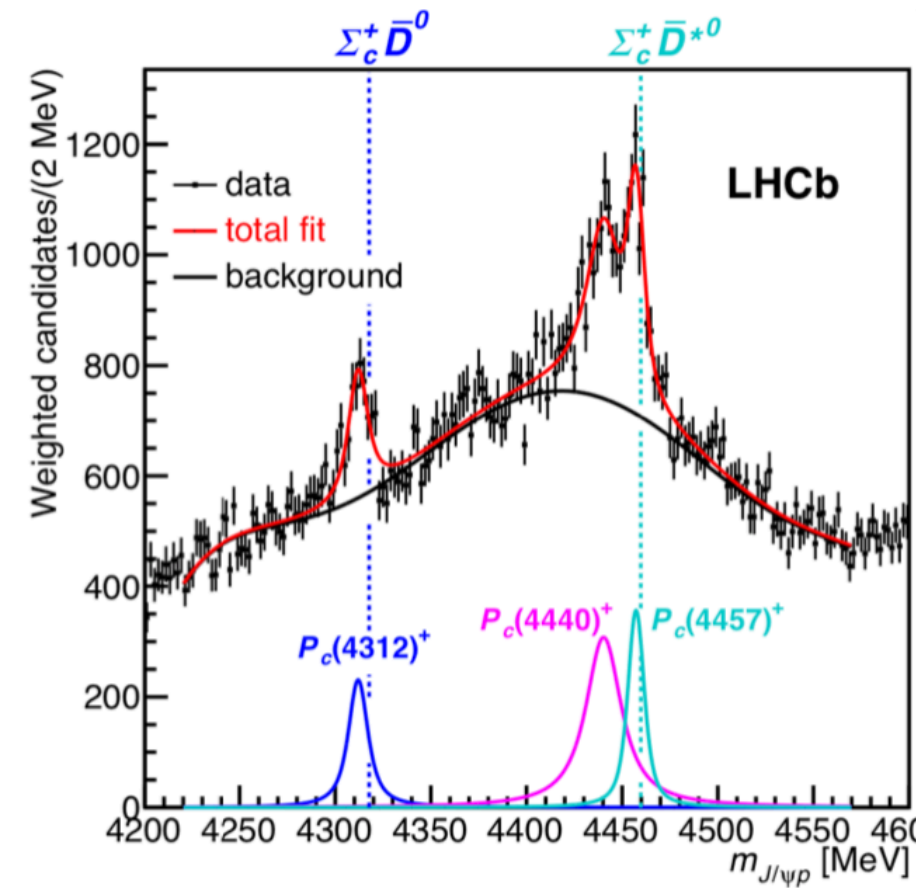
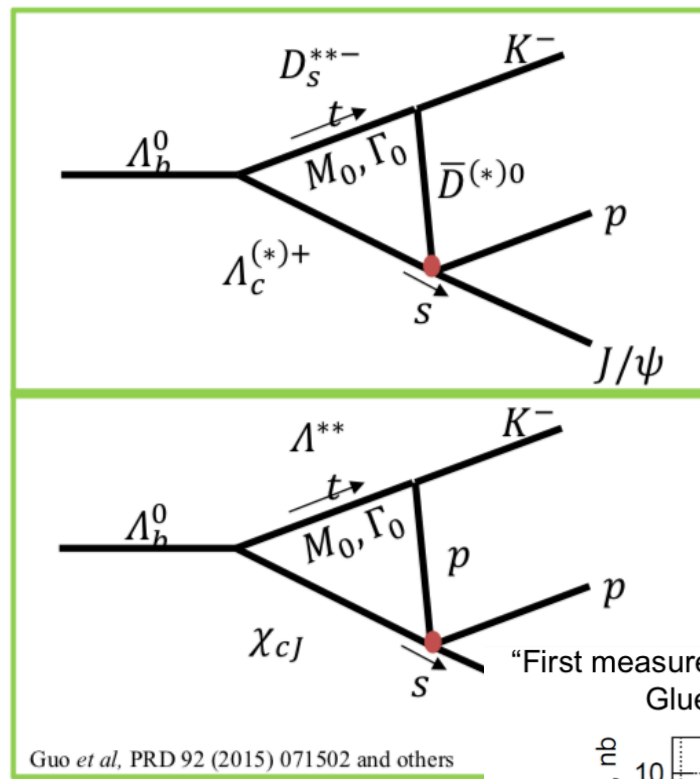
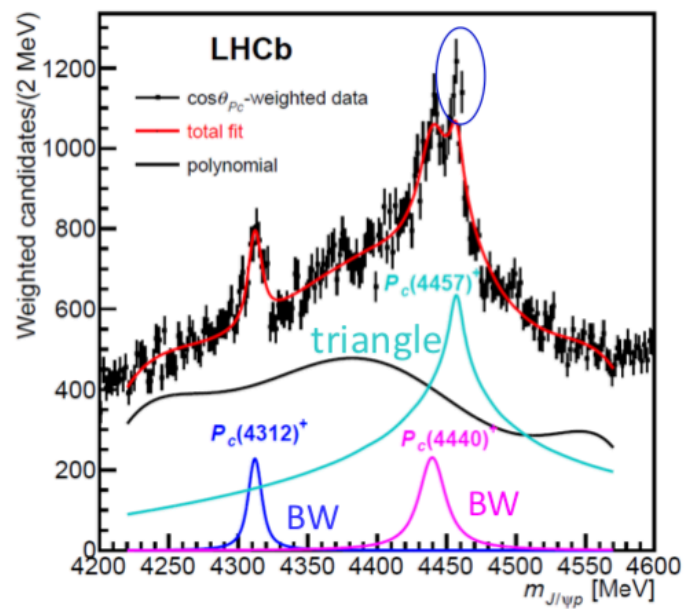
LHCb 2015 — Dalitz plot fit

LHCb 2019 — 10× stats — 1dim projection fit

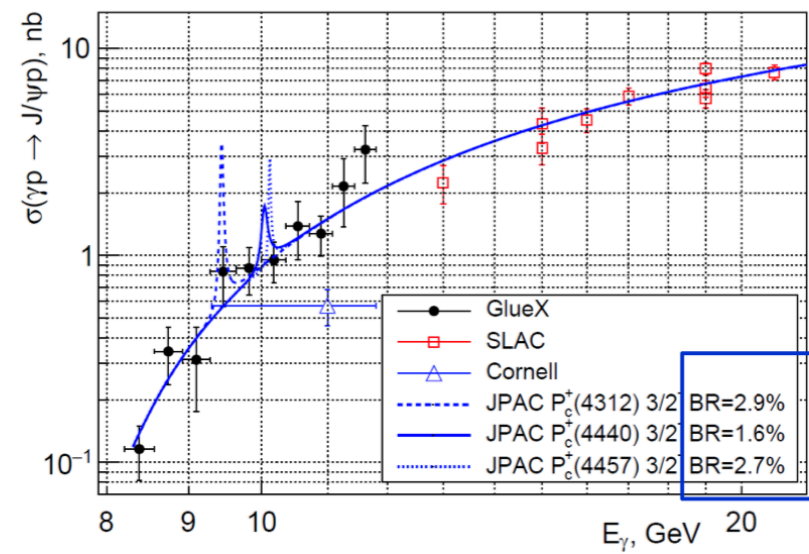


State	M [MeV]	Γ [MeV]	(95% CL)	$\mathcal{R}$ [%]
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$	(< 27)	$0.30 \pm 0.07^{+0.34}_{-0.09}$
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.1}$	(< 49)	$1.11 \pm 0.33^{+0.22}_{-0.10}$
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$	(< 20)	$0.53 \pm 0.16^{+0.15}_{-0.13}$

$P_c(4457)^+$ is right at the
 $\Lambda_c(2595)^+ \bar{D}^0$ threshold
 2BW + 1 triangle-diagram amplitudes + polynomial
 $\Gamma_0 = 159$ MeV [PDG]



“First measurement of near-threshold J/ψ exclusive photoproduction off the proton”
 GlueX Collaboration, **May 26, 2019**, arXiv:1905.10811 [nucl-ex]



GLUEX

“GlueX Physics” on 18/8
 (Sun.) at S1 by M. SHEPHERD

Model-dependent upper limits
 at 90% C.L. from JPAC model
 [PRD 94 (2016) 034002]

BES III has an active program to study light mesons ...

Light meson spectroscopy at e^+e^- machines

Beijiang Liu

Institute of High Energy Physics, CAS

... an example of what they're interested in

Tensor glueball candidate?

$$\Gamma(J/\psi \rightarrow \gamma G_{2+}) = 1.01(22) \text{ keV}$$
$$\Gamma(J/\psi \rightarrow \gamma G_{2+})/\Gamma_{tot} = 1.1 \times 10^{-2}$$

CLQCD, Phys. Rev. Lett. 111, 091601 (2013)

Experimental results

$$\text{Br}(J/\psi \rightarrow \gamma f_2(2340) \rightarrow \gamma \eta \eta) = (3.8^{+0.62+2.37}_{-0.65-2.07}) \times 10^{-5}$$

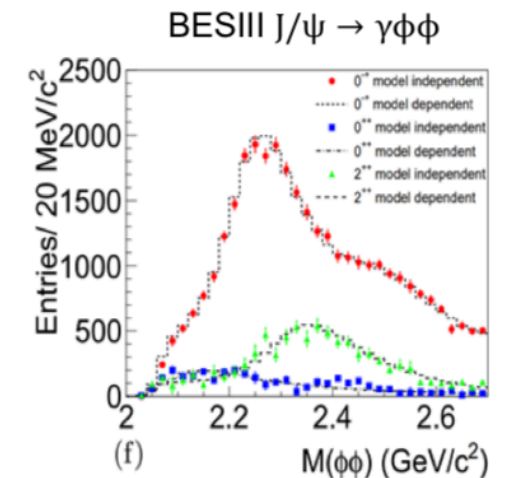
Phys.Rev. D87, 092009 (2013)

$$\text{Br}(J/\psi \rightarrow f_2(2340) \rightarrow \gamma \phi \phi) = (1.91 \pm 0.14^{+0.72}_{-0.73}) \times 10^{-4}$$

Phys.Rev. D93, 112011 (2016)

$$\text{Br}(J/\psi \rightarrow \gamma f_2(2340) \rightarrow \gamma K_S K_S) = (5.54^{+0.34+3.82}_{-0.40-1.49}) \times 10^{-5}$$

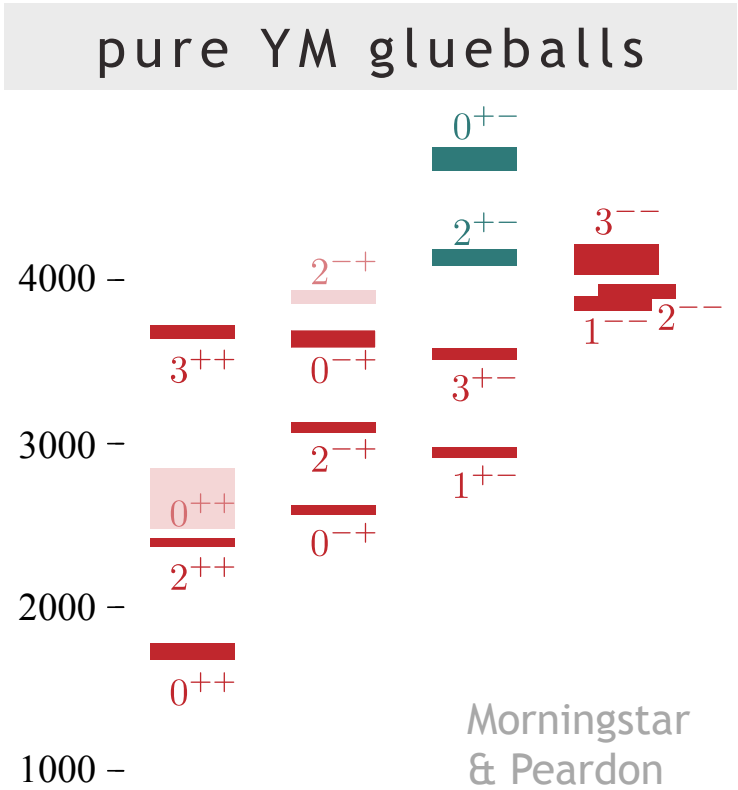
Phys.Rev. D98, 072003 (2018)



$f_2(2010)$, $f_2(2300)$ and $f_2(2340)$
stated in π^-p reactions are observed
with a strong production of $f_2(2340)$

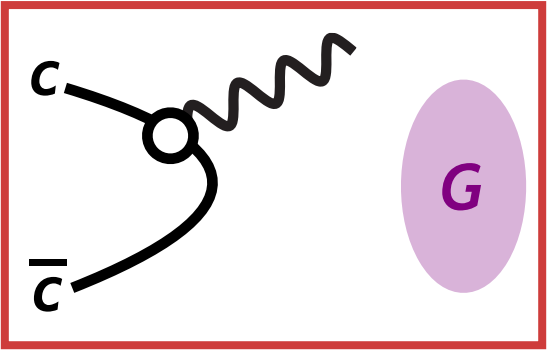
It is desirable to search for more decay modes

long been proposed to study glueballs in J/ψ radiative decay (“glue-rich”)

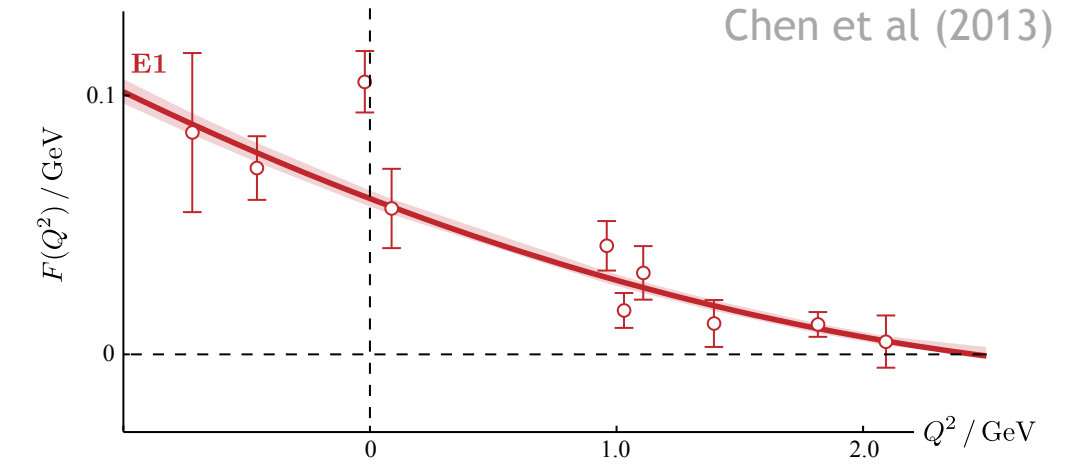


compute three-point correlation functions

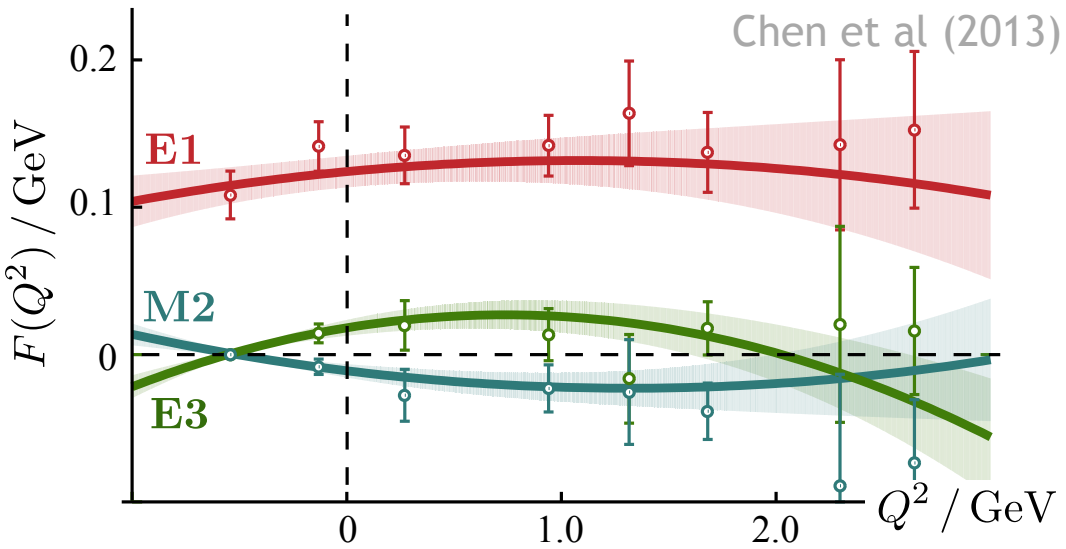
$$\langle 0 | \mathcal{O}_{J/\psi}(T) j_{\text{em}}^\mu(t) \mathcal{O}_G(0) | 0 \rangle$$



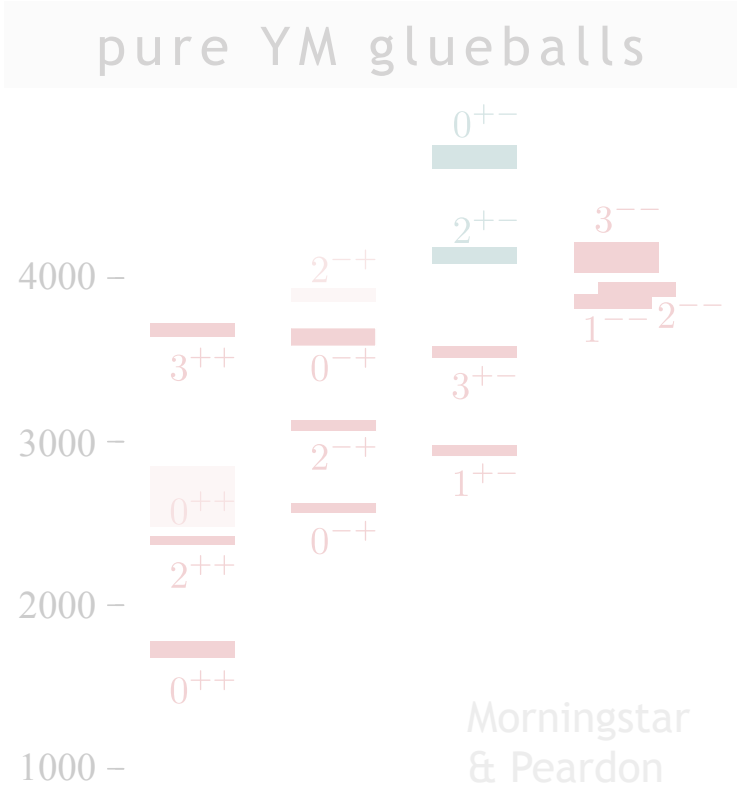
$J/\psi \rightarrow \gamma^* G[0^{++}]$



$J/\psi \rightarrow \gamma^* G[2^{++}]$



long been proposed to study glueballs in J/ψ radiative decay (“glue-rich”)

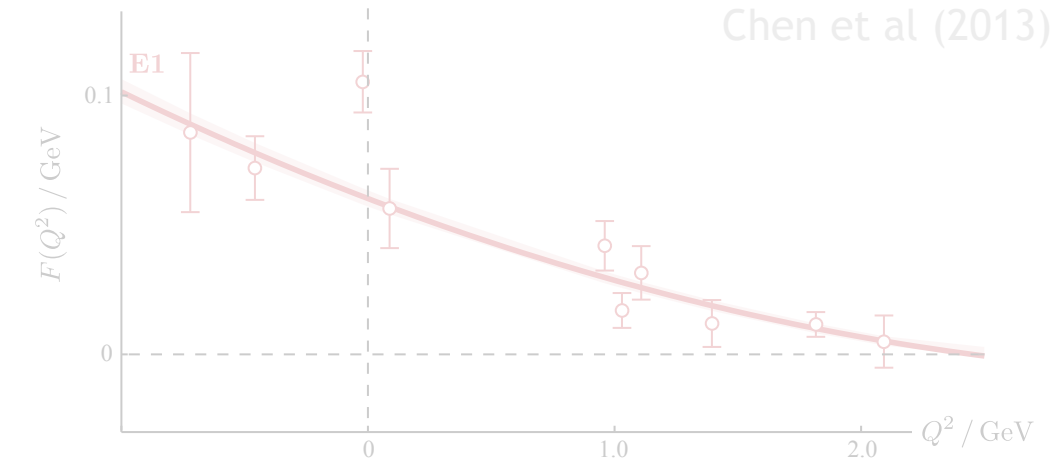


compute three-point correlation functions

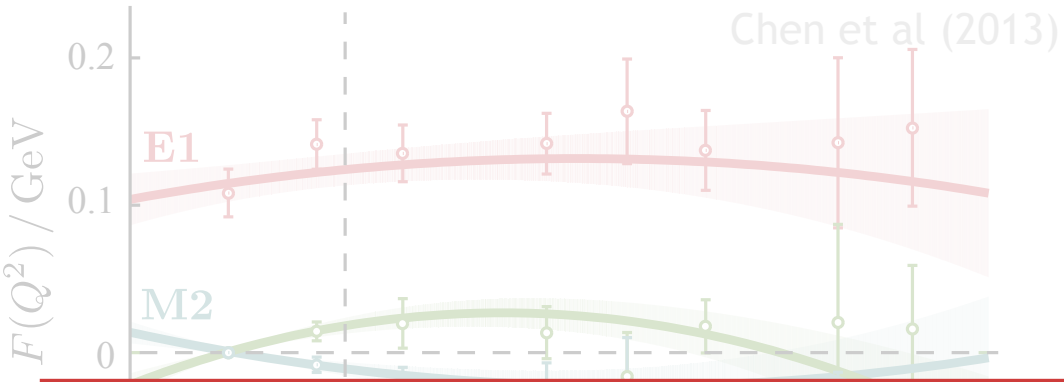
$$\langle 0 | \mathcal{O}_{J/\psi}(T) j_{\text{em}}^\mu(t) \mathcal{O}_G(0) | 0 \rangle$$



$$J/\psi \rightarrow \gamma^* G[0^{++}]$$



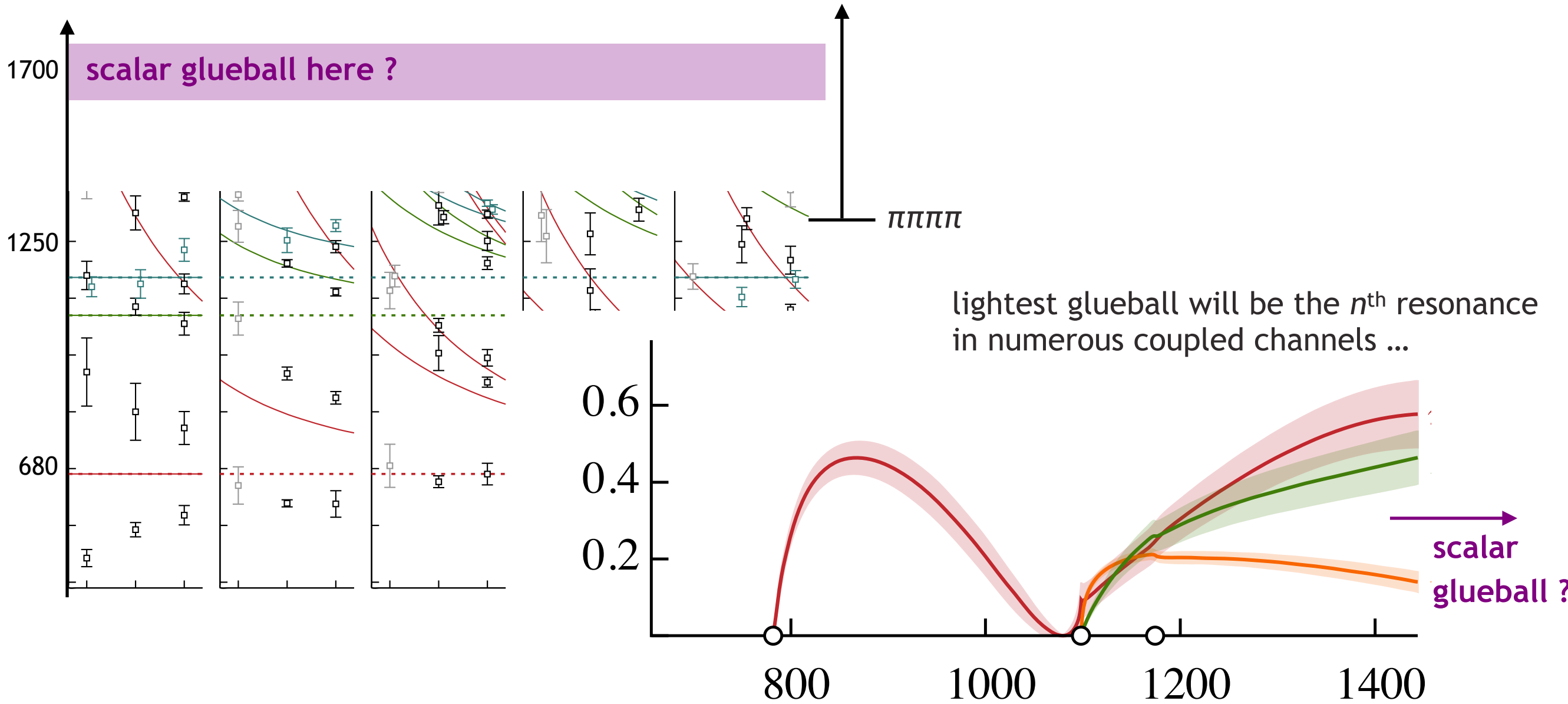
$$J/\psi \rightarrow \gamma^* G[2^{++}]$$



helpful to build intuition, but ...
this is in pure Yang-Mills – no light quarks !

as seen in Beijing’s plenary talk

glueballs in QCD are much more complicated beasts ...



lightest glueball will be the n^{th} resonance in numerous coupled channels ...

and then we need to couple it to an external current ...

this is too tough to start with ...
... back up and try something simpler ...

importance of developing QCD results for resonance decays

— decays being used to infer exotic structure

$\phi(2170)$

- Observed by Babar in $e^+e^- \rightarrow \gamma_{\text{ISR}}\phi f_0(980)$, confirmed by BESII and Belle
- Many interpretations
 - $s\bar{s}g$ hybrid
 - 3^3S_1 or 2^3D_1 strangeonium
 - Tetraquark
 - Molecular of $\Lambda\bar{\Lambda}$
 - $\phi f_0(980)$ resonance with FSI
 - Three body system of ϕKK

also parallel talk
Study of $\phi(2170)$ at BESIII

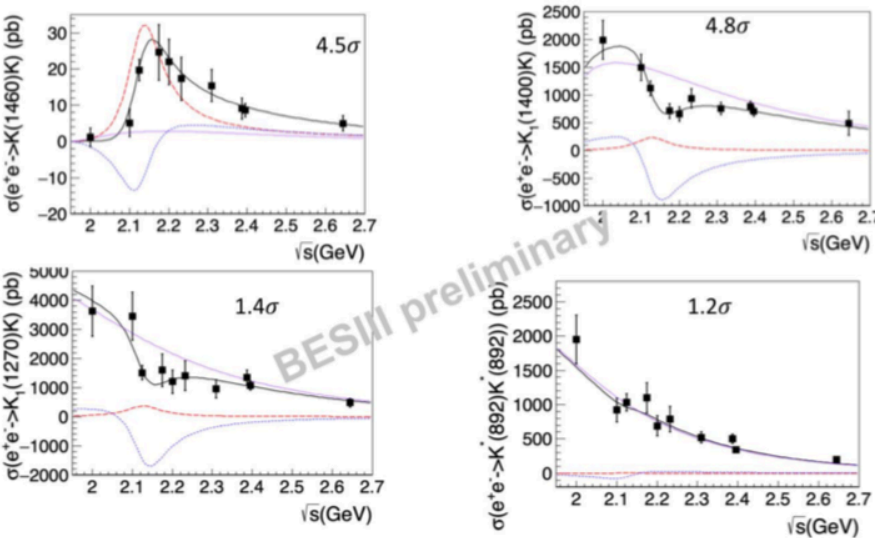
Wenbiao Yan (USTC)

On behalf of BESIII Collaboration

$e^+e^- \rightarrow K^+K^-\pi^0\pi^0$ at BESIII

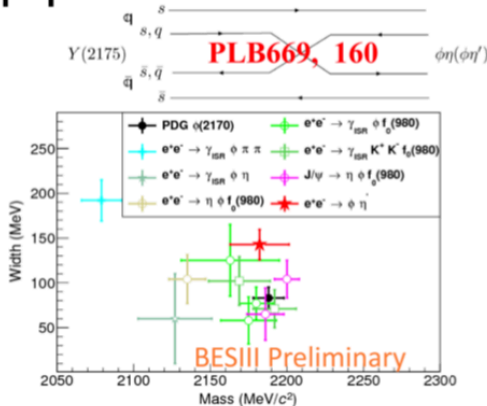
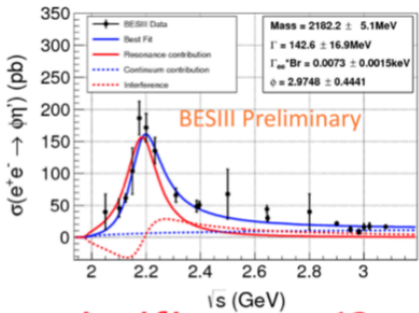
- $\phi(2170) \rightarrow KK\pi\pi$ has good discriminate power to several models
 - $K^*(892)K^*(892)$: $s\bar{s}g$ (forbidden), 3^3S_1 (favored)
 - $KK_1(1400)$: $s\bar{s}g$ (favored)
 - $KK(1460)$: $s\bar{s}g$ (forbidden), 2^3D_1 (favored)

$e^+e^- \rightarrow K^+K^-\pi^0\pi^0$ at BESIII



- $M = (2127 \pm 17 \pm 12) \text{ MeV}/c^2$ and $\Gamma = (107 \pm 32 \pm 28) \text{ MeV}$
- Non-observation in $K^*(892)K^*(892)$: deviates from the prediction of $3^3S_1 s\bar{s}$
- Observation in $KK(1460)$: inconsistent with the prediction of hybrid interpretation

$e^+e^- \rightarrow \phi\eta'$



$$\frac{\Gamma_{ee} \cdot Br^{\phi\eta}}{\Gamma_{ee} \cdot Br^{\phi\eta'}} = \frac{1.7 \pm 0.7^*}{7.3 \pm 1.5} = 0.23 \pm 0.11$$

* Babar PRD 77 092002

Decay modes	$2^3D_1 s\bar{s}$		$1^- s\bar{s}g$	$3^3S_1 s\bar{s}$
$\Gamma(\text{MeV})$	$3P_0$ model	Flux tube	Flux tube	$3P_0$ model
$\phi\eta$	0	0	1.2	21
$\phi\eta'$	2.9	2.8	0.4	11
Ratio($\phi\eta/\phi\eta'$)	0.0	0.0	3.0	2.0

more on the $\phi(2170)$ a.k.a $Y(2175)$ in a parallel talk

Is the $Y(2175)$ a strangeonium hybrid meson?

Wei Chen

Sun Yat-Sen University

HADRON 2019 · Guilin, China
August 16-21, 2019

Based on: **PRD98(2018),096020; PRD100(2019),034012.**

Collaborators: **J. Ho, R. Berg, T. G. Steele, D. Harnett**

Gaussian sum-rules analyses

The resulting predictions:

$$s_0^{\text{opt}} = 9.7 \pm 1.0 \text{ GeV}^2$$

$$m_2 = m_{\text{fit}} = 2.90 \pm 0.16 \text{ GeV}$$

No evidence for a significant strangeonium hybrid component of the $\phi(2170)$, which is consistent with the LQCD calculation (PRD84(2011),074023).

J^{PC}	1^{--}
$q\bar{q}$ operator	$\bar{q}\gamma_\mu q$
$n^{2S+1}L_J$	n^3S_1 $[q\bar{q}]$ spin-triplet
state	ρ, ω
$\bar{q}gq$ operator	$\bar{q}\frac{\lambda^a}{2}\gamma^\nu\gamma_5\tilde{G}_{\mu\nu}^a q$ $[q\bar{q}]$ spin-singlet

QCD Sum Rules

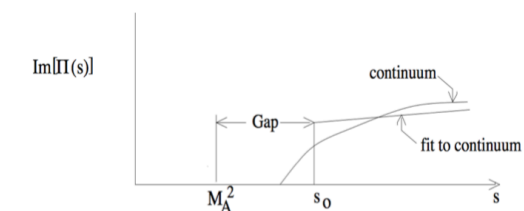
- Study **two-point correlation function** of current $J_\mu(x)$ with the same quantum numbers with hadron state:

$$\Pi_{\mu\nu}(q^2) = i \int d^4x e^{iq \cdot x} \langle \Omega | T[J_\mu(x) J_\nu^\dagger(0)] | \Omega \rangle$$

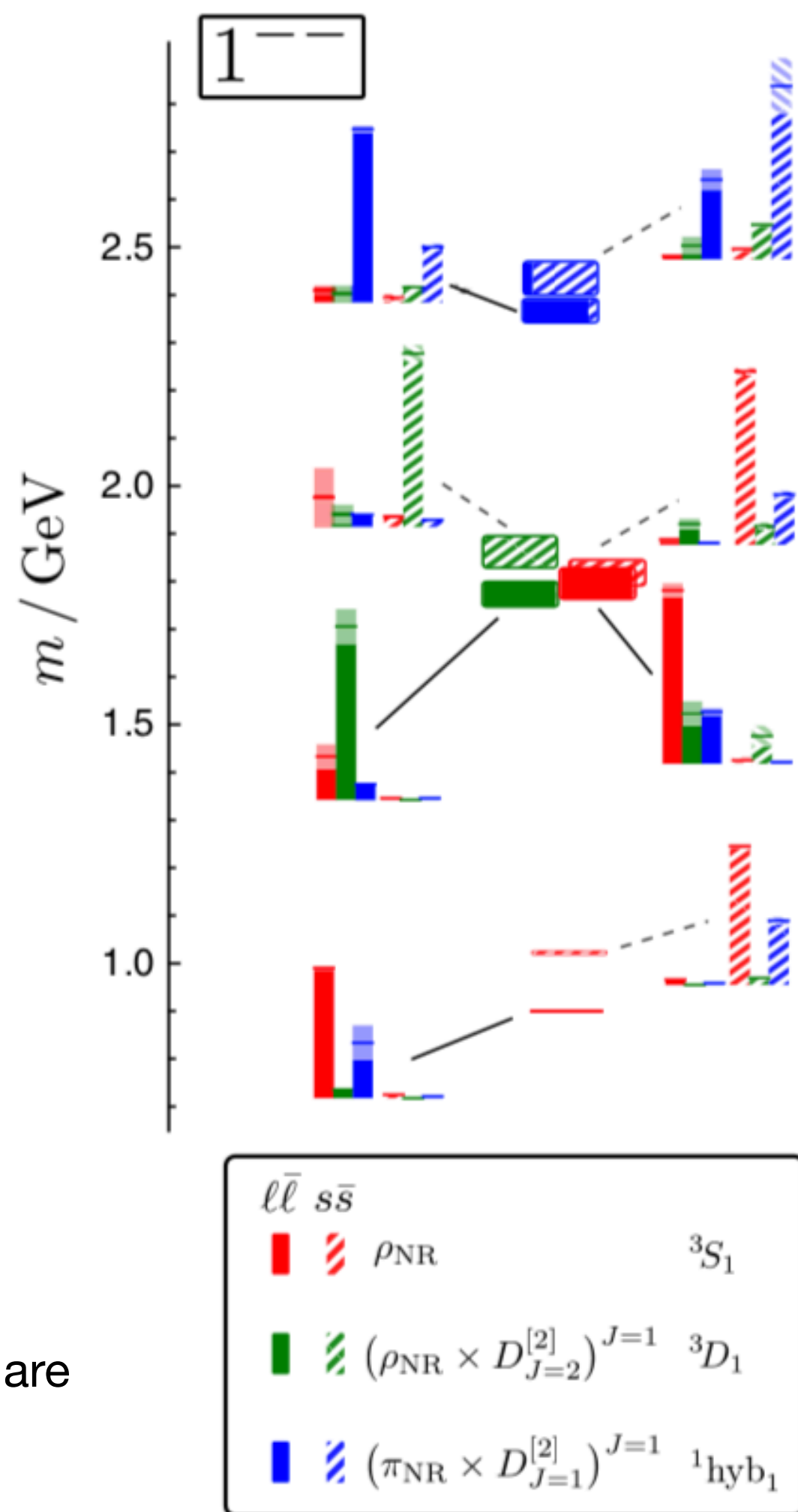
- Classify states $|X\rangle$ by coupling to current $\langle \Omega | J_\mu(x) | X \rangle \neq 0$
- Hadron level:** described by the **dispersion relation**

$$\Pi(q^2) = \frac{(q^2)^N}{\pi} \int \frac{\text{Im}\Pi(s)}{s^N(s - q^2 - i\epsilon)} ds + \sum_{n=0}^{N-1} b_n (q^2)^n,$$

$$\rho(s) = \frac{1}{\pi} \text{Im}\Pi(s) = \sum_n \delta(s - m_n^2) \langle 0 | J | n \rangle \langle n | J^\dagger | 0 \rangle$$



No evidence for a significant strangeonium hybrid component of the $\phi(2170)$, which is consistent with the LQCD calculation (PRD84(2011),074023).



somehow lattice QCD and sum rules communities are speaking in different languages ...

Simon Eidelman

Budker Institute of Nuclear Physics SB RAS
and Novosibirsk State University, Novosibirsk, Russia
and Lebedev Physical Institute RAS, Moscow, Russia

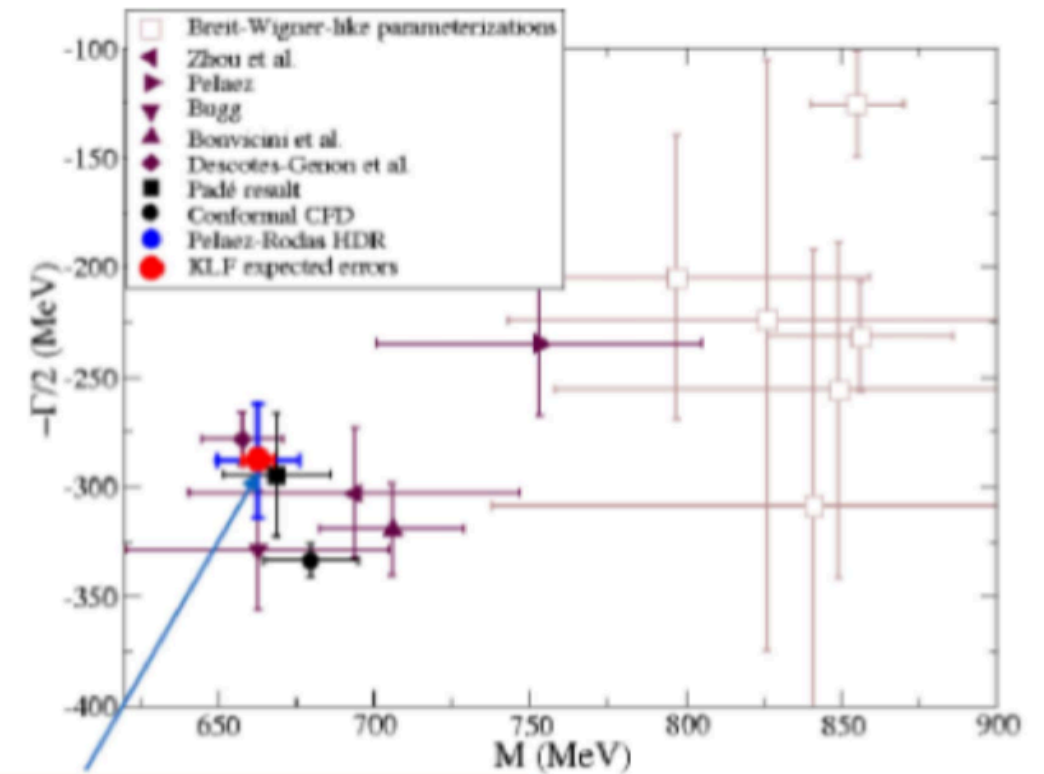
a nice encyclopaedic summary of current and future facilities, but ...

Simon Eidelman

Budker Institute of Nuclear Physics SB RAS
and Novosibirsk State University, Novosibirsk, Russia
and Lebedev Physical Institute RAS, Moscow, Russia

κ properties at KLF

Mass and width of the $K_0^*(700)$ (κ)



100 days of running

Measurement with KLF will reduce:
Uncertainty in the mass by a factor of two!
Uncertainty in the width by a factor of five!

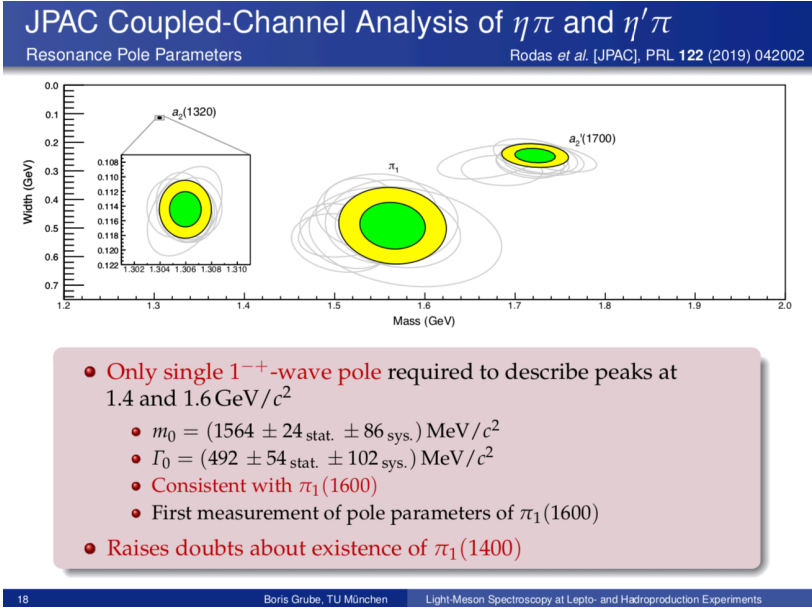
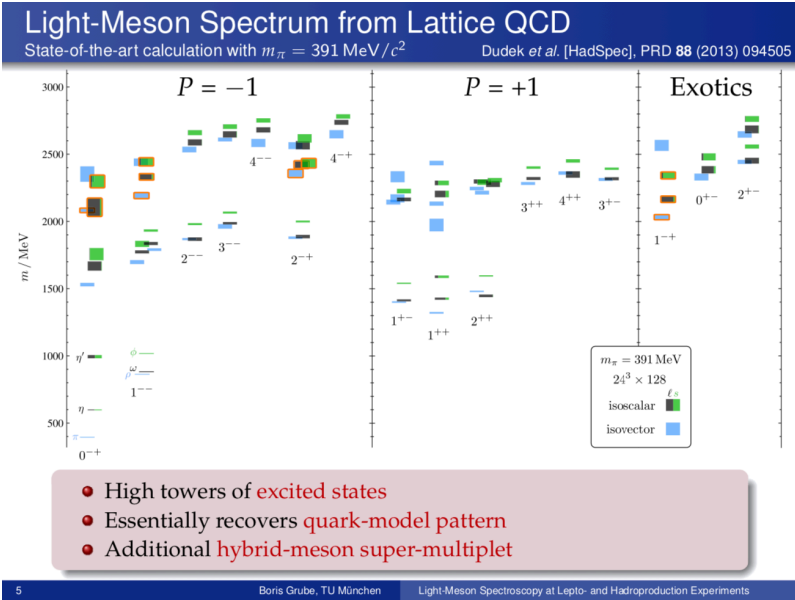
“a lie can get halfway around the world before the truth has got its boots on”

JLab theory group results having an impact in the broader community

Light-Meson Spectroscopy at Lepto- and Hadroproduction Experiments

Boris Grube

Institute for Hadronic Structure and Fundamental Symmetries
Technische Universität München
Garching, Germany



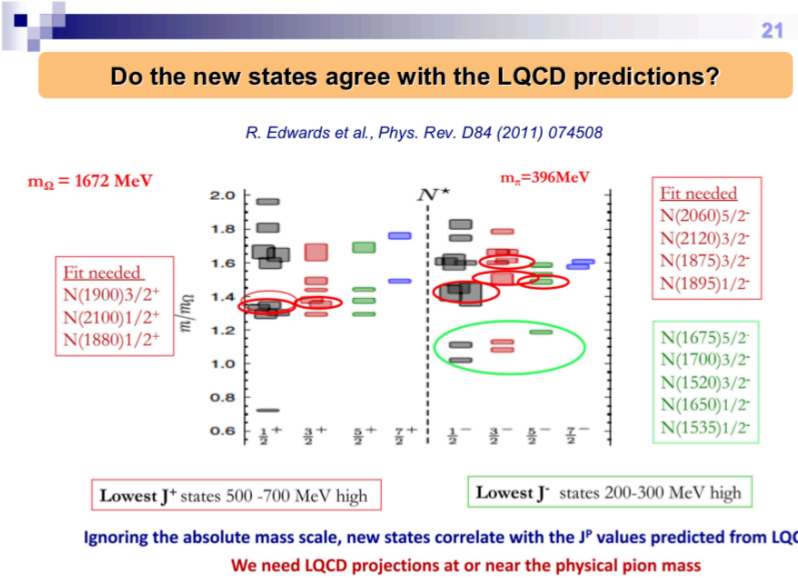
Review of light baryon spectroscopy

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Stefan Diehl

Justus Liebig University Giessen
University of Connecticut

August 21, 2019



Relativistic effects in radiative charmonium transitions: A covariant quark model approach

Presenter:
Tomohito MAEDA (Nihon Univ.)

Collaborators:
Toshihiko KOMADA, Masuho ODA, Kenji YAMADA (Nihon Univ.)

- Note that similar results are reported in several lattice calculations J. J. Dudek, *et al.*, (2006, 2009)
Y. Chen *et al.*, (2011)
D. Becirevic and F. Sanfilippo, (2012)

Dispersive techniques for processes $\gamma^* \gamma^* \rightarrow \pi\pi, \pi\eta$

Igor Danilkin

August 20, 2019, Gulin, China

Right-hand cuts (hadronic input)

Coupled-channel Omnès formalism
 $t(s) = U(s) + \int_R \frac{ds'}{\pi} \frac{\rho(s') t(s')^2}{s' - s}$
 $t(s) = \frac{N(s)}{D(s)} = \Omega(s) N(s)$ Chew, Mandelstam (1960)
Model independent form of the left-hand cuts: conformal mapping expansion
 $U(s) = \sum_k C_k \xi(s)^k$ I.D., Lutz, Gasparyan (2011)
Coefficients C_k matched to $SU(3)$ Chiral Perturbation Theory at threshold

STRONG INTERACTION STABLE HEAVY TETRAQUARKS FROM THE LATTICE

Kim Maltman, York University
(加拿大, 约克大学)

with
B. Colquhoun, A. Francis, J. Hudspith, R. Lewis

➤ For completeness: the $I(J^P) = 0(1^+) u d \bar{c} \bar{c}$ channel

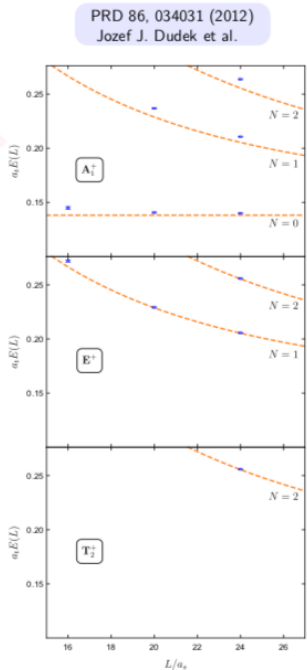
❖ HadSpec [1709.01417]

- $n_f=2+1$, anisotropic clover + improved Symanzik gauge, $m_\pi = 391 \text{ MeV}$
- large “meson-meson” + tetraquark basis
- No evidence for $u d \bar{c} \bar{c}$ or $\ell s \bar{c} \bar{c}$ bound tetraquarks

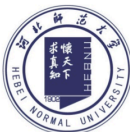
people are using hadron spectrum collaboration lattice QCD results
in their theory calculations

Partial Wave Mixing in Hamiltonian Effective Field Theory (HEFT)

Yan Li¹ Jia-jun Wu¹ Curtis D. Abell² Derek B. Leinweber²
Anthony W. Thomas^{2,3}



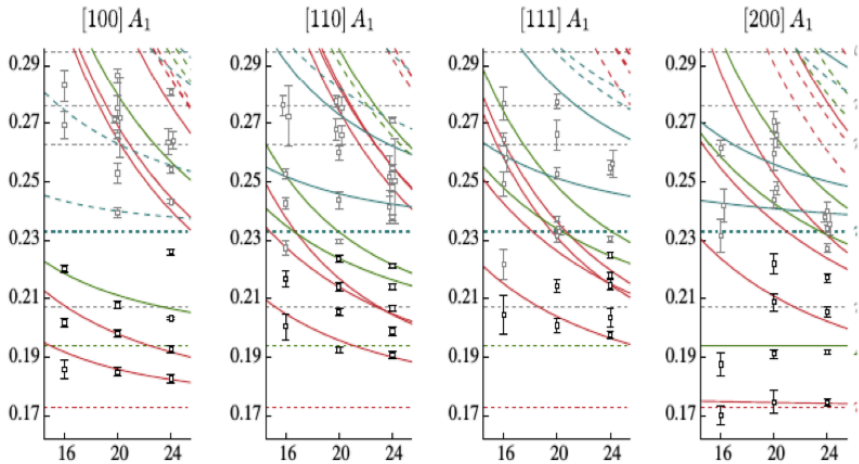
Determination of resonance properties from
lattice energy levels using chiral EFT



Zhi-Hui Guo (郭志辉)

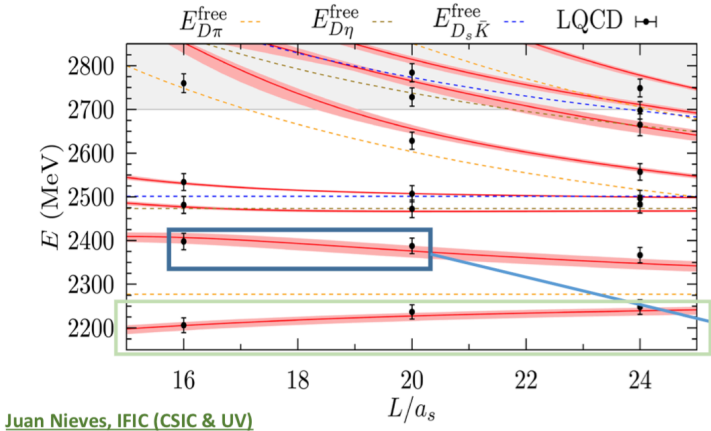
Hebei Normal University (河北师范大学)

E.g. Pi-eta, KKbar, Pi-eta' scattering
[Dudek, Edwards, Wilson, PRD'16]



Theoretical review of heavy-light spectroscopy

Juan M Nieves (IFIC, CSIC & U. Valencia)



LQCD: G. Moir et al., JHEP 1610 (2016) 011 (Hadron Spectrum Collaboration).
 $D\pi, D\eta, D_s\bar{K}$ coupled-channels

Juan Nieves, IFIC (CSIC & UV)

Quark-mass dependence
of light meson masses
and decay constants

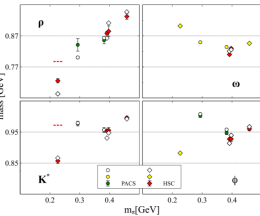
Xiao-Yu Guo*, Matthias F.M. Lutz

[PRD98,056005; NPA988,36; NPA988,48]

GSI Helmholtzzentrum, Darmstadt

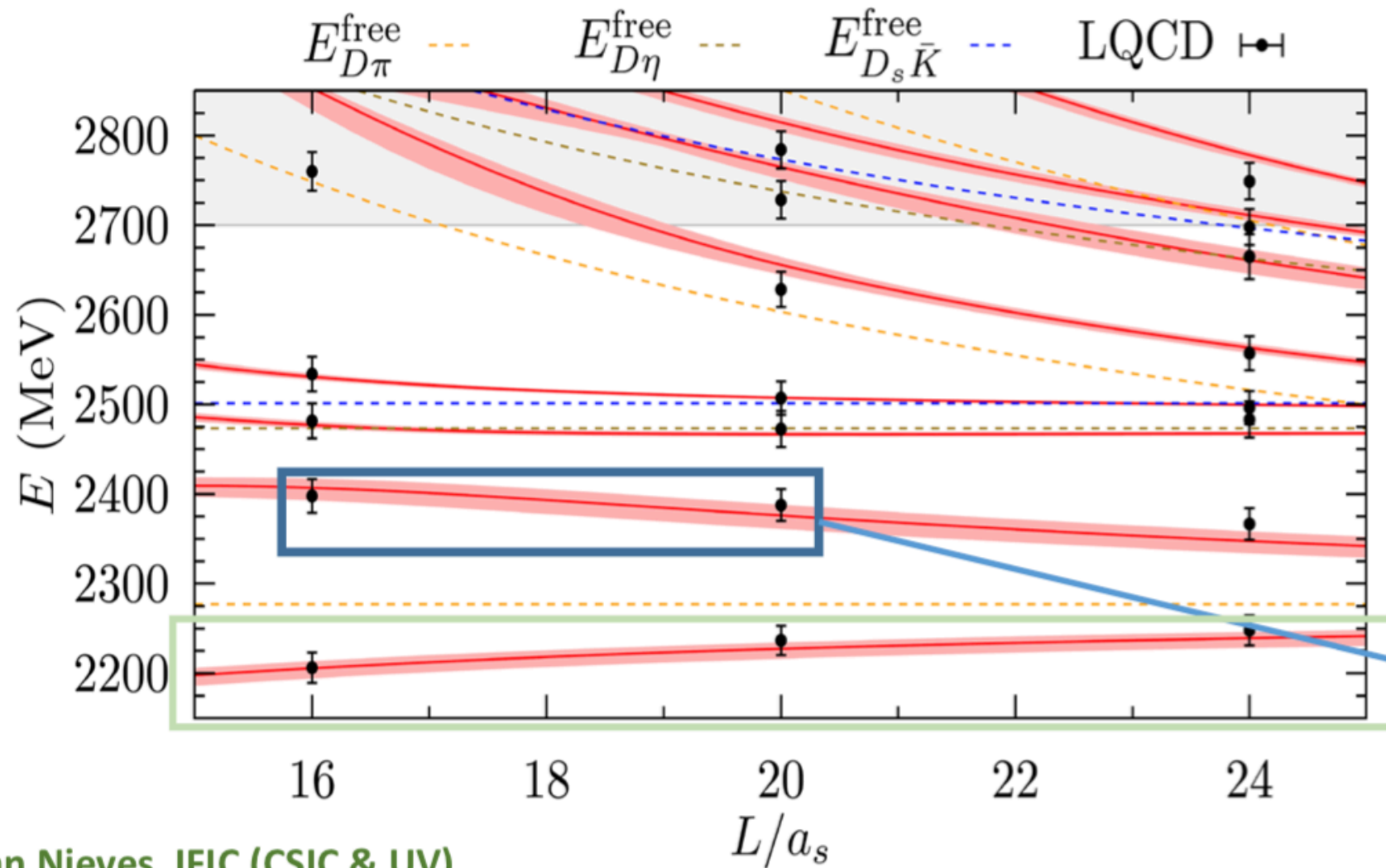
A global fit to QCD lattice data

- masses from PACS-CS [Phys.Rev.D79,034503]
 - energy levels from quark-antiquark interpolators
 - levels are well reproduced
 - predictions for ω -meson masses on the ensembles
- masses from HSC [Phys.Rev.D79,034503]
 - levels from quark-antiquark interpolators well reproduced
 - level from πK interpolators well reproduced [Nucl.Phys.B932,29]
 - tension in level from $\pi\pi$ interpolators, not well separated from the two-body $\pi\pi$ state (dashed line) [Nucl.Phys.B910,842]



Theoretical review of heavy-light spectroscopy

Juan M Nieves (IFIC, CSIC & U. Valencia)



✓ Level below threshold, associated with a bound state.

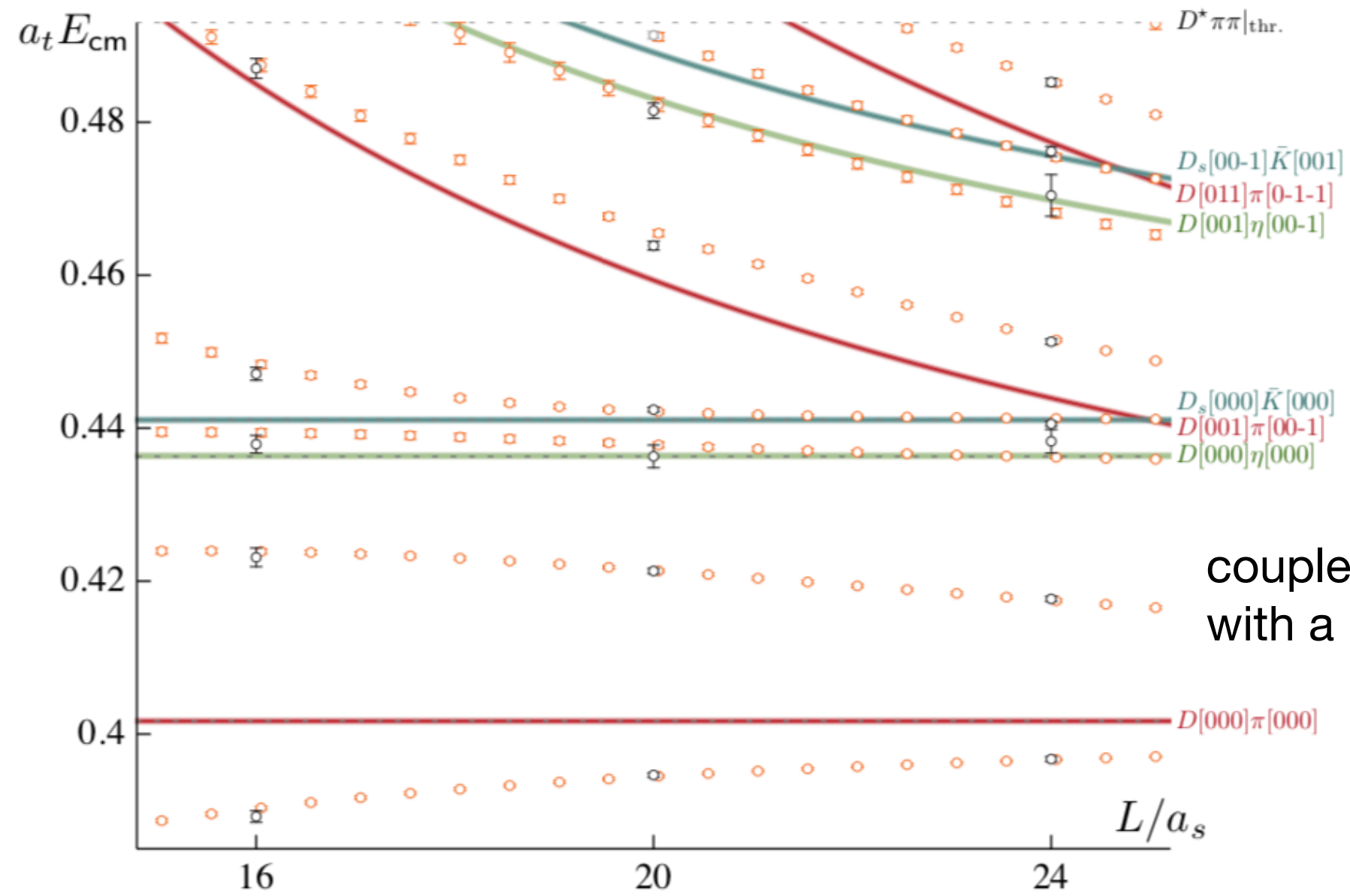
✓ Second level, lying between the $D\pi$ and $D\eta$ thresholds, is very shifted with respect to both of them, hinting at the presence of a **Resonance?**

very frustrating to see wrong statements like this!

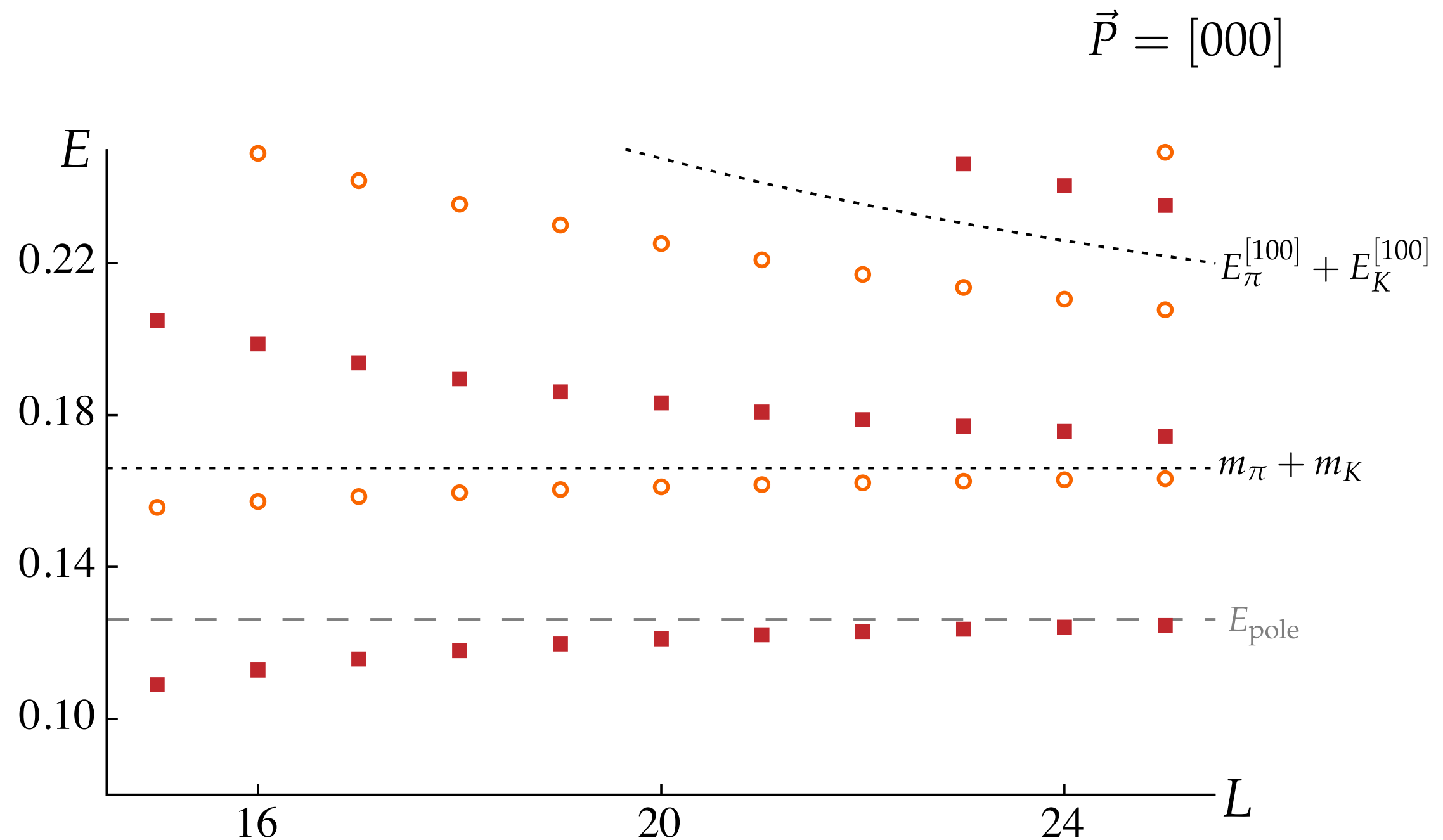
Juan Nieves, IFIC (CSIC & UV)

LQCD: G. Moir et al., JHEP 1610 (2016) 011 (Hadron Spectrum Collaboration).

$D\pi, D\eta, D_s\bar{K}$ coupled-channels



coupled-channel amplitude description
with a bound-state and no resonance



hadron spectroscopy from lattice QCD

Jozef Dudek

now routinely done, calculations with

very light quarks (physical pion mass)

QED effects included

breaking of isospin symmetry ($m_u \neq m_d$)

systematic control,
but only for the **simplest observables**

now routinely done, calculations with

very light quarks (physical pion mass)

QED effects included

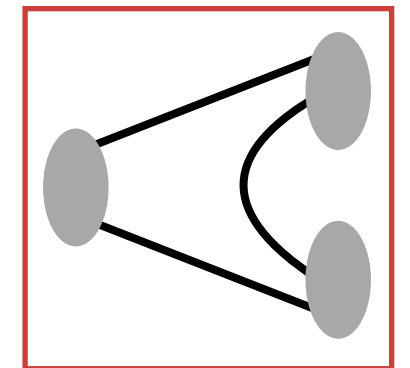
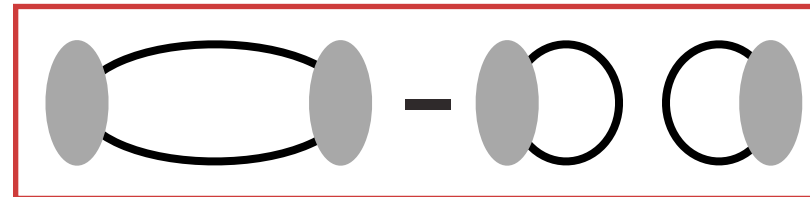
breaking of isospin symmetry ($m_u \neq m_d$)

systematic control,
but only for the simplest observables

more relevant to excited spectroscopy

large diverse basis of hadron operators (including multi-hadrons)

inclusion of $q\bar{q}$ annihilation effects (access to isoscalar mesons, hadron decays)



‘variational’ analysis of matrices of correlation functions

(reliable determination of many excited states)

much of this progress follows from

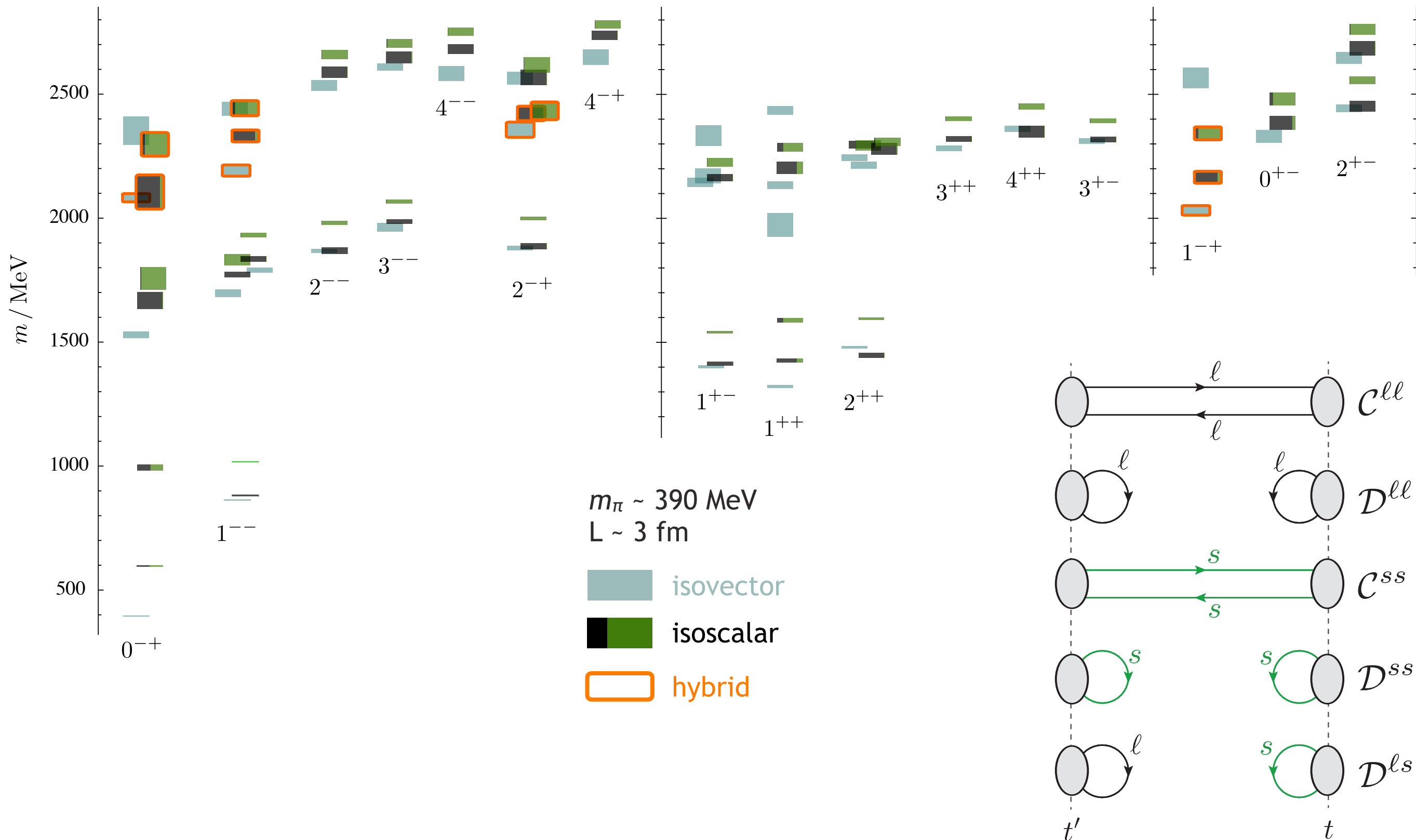
PHYSICAL REVIEW D 80, 054506 (2009)

Peardon et al

Novel quark-field creation operator construction for hadronic physics in lattice QCD

‘distilled’ and matured for ten years

diagonalizing a large basis of operators $\sim \bar{\psi} \Gamma \overleftrightarrow{D} \dots \overleftrightarrow{D} \psi$



(quasi) flavor-exotic hadrons

tempting to make use of these tools to study contemporary exotic hadrons

e.g. $Z_c(3900)$, a resonance in $J/\psi \pi$? — probably $J^P=1^+$

compute correlation functions using a basis of

tetraquark-like $\psi(\mathbf{x})\psi(\mathbf{x})\bar{\psi}(\mathbf{x})\bar{\psi}(\mathbf{x})$ with appropriate spin, color coupling

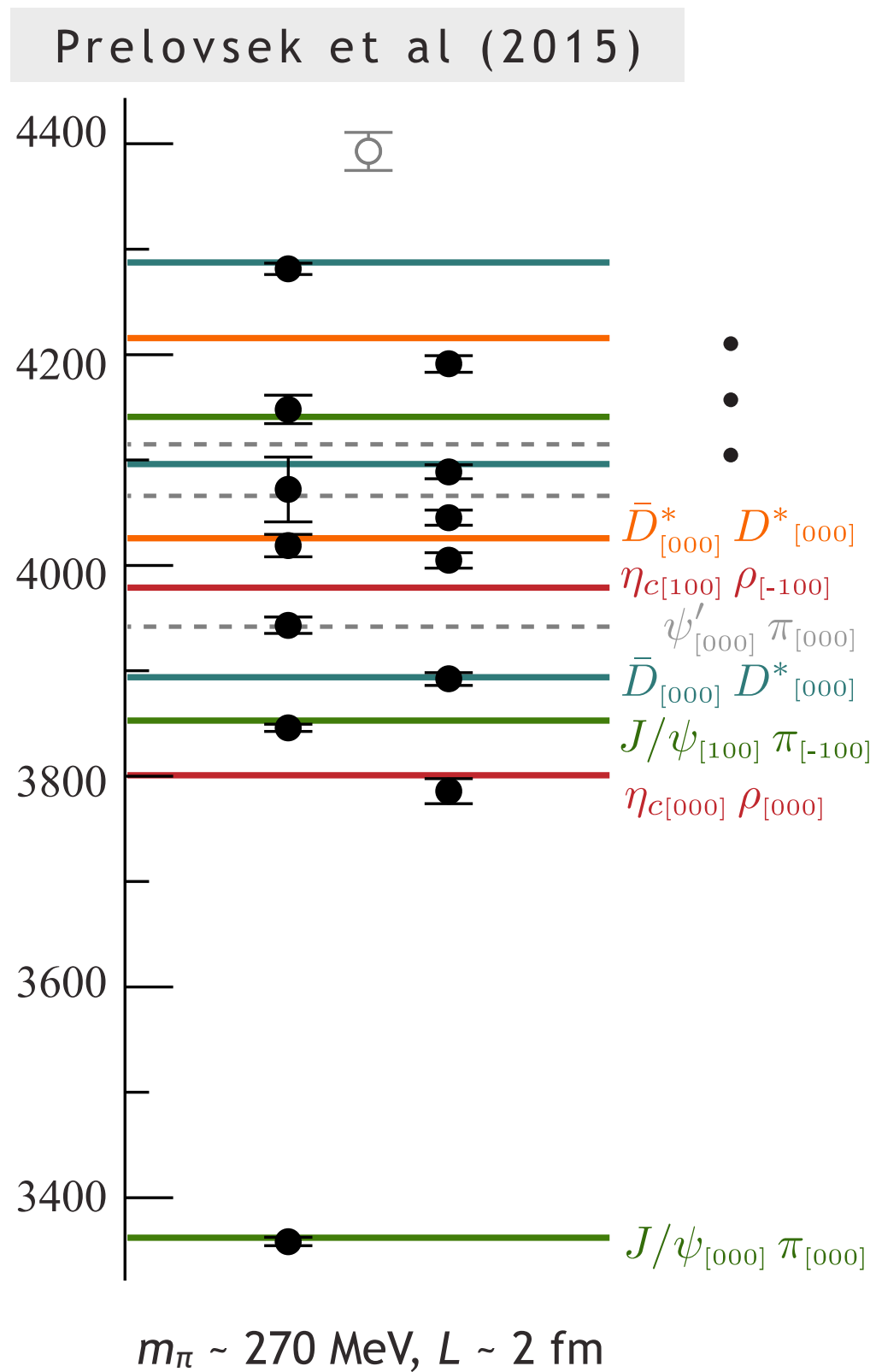
and

meson-meson-like $\left(\sum_{\mathbf{x}} e^{i\mathbf{p}_1 \cdot \mathbf{x}} \bar{\psi} \Gamma \psi(\mathbf{x}) \right) \left(\sum_{\mathbf{y}} e^{i\mathbf{p}_2 \cdot \mathbf{y}} \bar{\psi} \Gamma \psi(\mathbf{y}) \right)$
operators

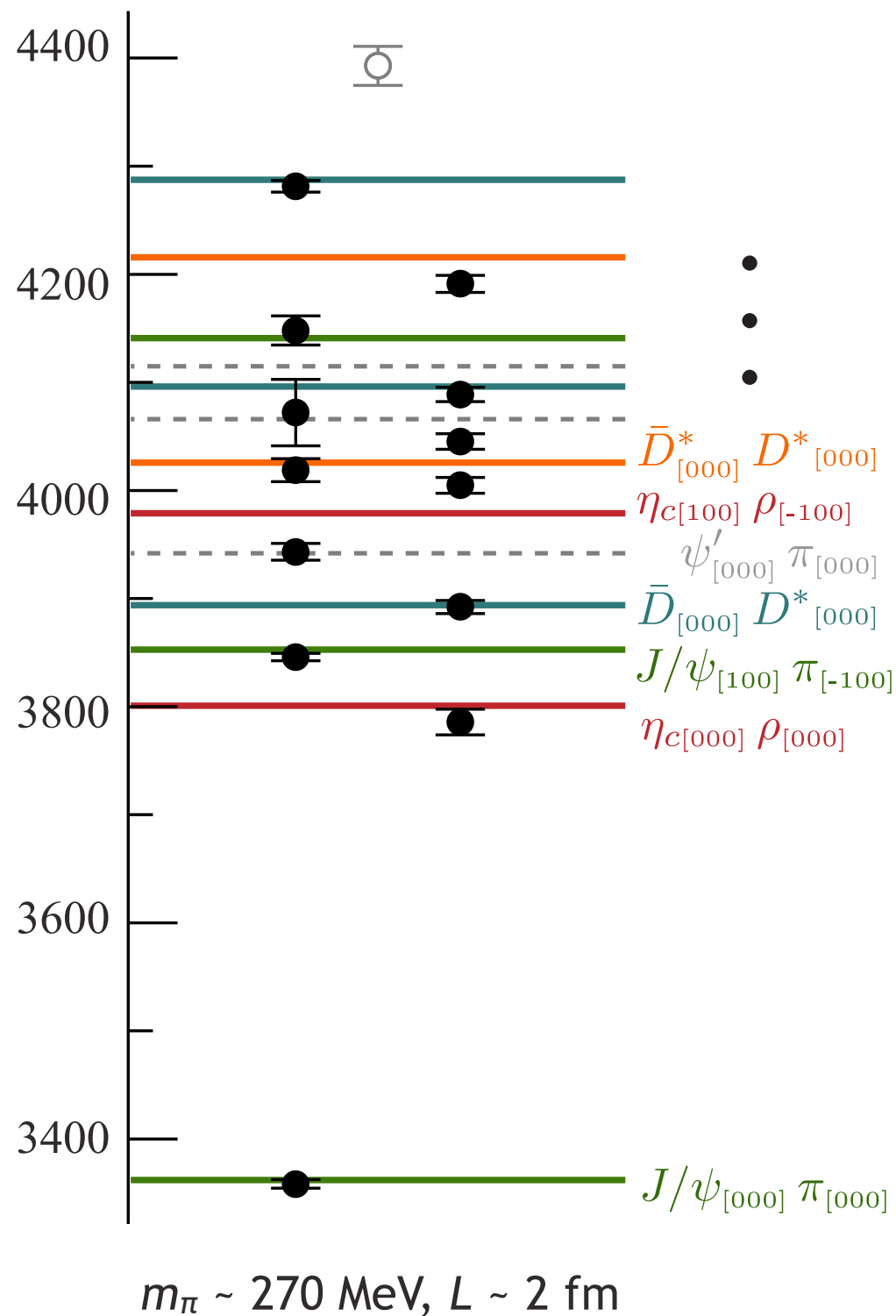
for non-interacting mesons,
expect a spectrum

$$\sqrt{m^2 + \mathbf{p}_1^2} + \sqrt{m^2 + \mathbf{p}_2^2}$$

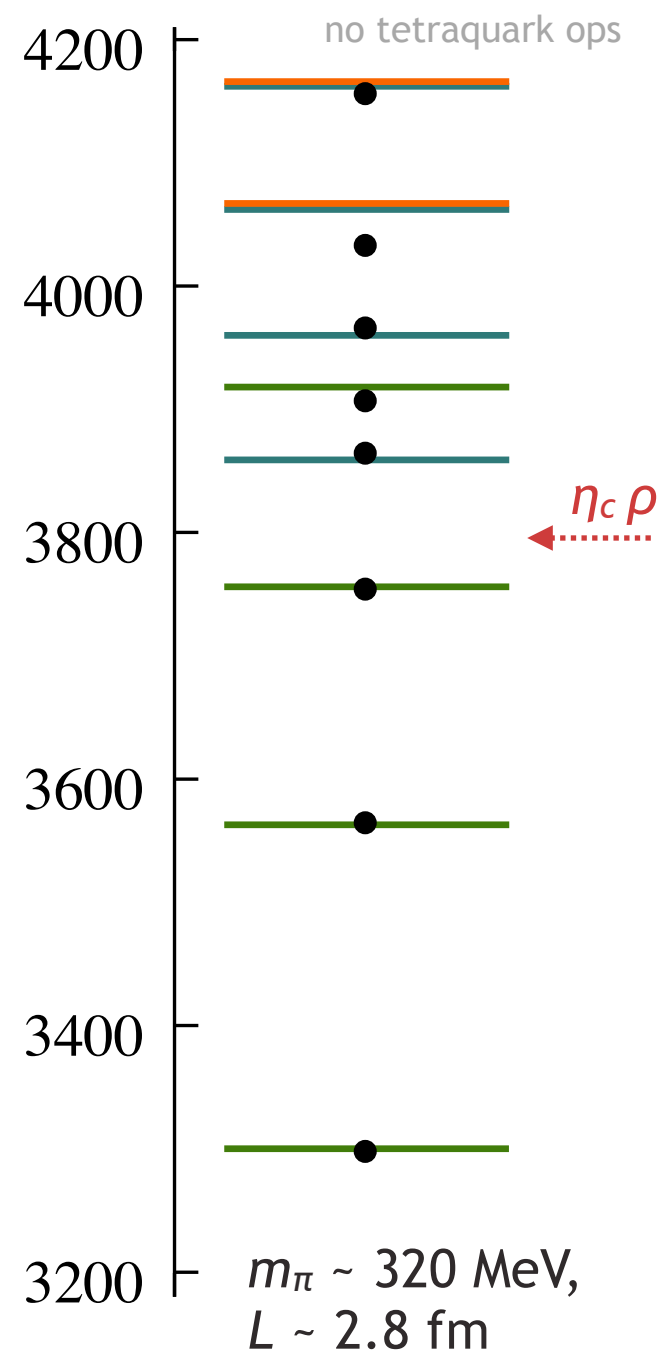
$$\mathbf{p} = \frac{2\pi}{L} (n_x, n_y, n_z)$$



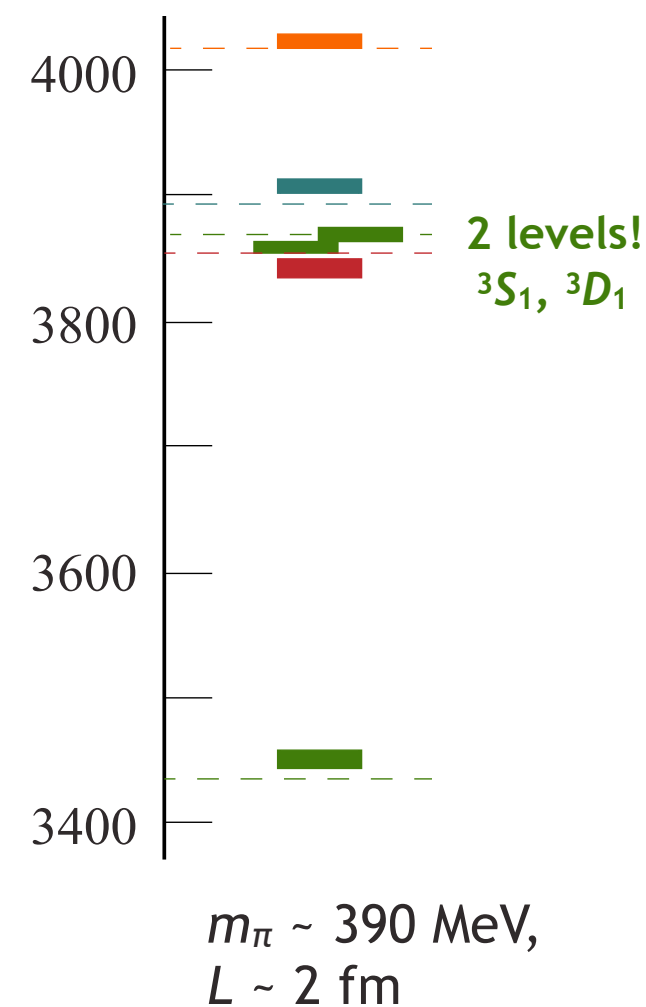
Prelovsek et al (2015)



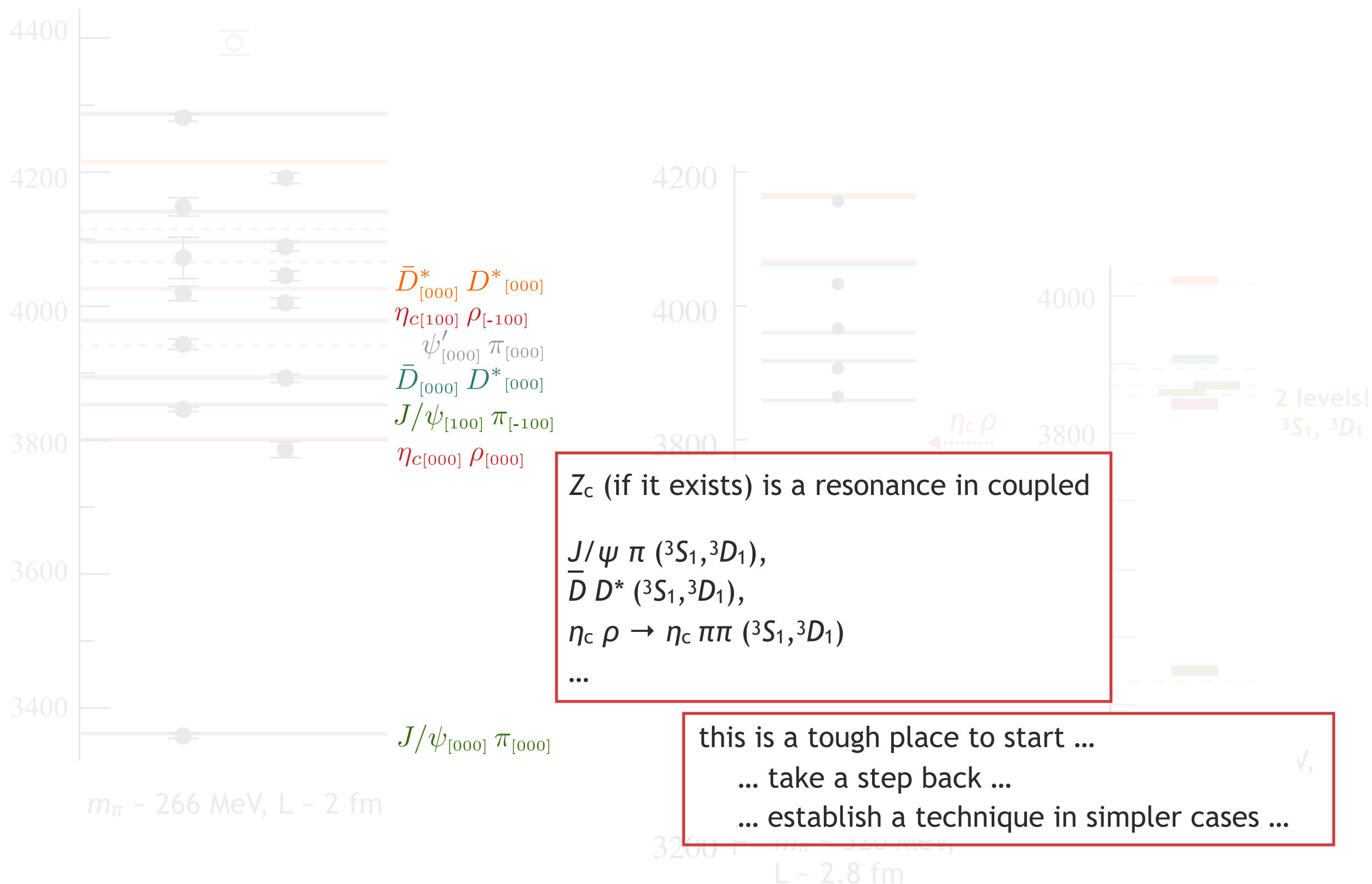
Chen et al (2019)



hadspec (2017)



see also HALQCD approach ...



‘scattering’ in a finite volume

the important feature of the lattice calculation is the **periodic cubic boundary**

discrete spectrum in a finite-volume $\leftrightarrow$ scattering amplitudes in infinite volume “Lüscher method”

e.g. in elastic case $E_n(L) \rightarrow \delta(E_n)$

recent pedagogic review

REVIEWS OF MODERN PHYSICS, VOLUME 90, APRIL–JUNE 2018

Scattering processes and resonances from lattice QCD

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Ross D. Young[‡]

Special Research Center for the Subatomic Structure of Matter (CSSM),
Department of Physics, University of Adelaide, Adelaide 5005, Australia

 (published 18 April 2018)

job of the lattice is to compute the discrete spectrum

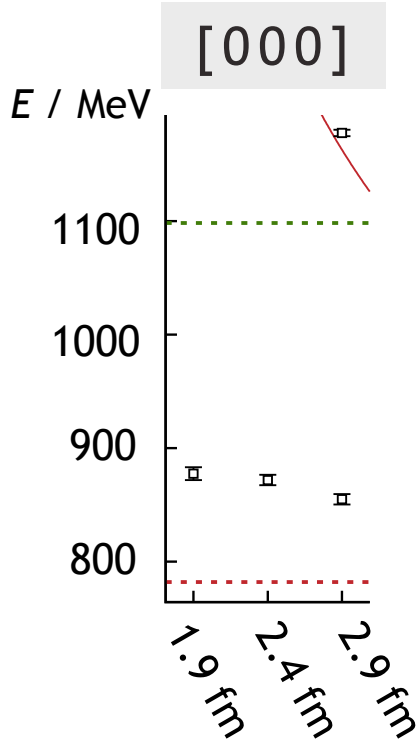
e.g. $\pi\pi$ scattering in P -wave — expect the ρ resonance ...

operator basis :

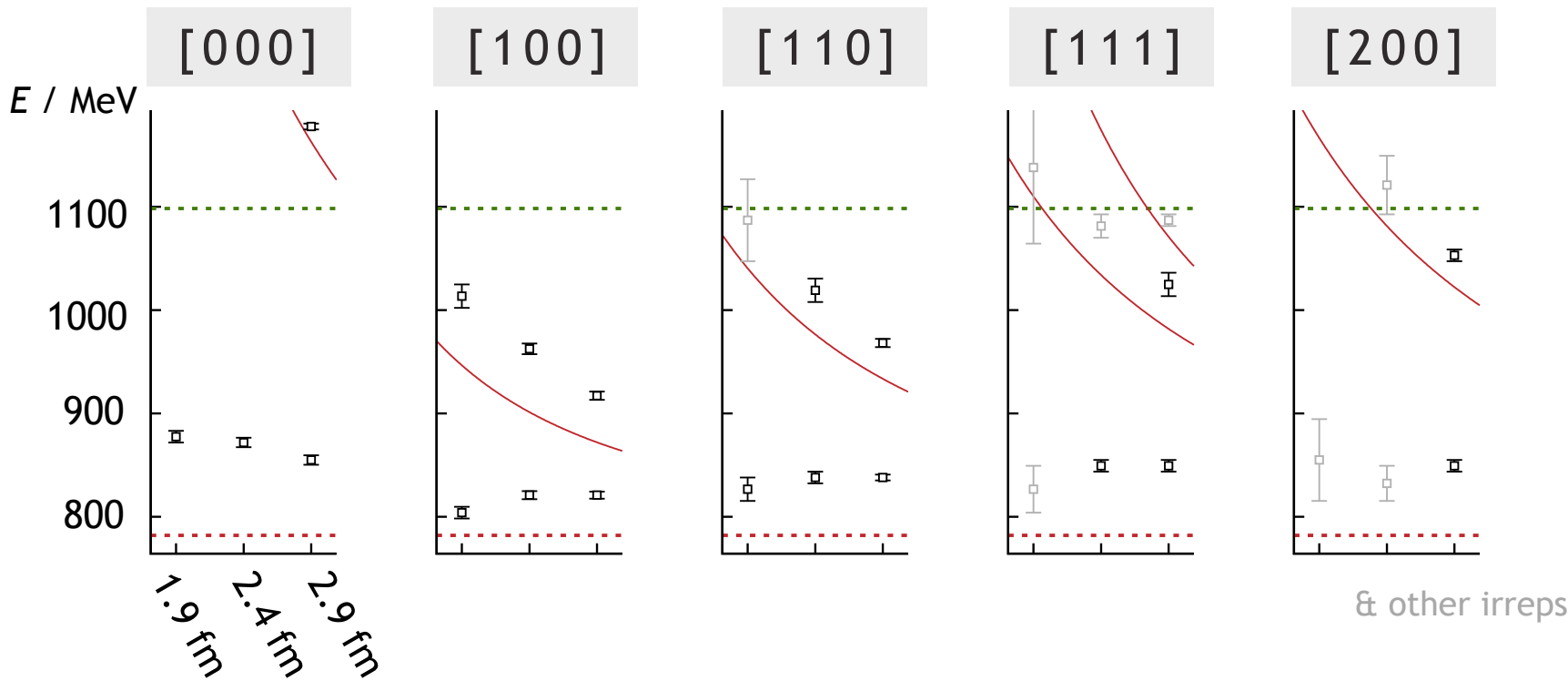
$\bar{\psi}\Gamma\overleftrightarrow{D}\dots\overleftrightarrow{D}\psi$ “ $q\bar{q}$ -like”

$\pi(\mathbf{p}_1)\pi(\mathbf{p}_2)$

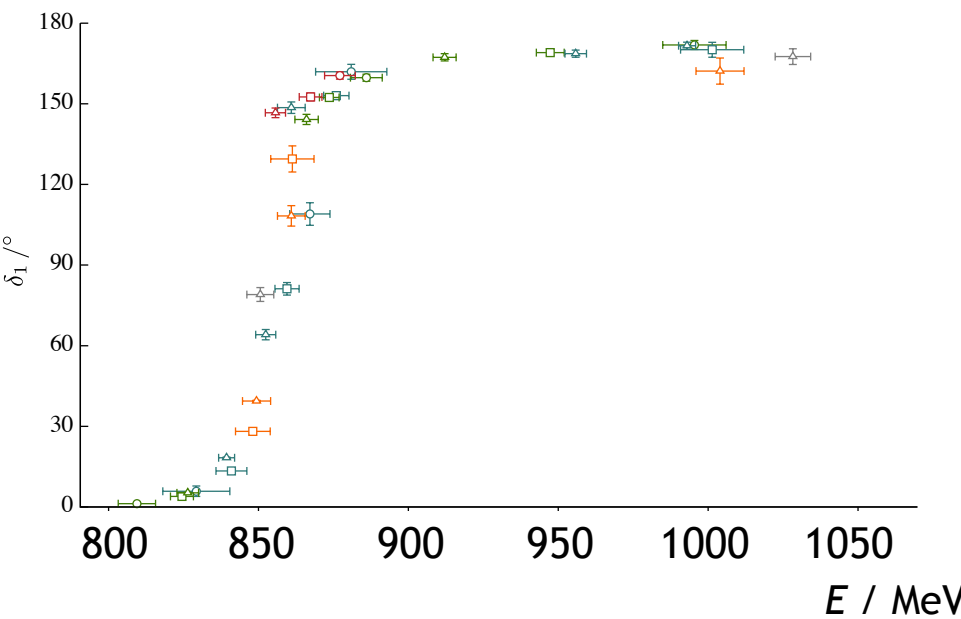
PRD87 034505 (2013) $m_\pi \sim 391 \text{ MeV}$



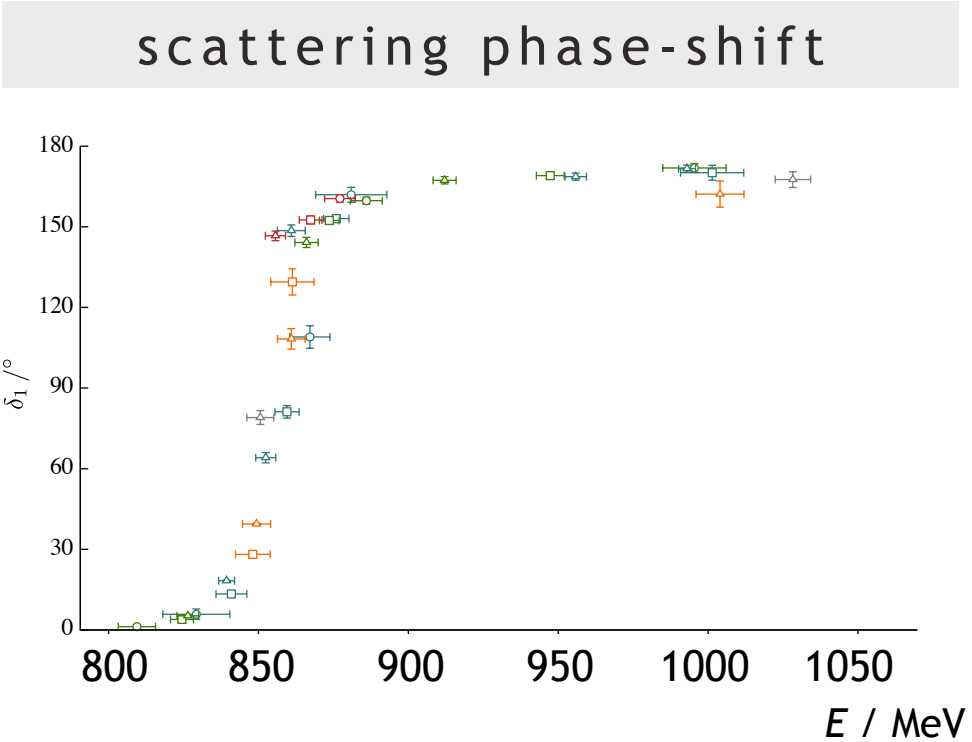
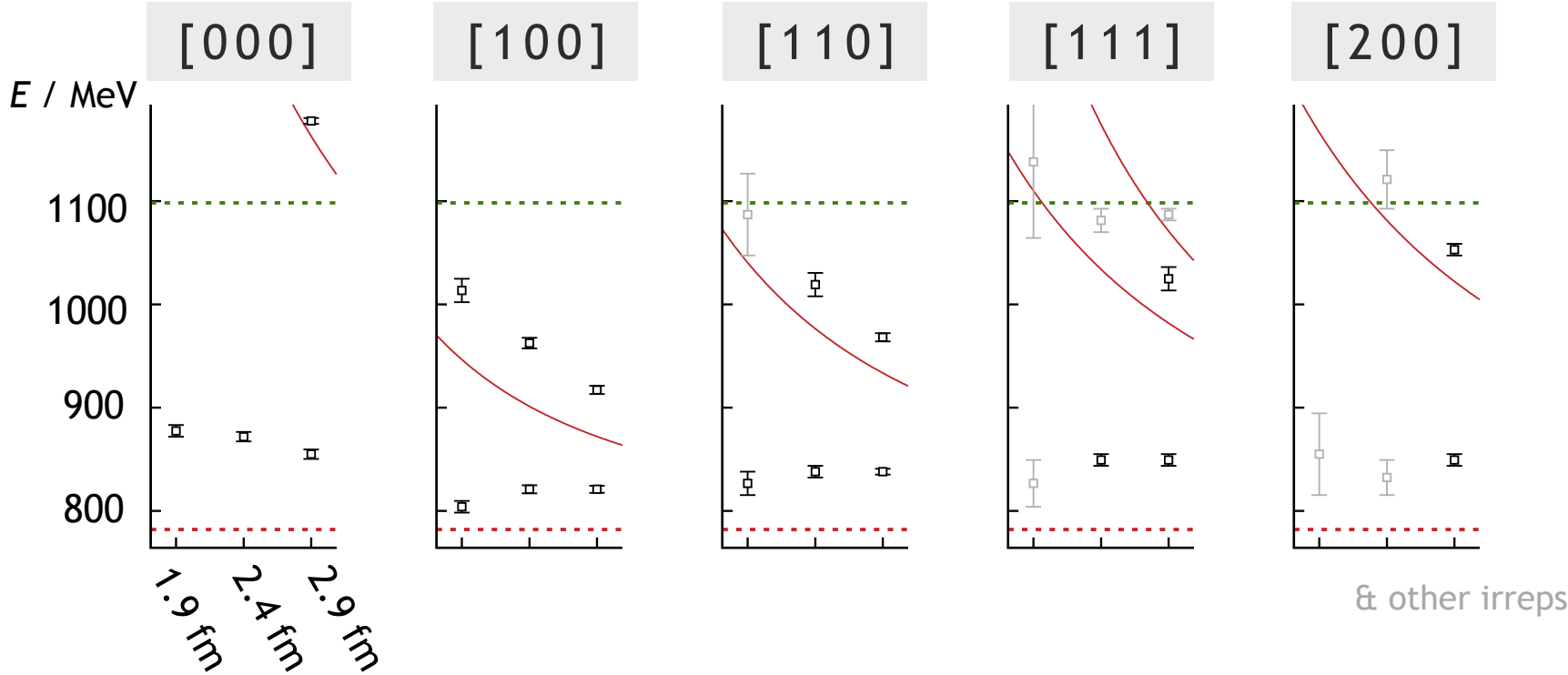
PRD87 034505 (2013) $m_\pi \sim 391$ MeV



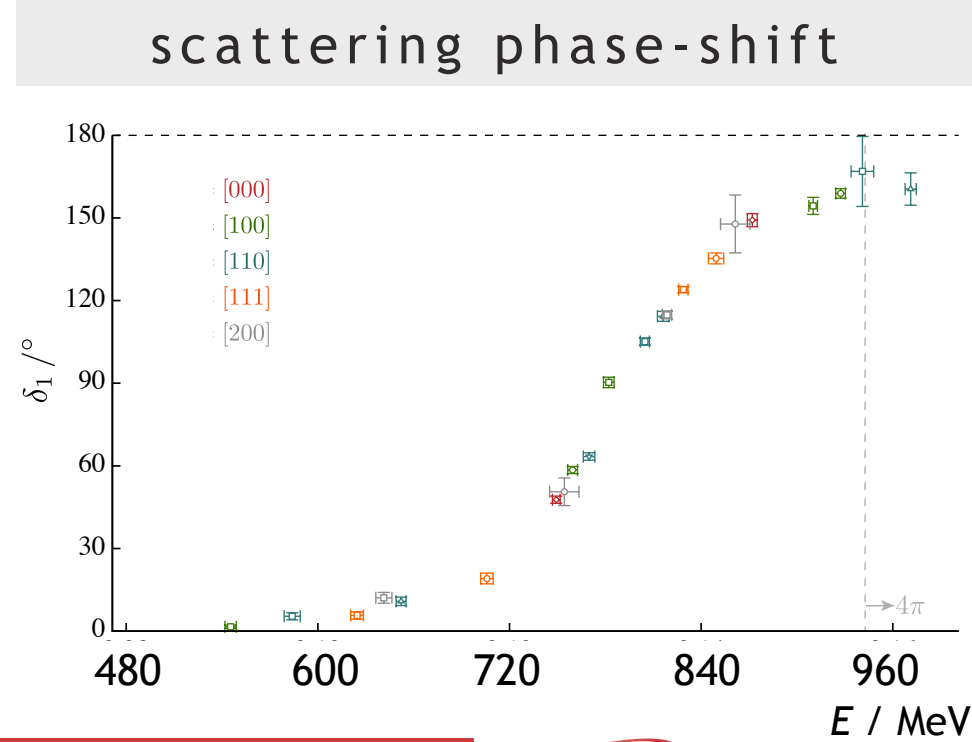
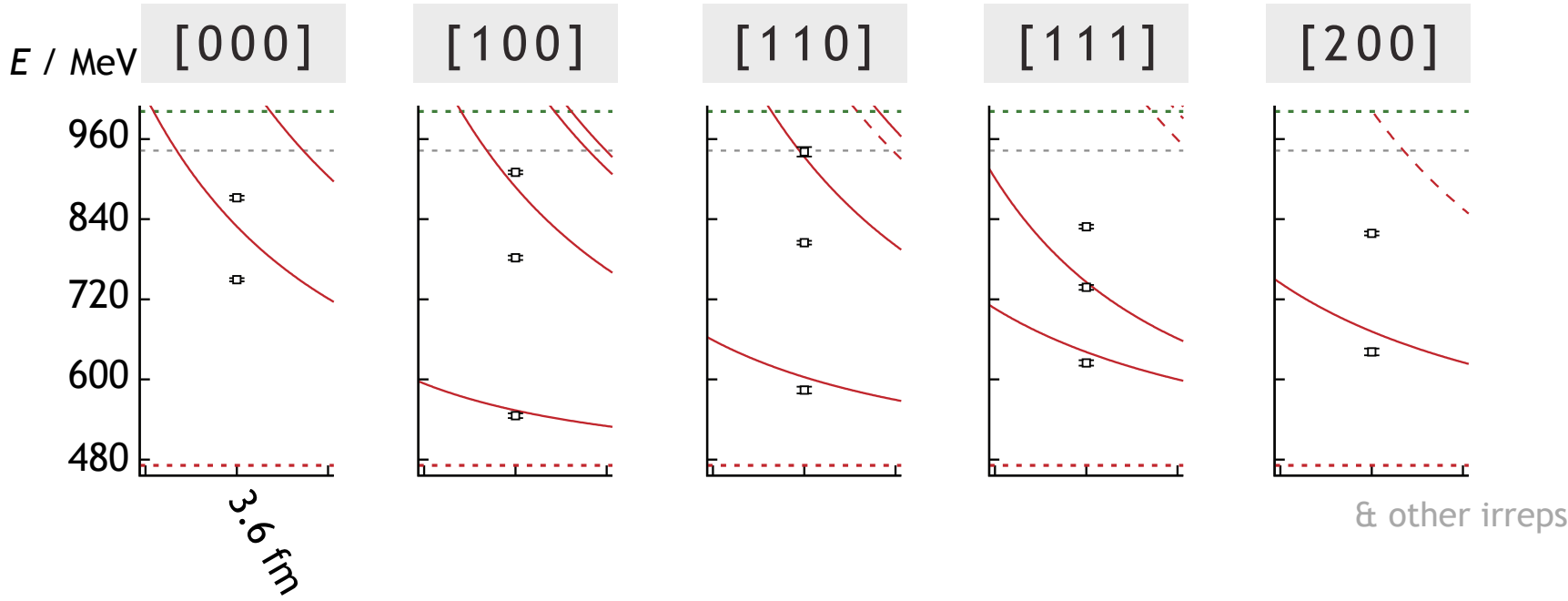
scattering phase-shift

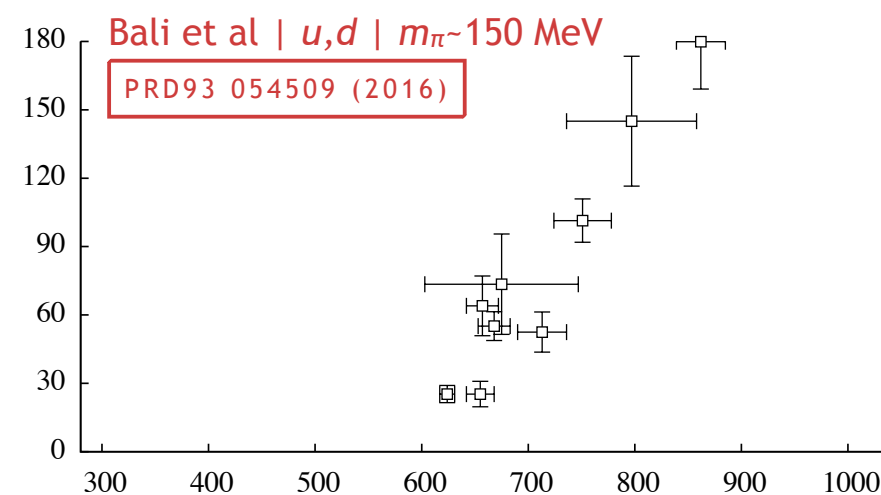
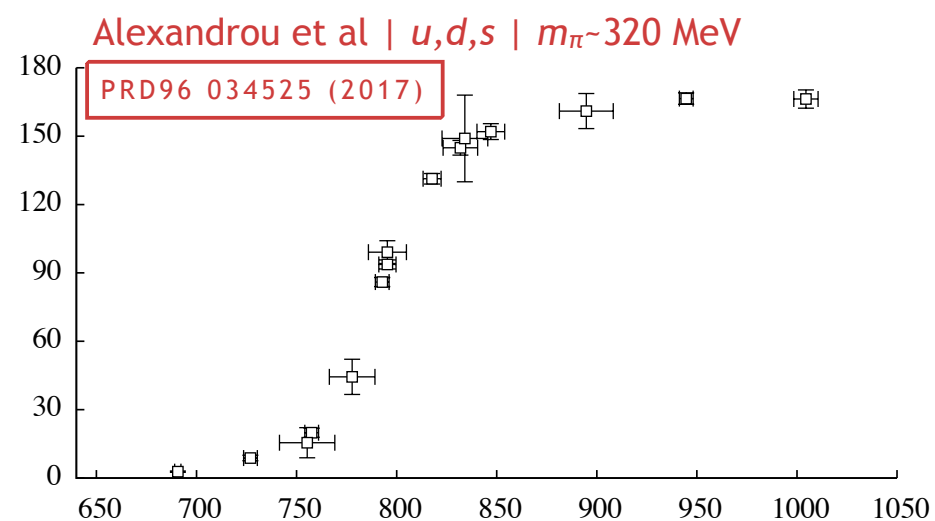
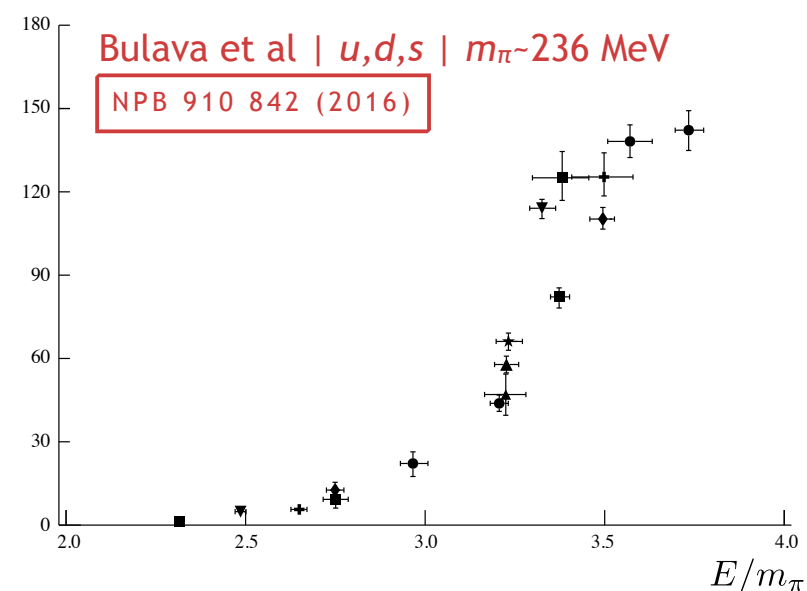
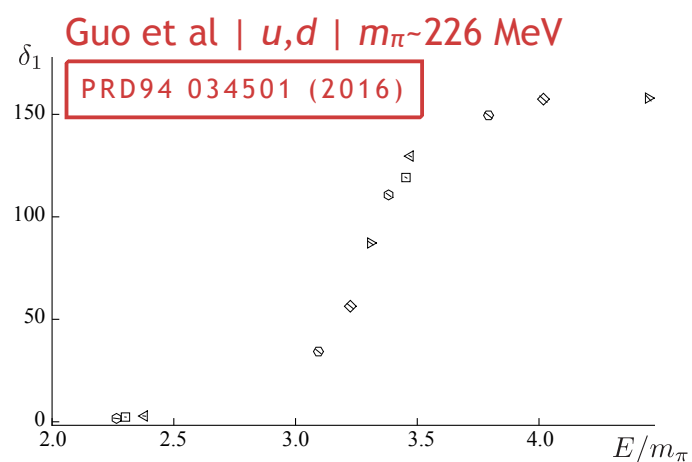
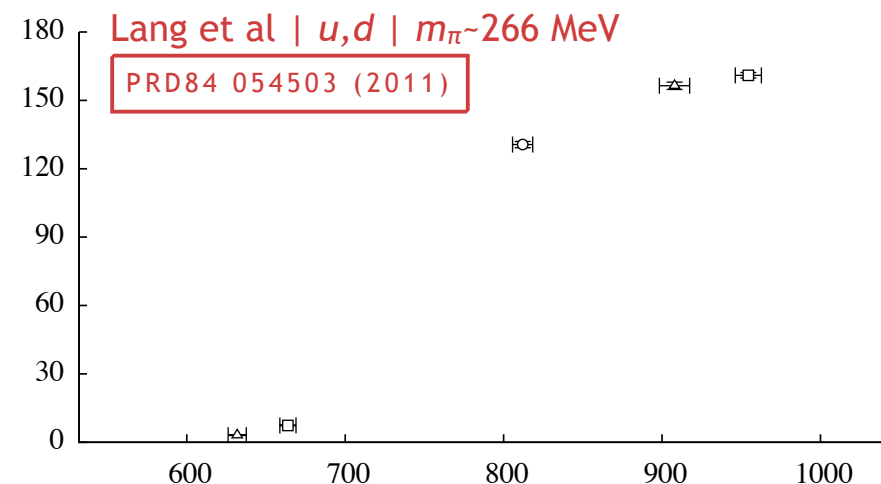
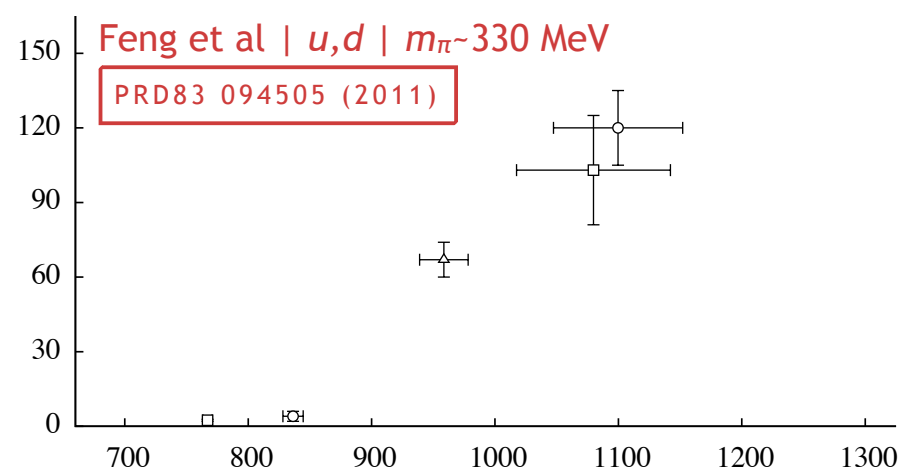


PRD87 034505 (2013) $m_\pi \sim 391$ MeV



PRD92 094502 (2015) $m_\pi \sim 236$ MeV





$$0 = \det \left[\mathbf{1} + \underbrace{i\rho(E)}_{\text{phase-space}} \cdot \underbrace{\mathbf{t}(E)}_{\text{scattering matrix}} \cdot \left(\mathbf{1} + \underbrace{i\mathcal{M}(E, L)}_{\text{matrix of known finite-volume functions}} \right) \right]$$

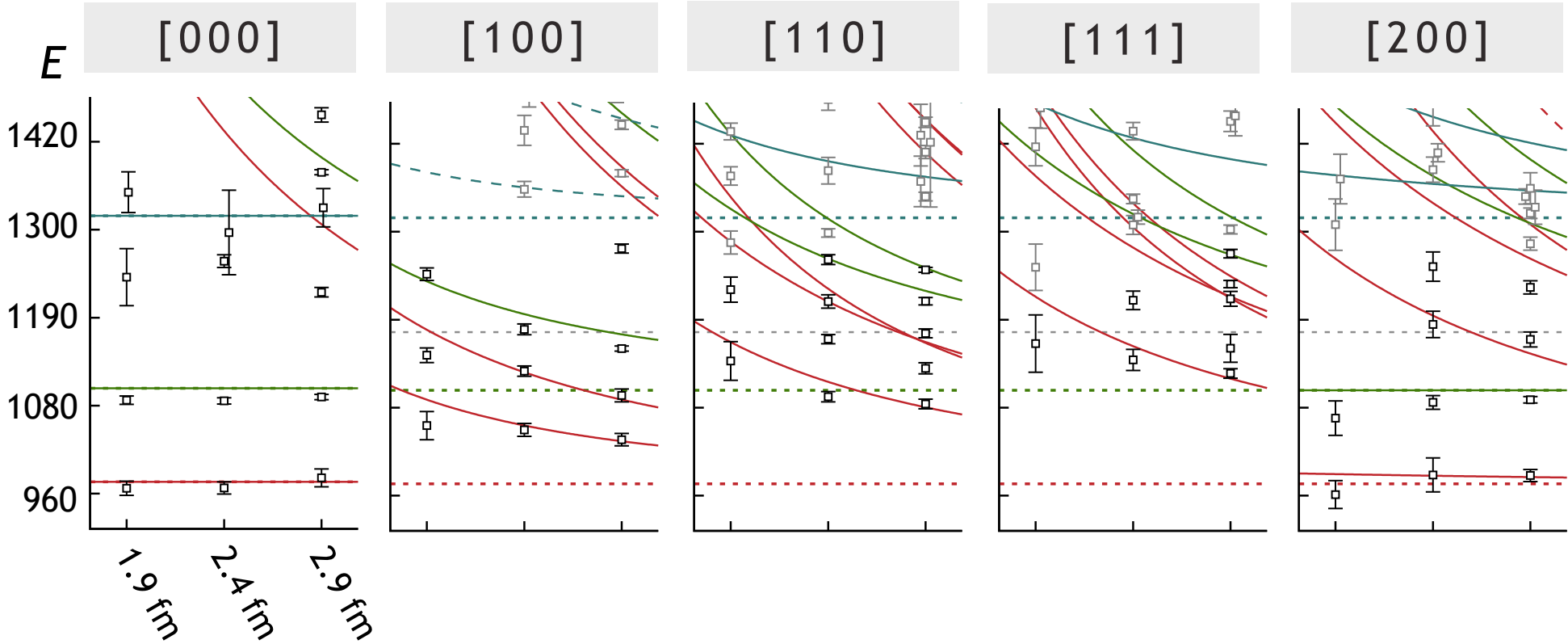
solved by some discrete $E_n(L)$

one approach: parameterize the energy dependence of $\mathbf{t}(E)$

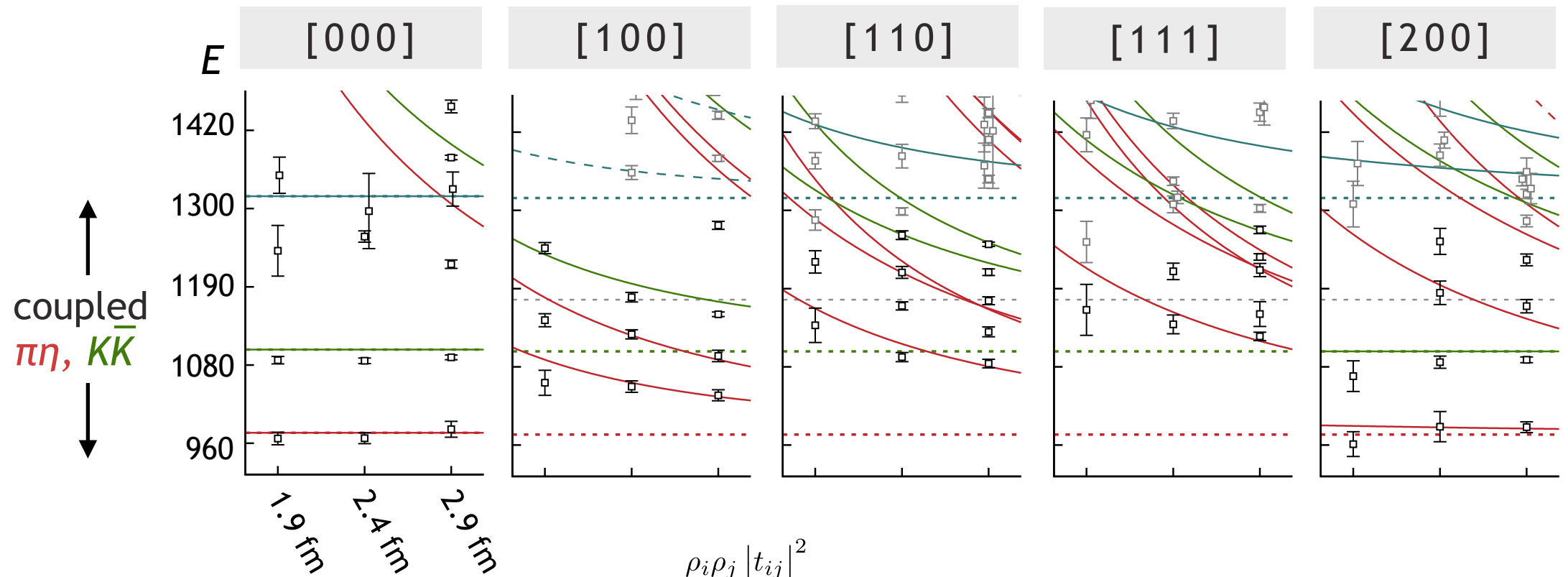
fit parameters by describing lattice spectrum

$m_\pi \sim 391$ MeV

↑
coupled
 $\pi\eta, K\bar{K}$
↓



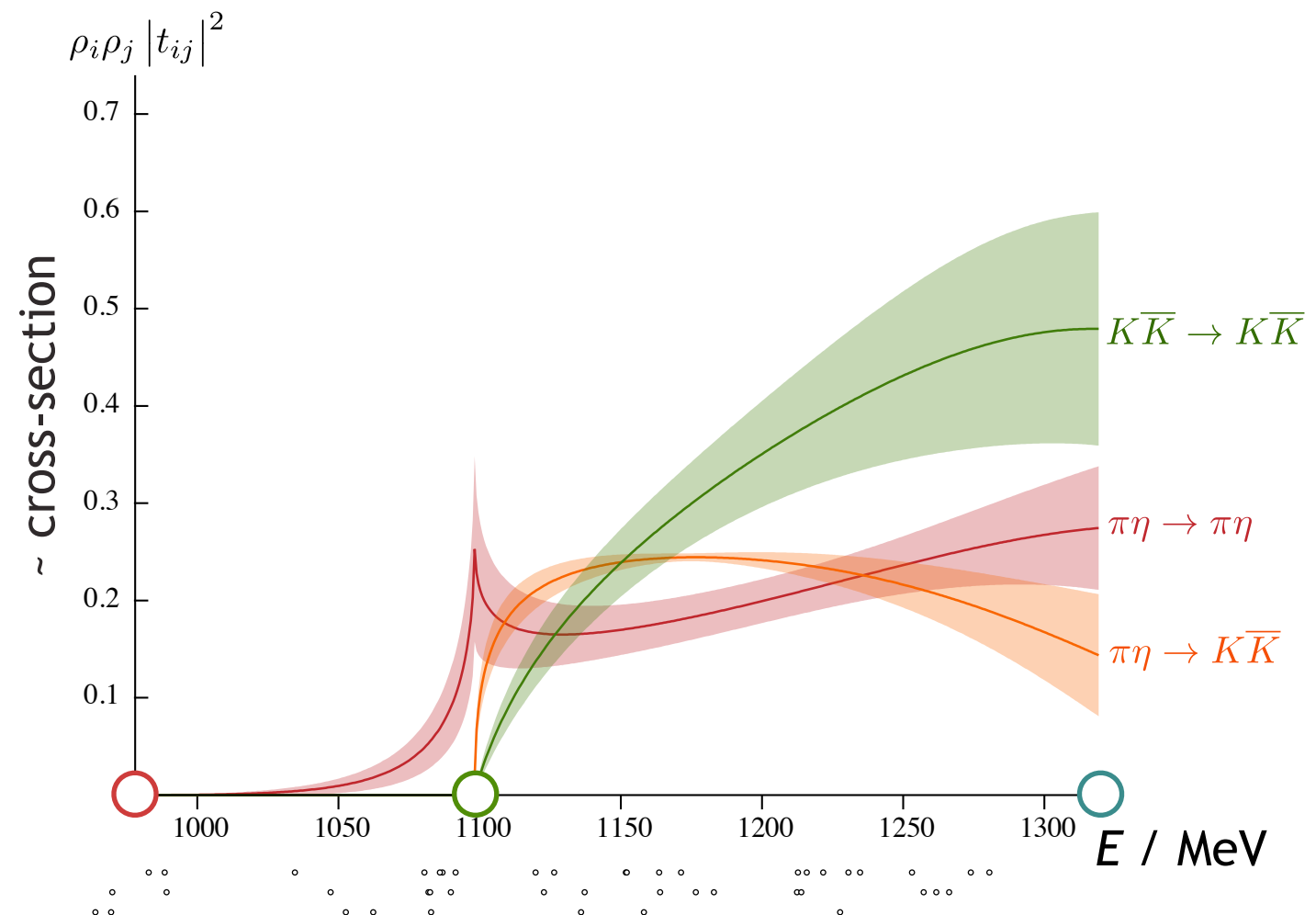
$m_\pi \sim 391$ MeV

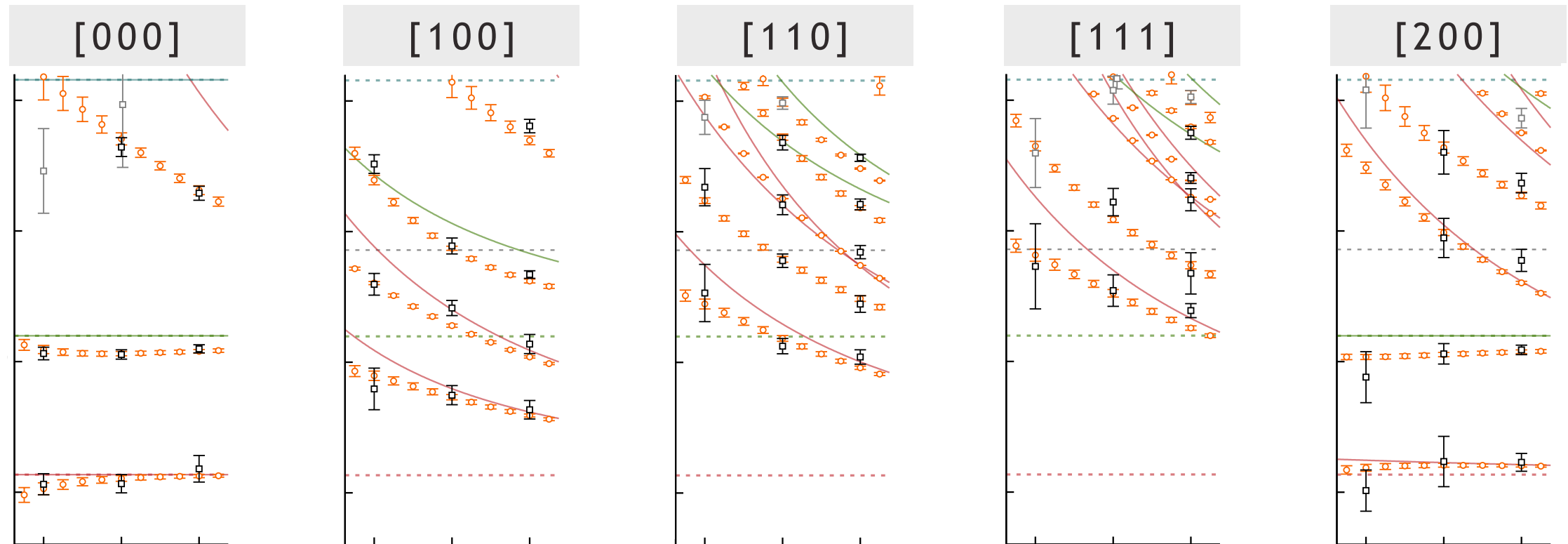


$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I} \quad \text{Im } \mathbf{I} = -\rho$$

$$\mathbf{K} = \frac{1}{m^2 - s} \begin{bmatrix} g_{\pi\eta}^2 & g_{\pi\eta}g_{K\bar{K}} \\ g_{\pi\eta}g_{K\bar{K}} & g_{K\bar{K}}^2 \end{bmatrix} + \begin{bmatrix} \gamma_{\pi\eta,\pi\eta} & \gamma_{\pi\eta,K\bar{K}} \\ \gamma_{\pi\eta,K\bar{K}} & \gamma_{K\bar{K},K\bar{K}} \end{bmatrix}$$

& Chew-Mandelstam phase-space



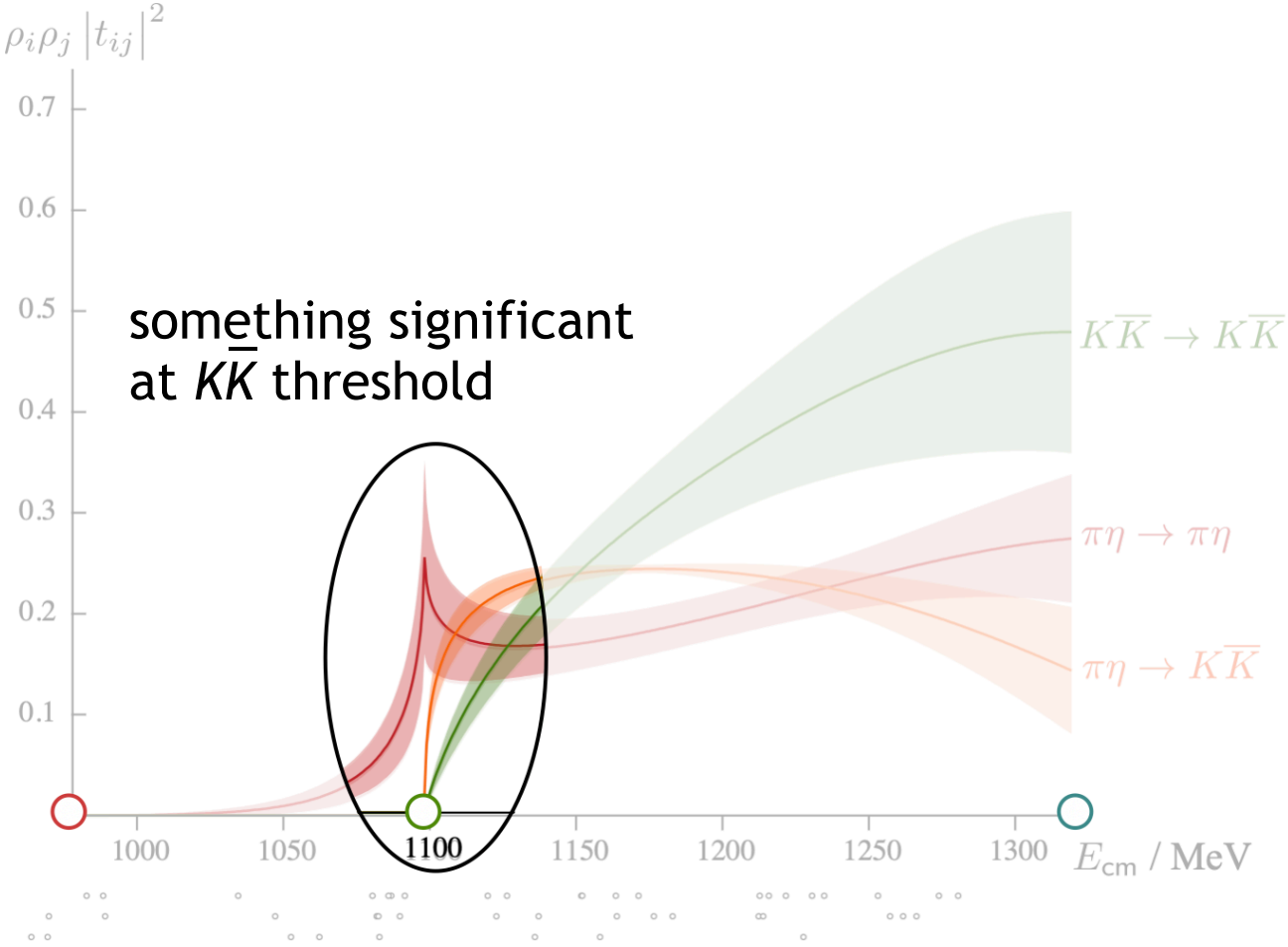


parameterized t-matrix describes the finite volume spectrum well $\chi^2/N_{\text{dof}} = \frac{58.0}{47 - 6} = 1.41$

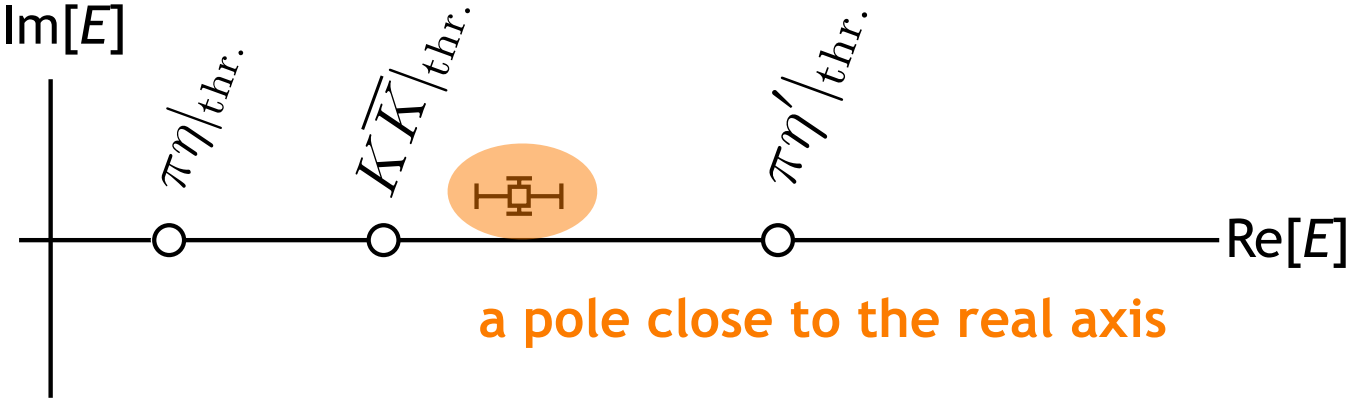
$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I} \quad \text{Im } \mathbf{I} = -\rho$$

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& Chew-Mandelstam phase-space



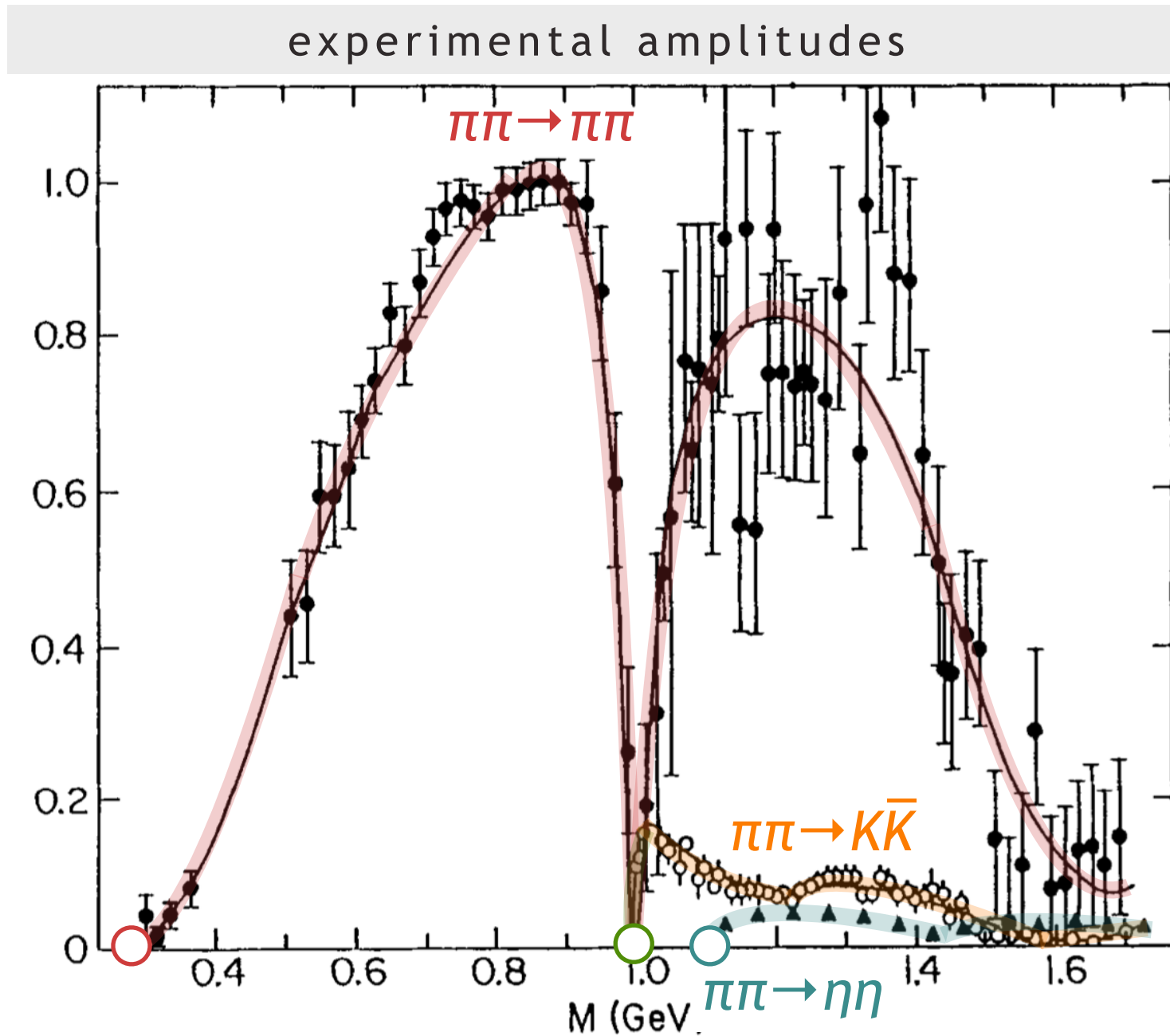
$$t_{ij}(s) \sim \frac{c_i c_j}{s_0 - s}$$
$$\sqrt{s_0} = m_R \pm \frac{i}{2}\Gamma_R$$



$$m_{a_0} = 1177(27) \text{ MeV}$$
$$\Gamma_{a_0} = 49(33) \text{ MeV}$$
$$\left| \frac{c_{K\bar{K}}}{c_{\pi\eta}} \right| = 1.3(4)$$

$m_\pi \sim 391 \text{ MeV}$

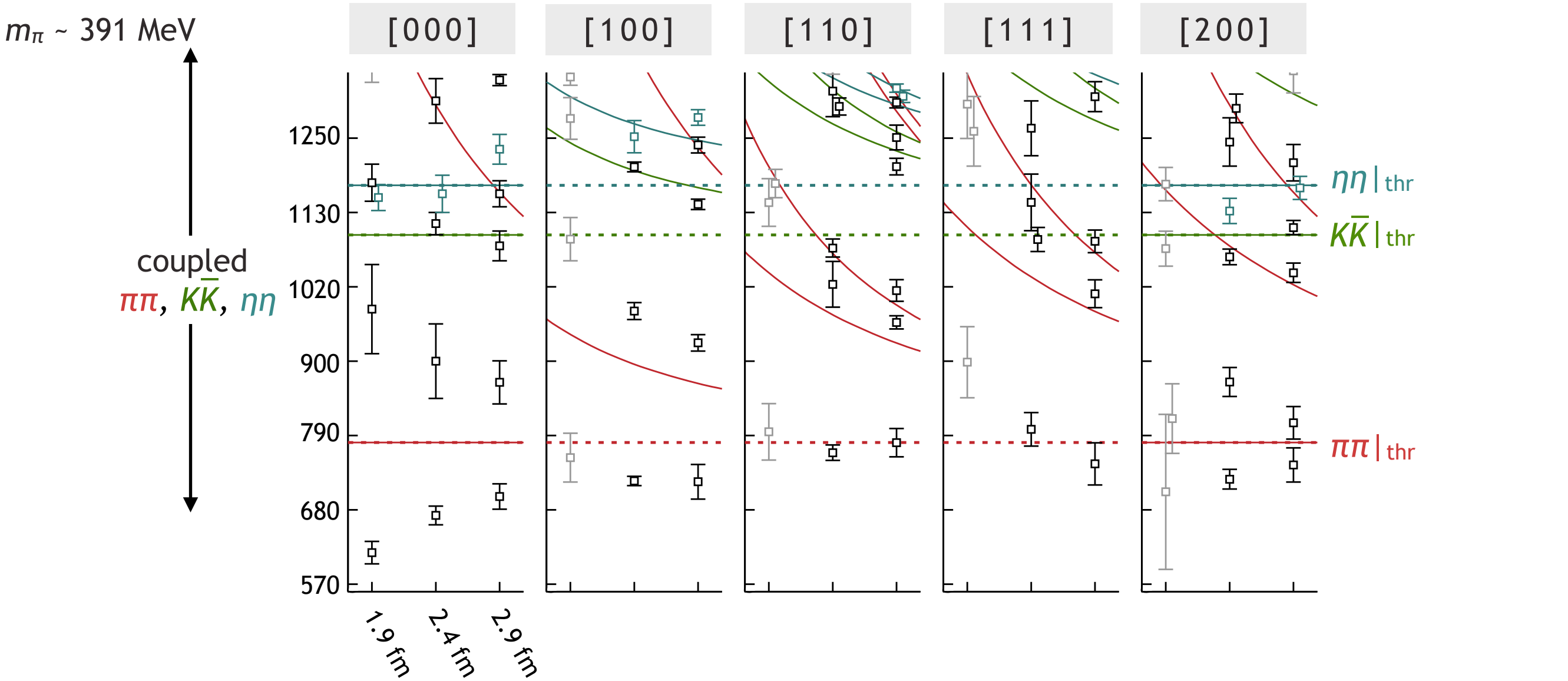
uncertainties include spread over different parameterizations

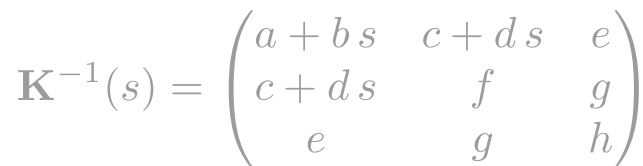


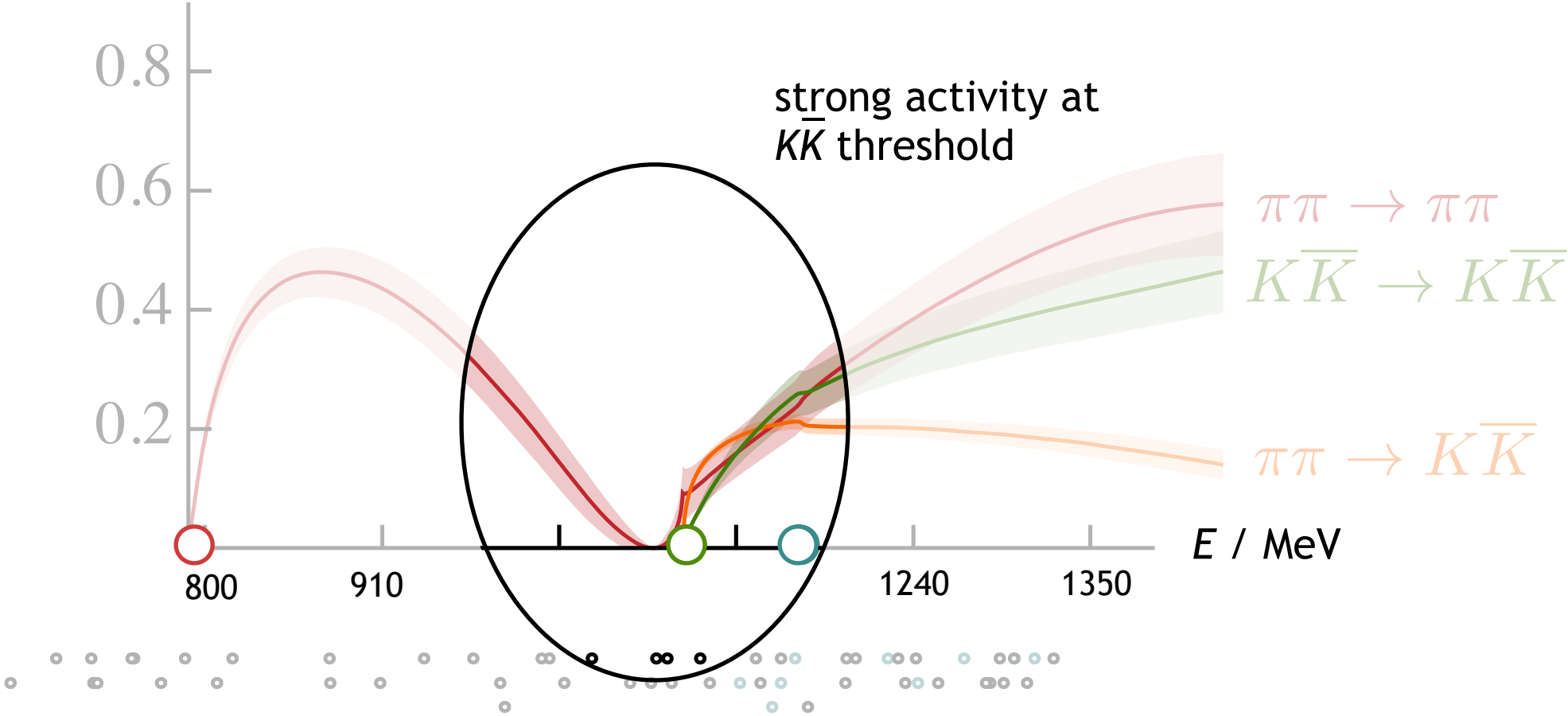
combination of broad σ resonance
and narrow $f_0(980)$ at $K\bar{K}$ threshold

two low lying resonances ...

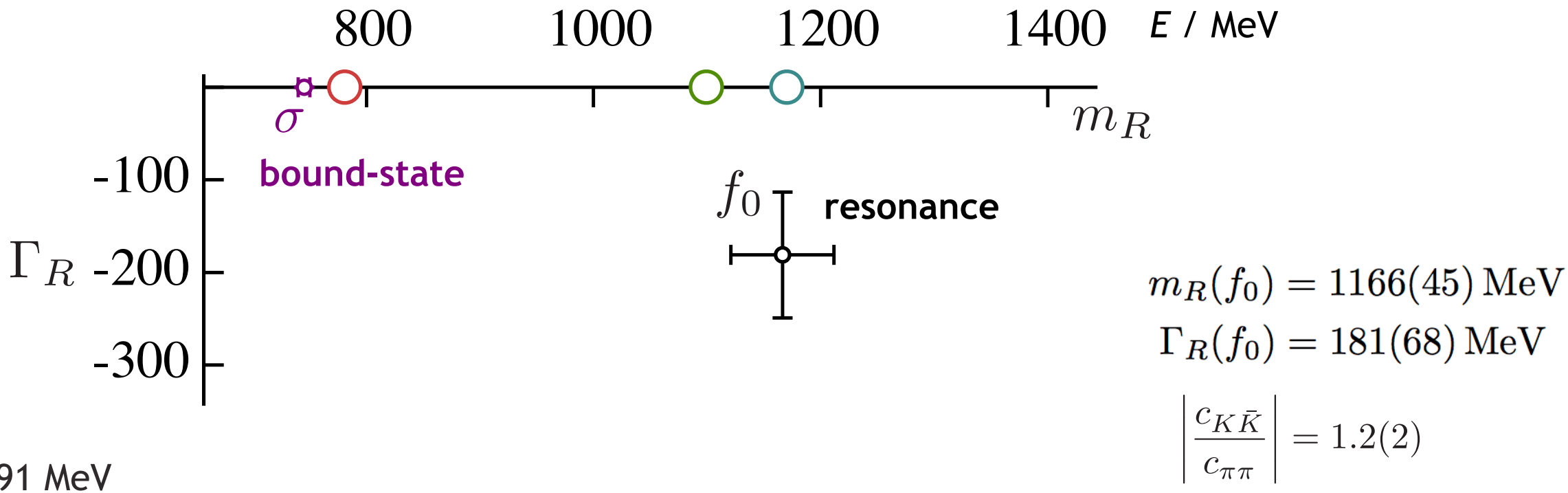
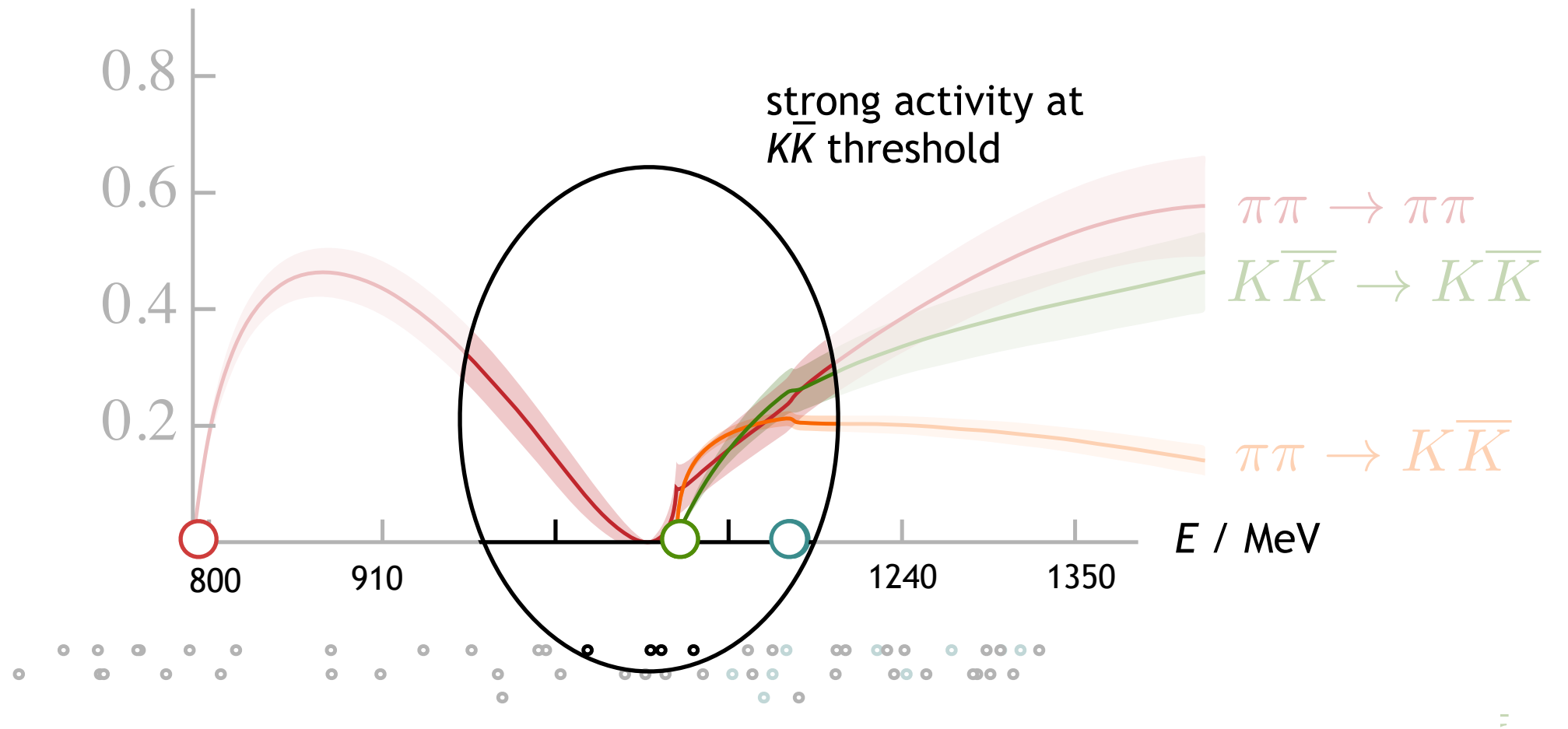
... start at a heavier quark mass ...





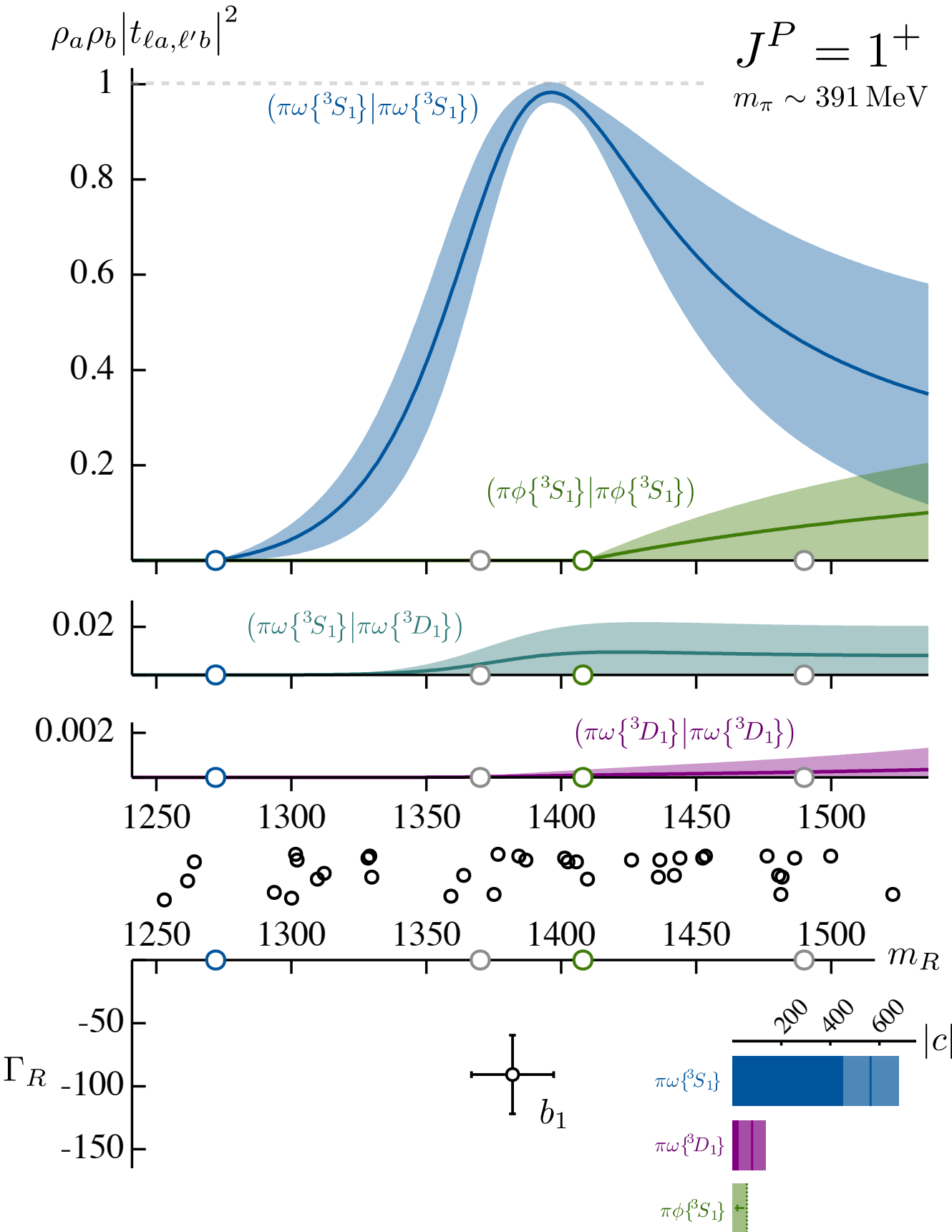


$m_\pi \sim 391 \text{ MeV}$



$m_\pi \sim 391 \text{ MeV}$

coupled
 $\pi\omega$ ($^3S_1, ^3D_1$), $\pi\phi$



clear b_1 resonance

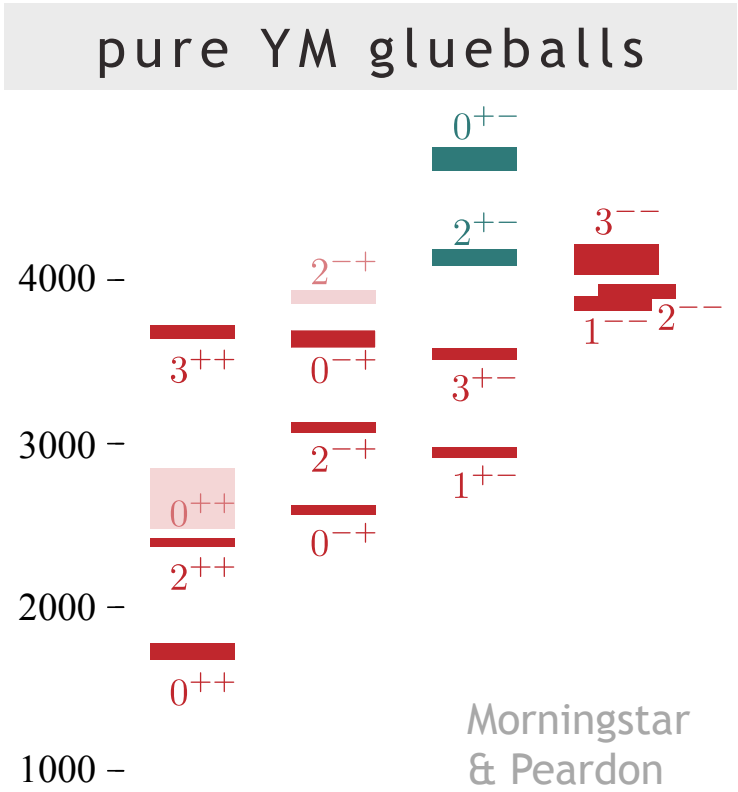
- strong $\pi\omega$ S -wave
- weak $\pi\omega$ D -wave
- negligible $\pi\phi$

so the technology is coming together, will soon be ready to handle complex cases like the $Z_c(3900)$

what about ‘exotic hadrons’ in the light sector ?

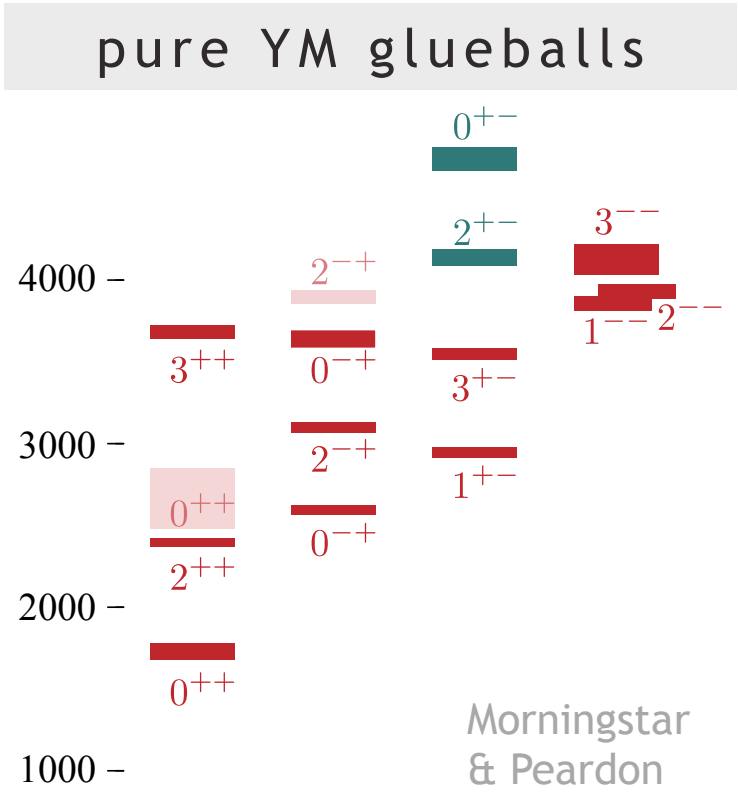
long been proposed to study glueballs in J/ψ radiative decay (“glue-rich”)

+ high quality
new data from BES III
as seen in Beijing’s plenary talk



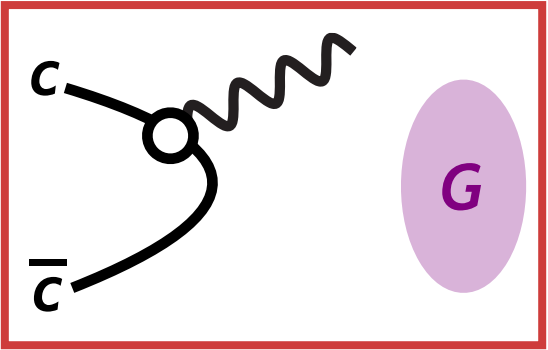
accessed using
Wilson loop operators

long been proposed to study glueballs in J/ψ radiative decay (“glue-rich”)

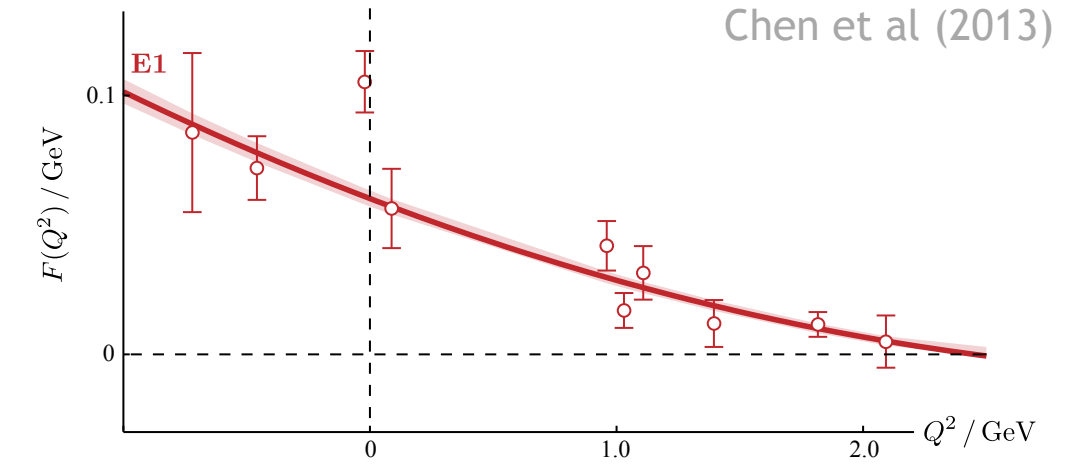


compute three-point correlation functions

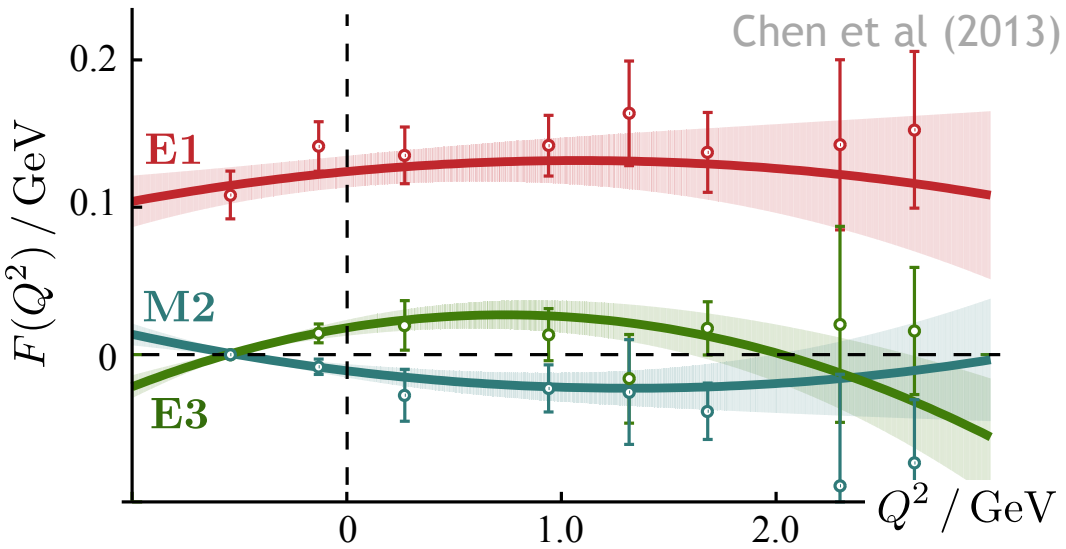
$$\langle 0 | \mathcal{O}_{J/\psi}(T) j_{\text{em}}^\mu(t) \mathcal{O}_G(0) | 0 \rangle$$



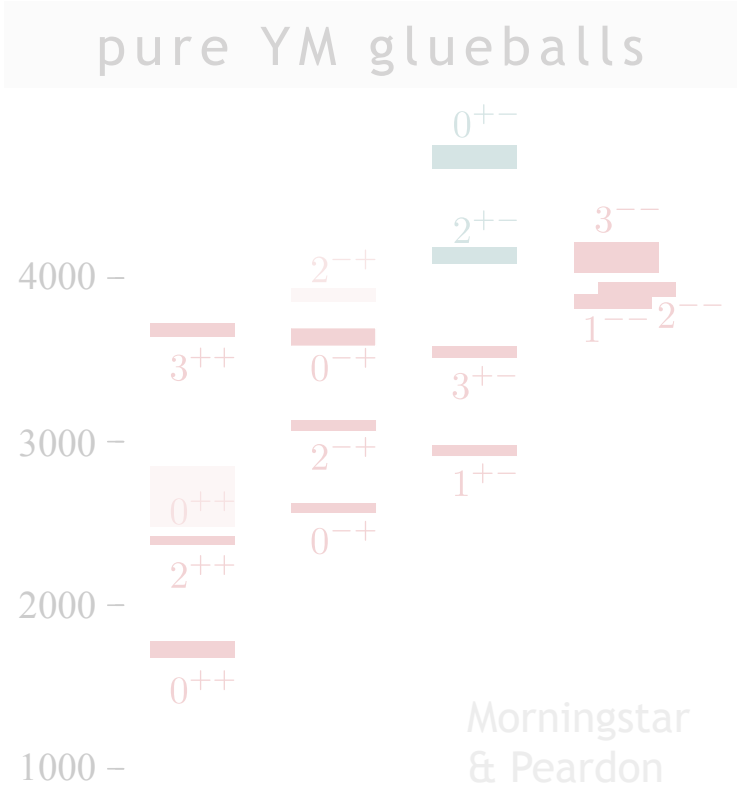
$J/\psi \rightarrow \gamma^* G[0^{++}]$



$J/\psi \rightarrow \gamma^* G[2^{++}]$



long been proposed to study glueballs in J/ψ radiative decay (“glue-rich”)

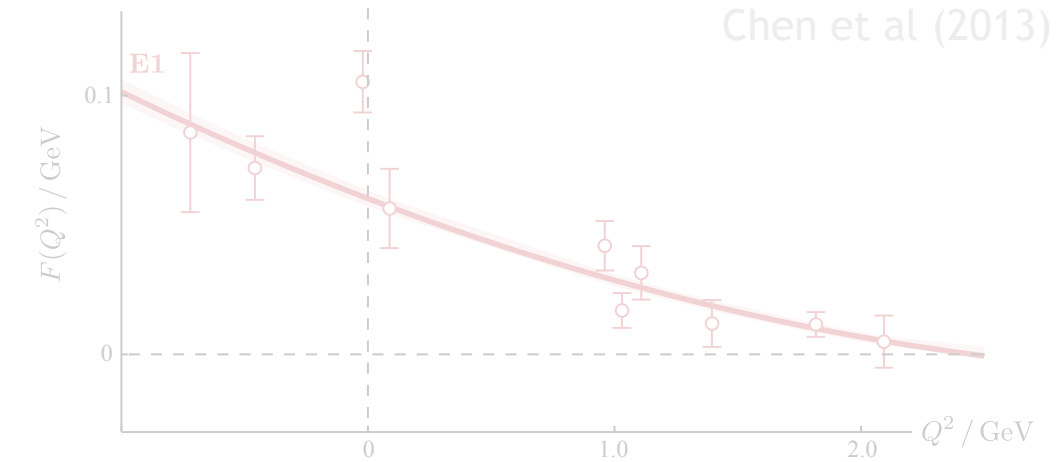


compute three-point correlation functions

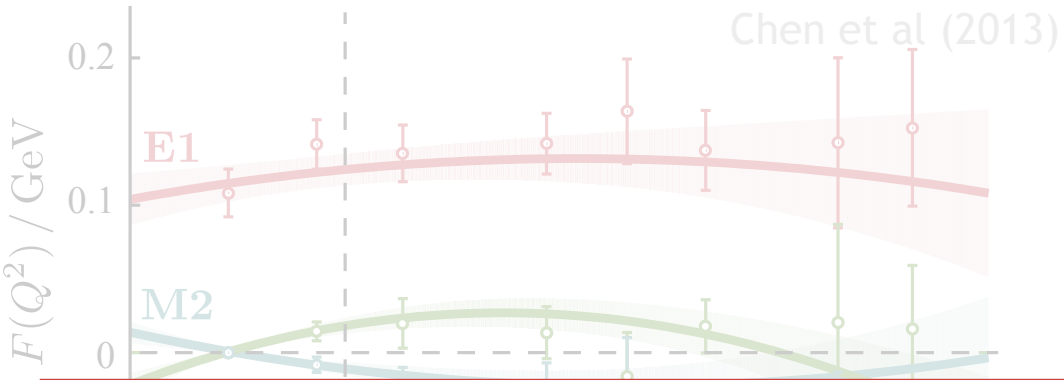
$$\langle 0 | \mathcal{O}_{J/\psi}(T) j_{\text{em}}^\mu(t) \mathcal{O}_G(0) | 0 \rangle$$



$$J/\psi \rightarrow \gamma^* G[0^{++}]$$



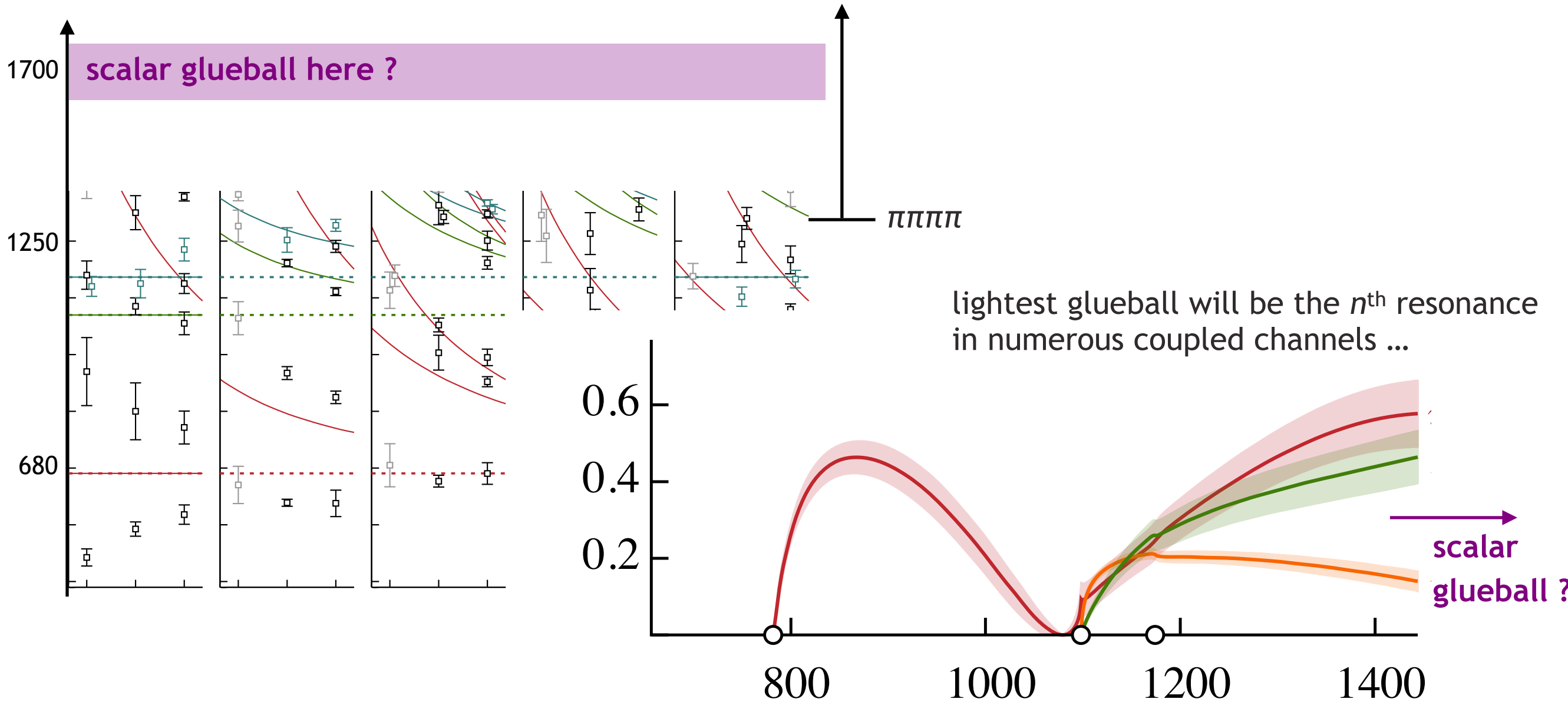
$$J/\psi \rightarrow \gamma^* G[2^{++}]$$



helpful to build intuition, but ...
this is in pure Yang-Mills — no light quarks !

as seen in Beijing’s plenary talk

glueballs in QCD are much more complicated beasts ...

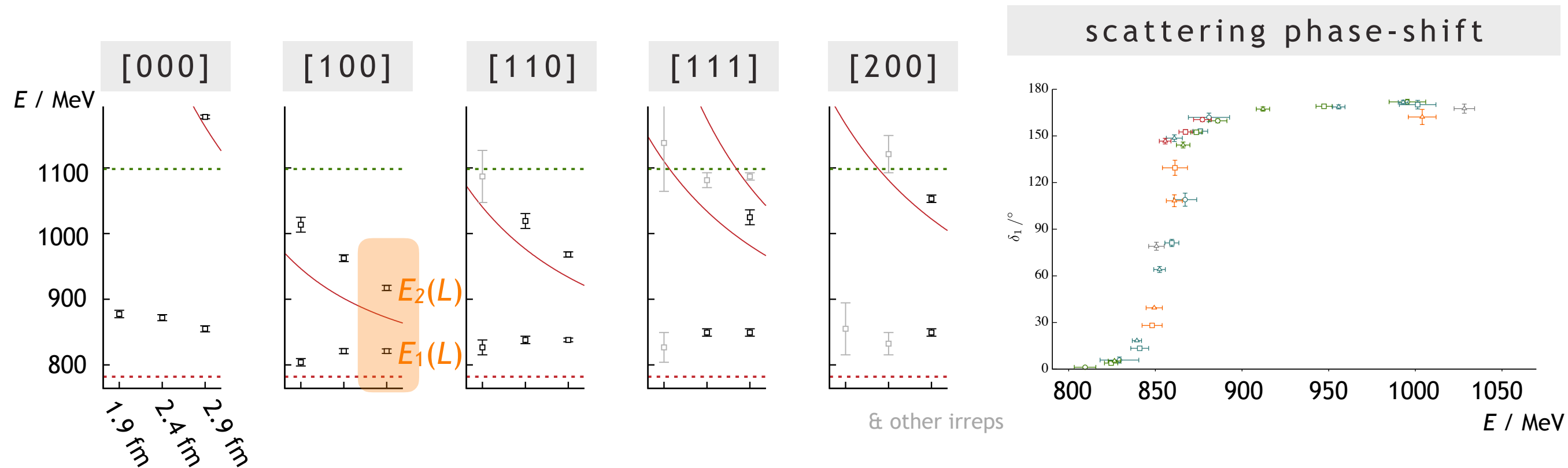


lightest glueball will be the n^{th} resonance in numerous coupled channels ...

and then we need to couple it to an external current ...

this is too tough to start with ...
... back up and try something simpler ...

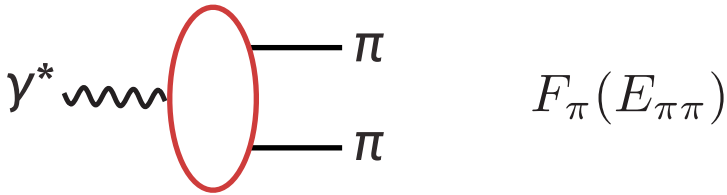
compute matrix elements for each finite-volume eigenstate
(no single one of which is the resonance)



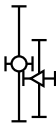
$$\text{e.g. } \langle 0 | j_{\text{em}}^\mu | E_n(L) \rangle \quad \text{or} \quad \langle \pi | j_{\text{em}}^\mu | E_n(L) \rangle$$

and there is a formalism which converts this
to an infinite volume amplitude as a function of $E_{\pi\pi} \dots$

$\gamma^* \rightarrow \pi\pi$

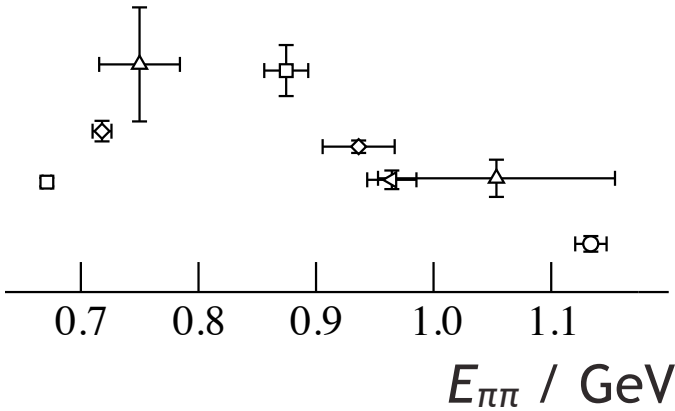


$F_\pi(E_{\pi\pi})$



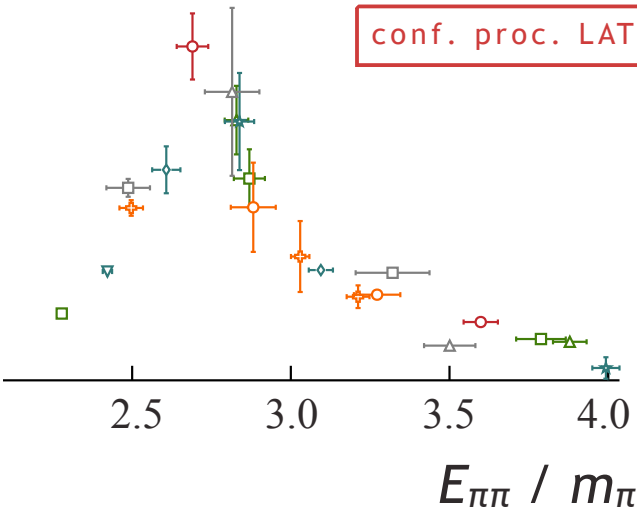
Feng et al | u,d,s | $m_\pi \sim 290$ MeV

PRD97 054513 (2018)

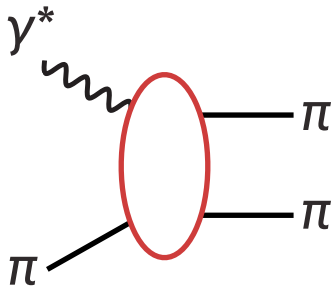


Bulava et al | u,d,s | $m_\pi \sim 280$ MeV

conf. proc. LAT '15



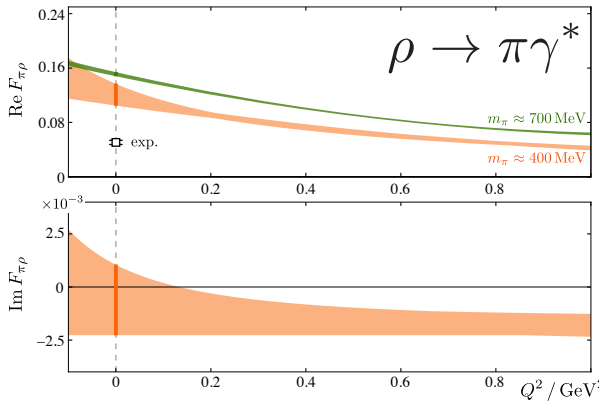
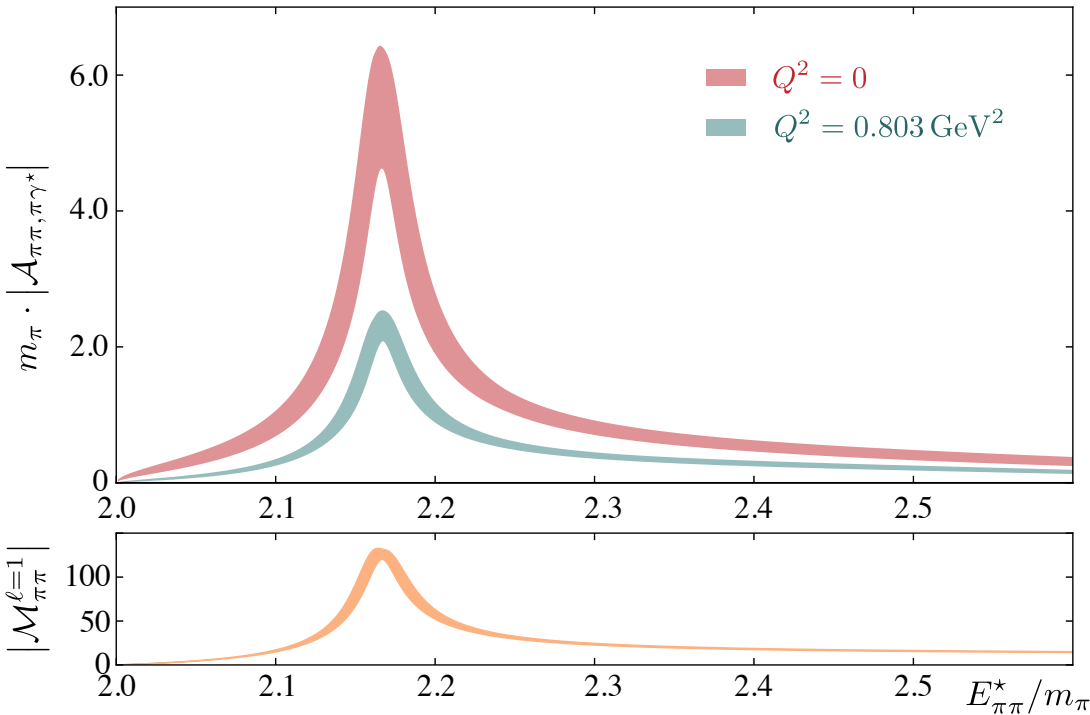
$\gamma^* \pi \rightarrow \pi\pi$

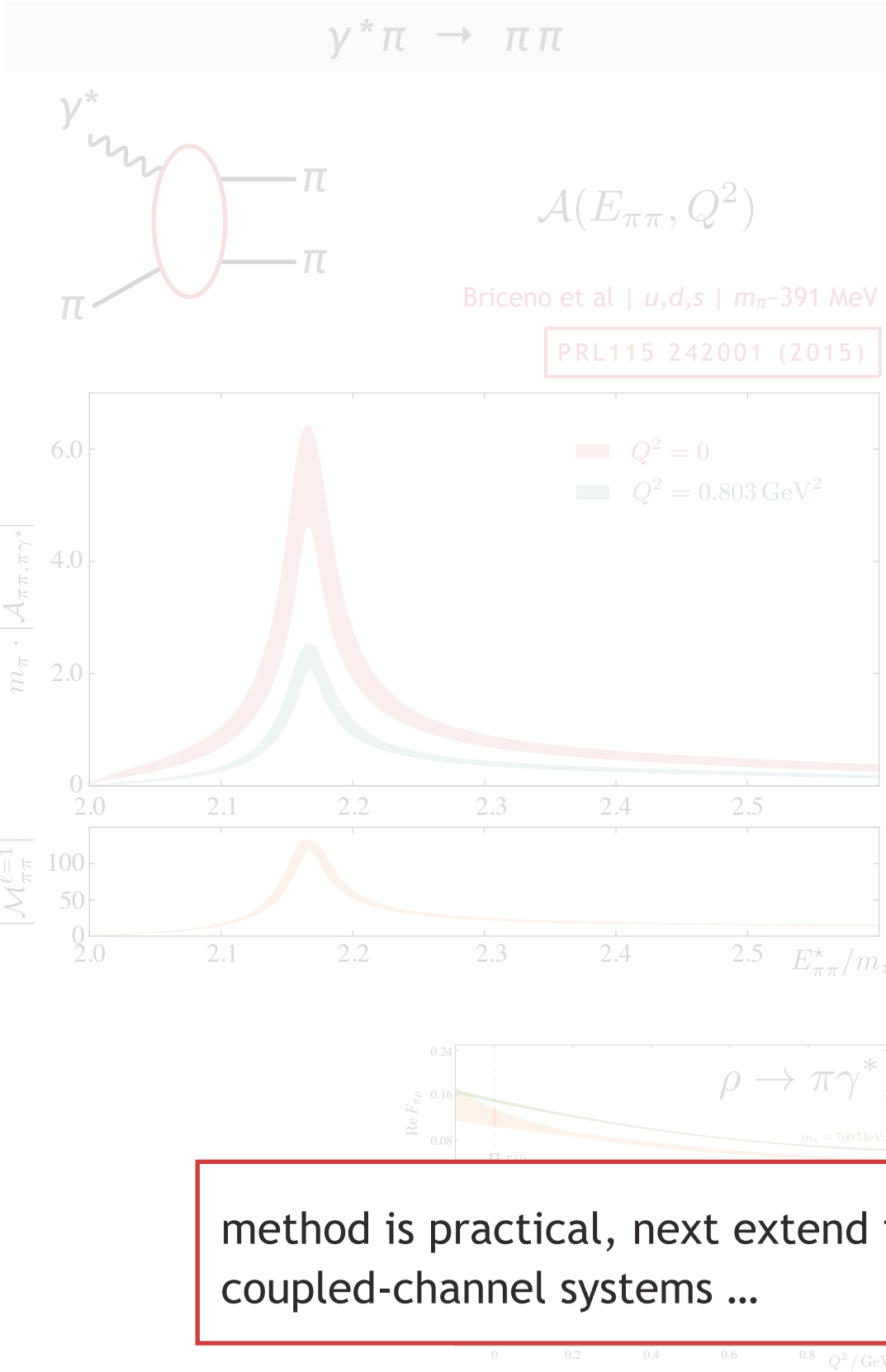
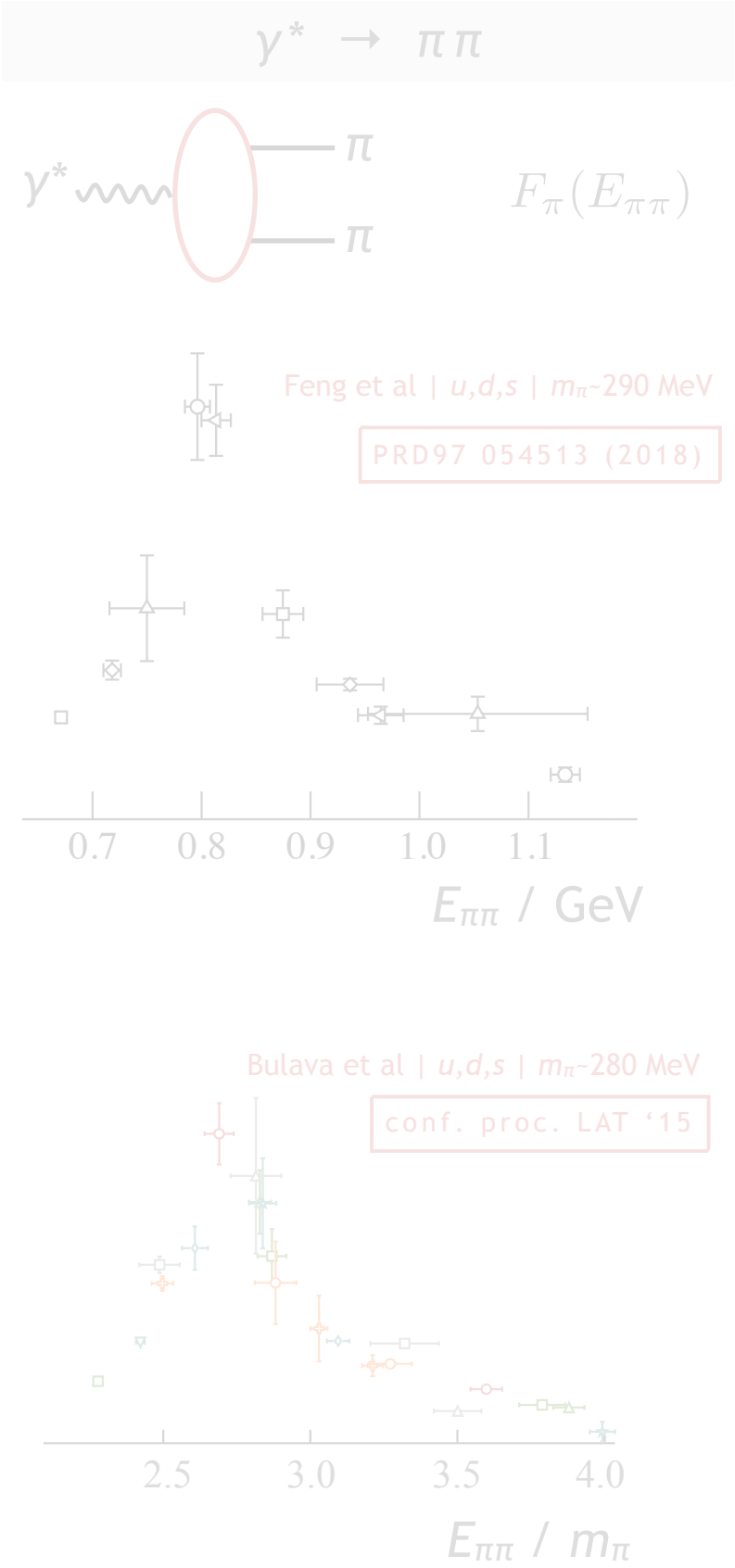


$\mathcal{A}(E_{\pi\pi}, Q^2)$

Briceno et al | u,d,s | $m_\pi \sim 391$ MeV

PRL115 242001 (2015)

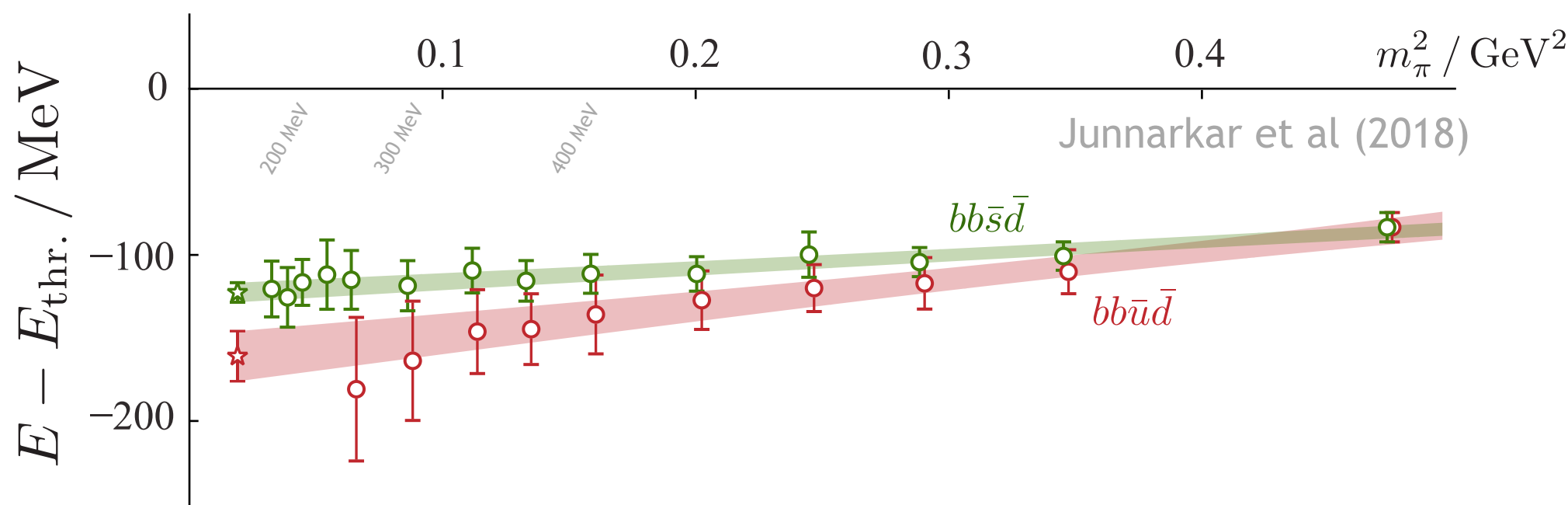




method is practical, next extend to coupled-channel systems ...

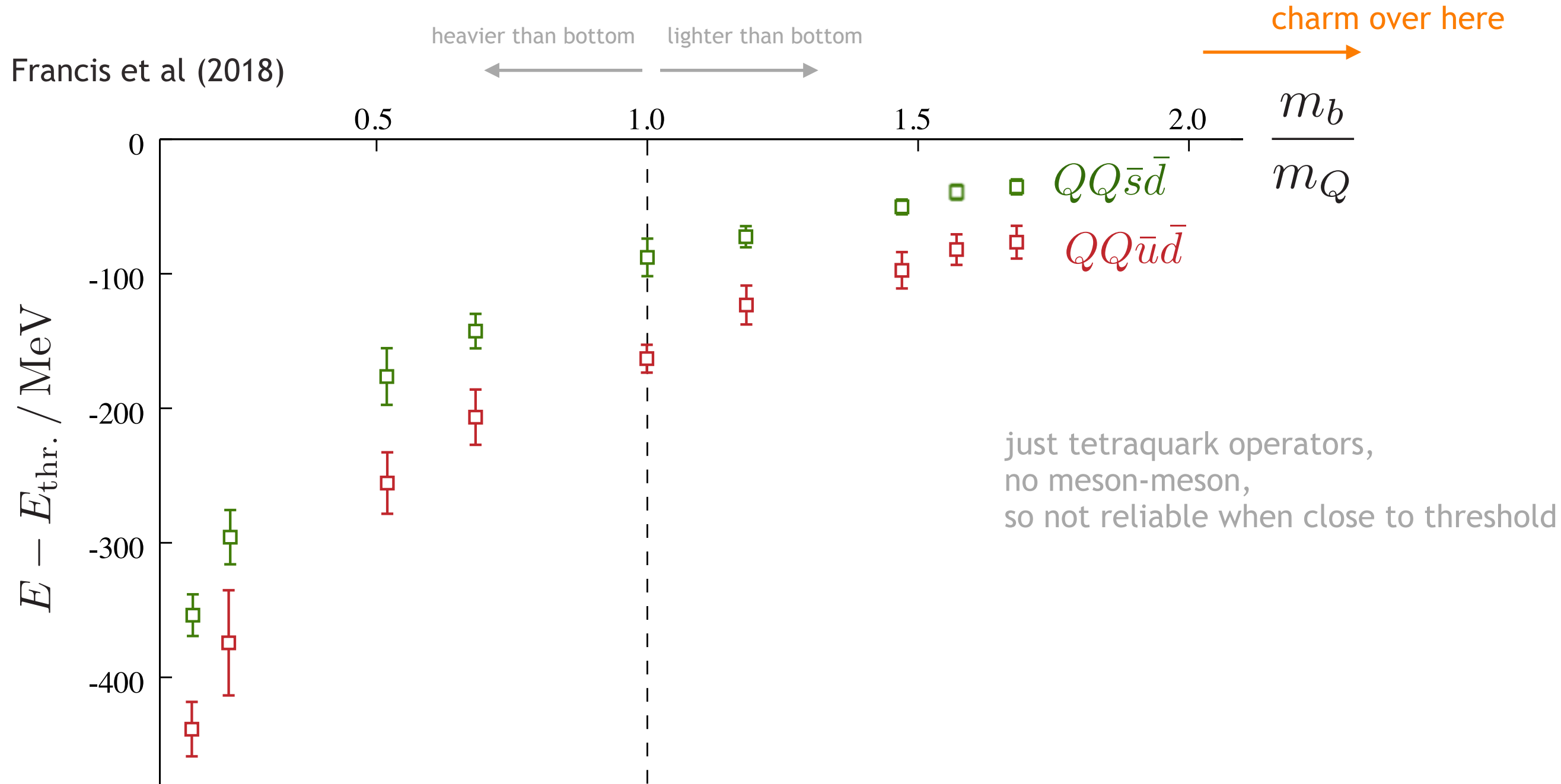
one recent observation in lattice QCD that has a good chance of being robust:

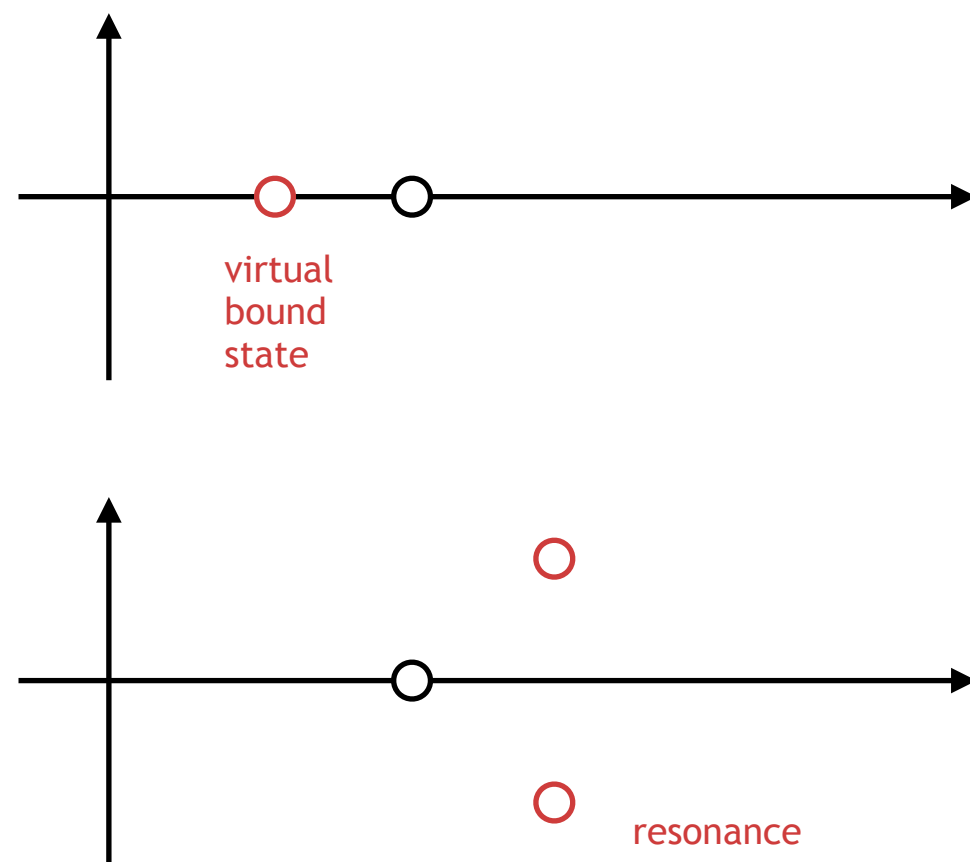
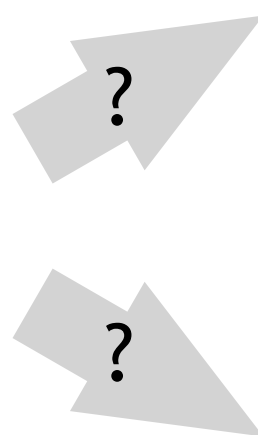
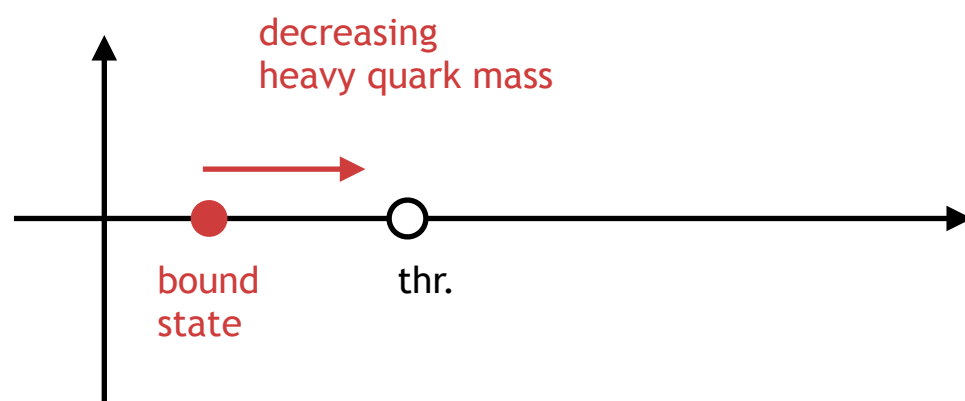
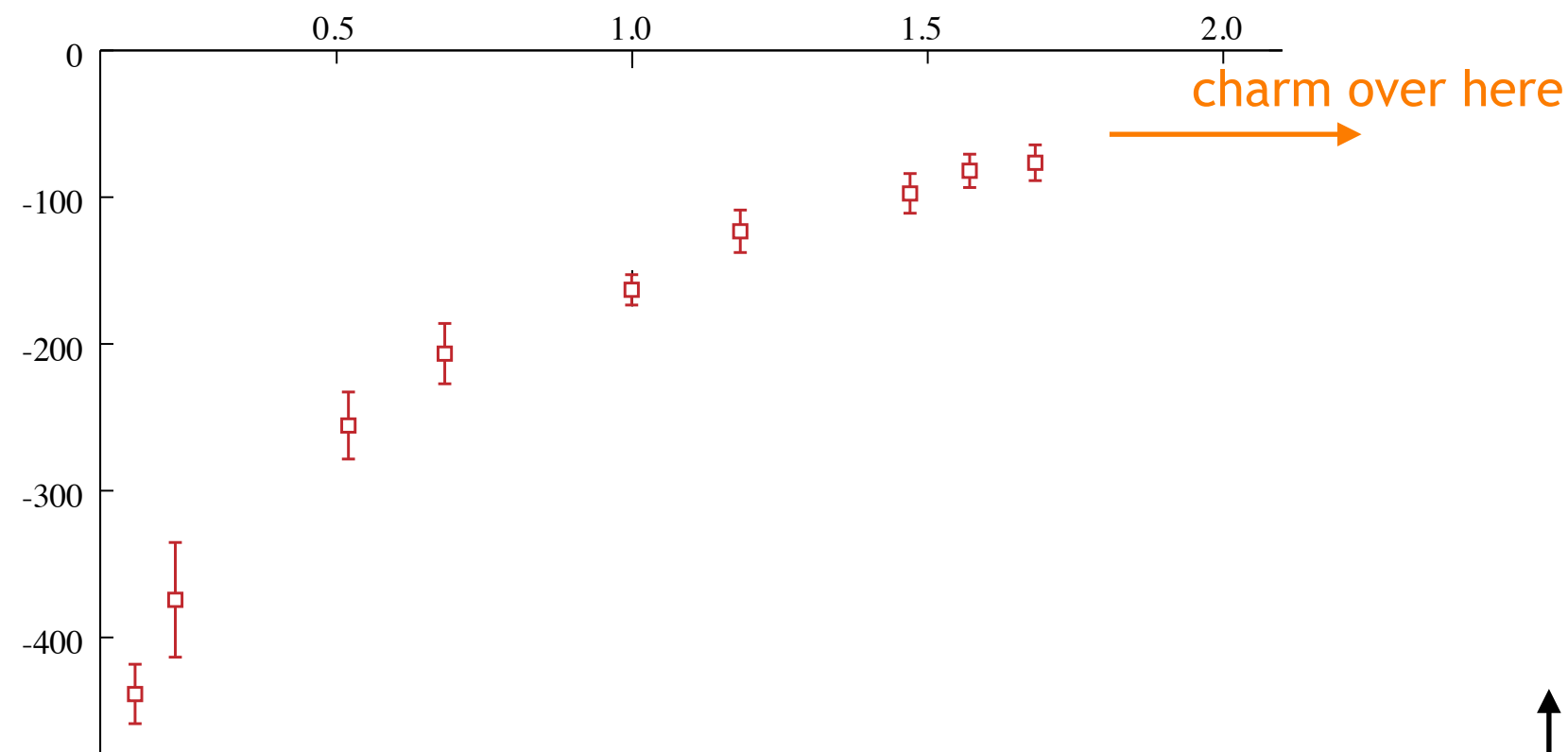
a double-bottom bound-state $bb\bar{u}\bar{d}$ ($I=0, J^P=1^+$, lying well below $B B^*$ threshold)
and probably a strange partner



Francis et al (2017) $|\Delta E(bb\bar{u}\bar{d})| \sim 100 - 200 \text{ MeV}$
Junnarkar et al (2018) $|\Delta E(bb\bar{s}\bar{d})| \sim 90 - 120 \text{ MeV}$
Leskovec et al (2018)

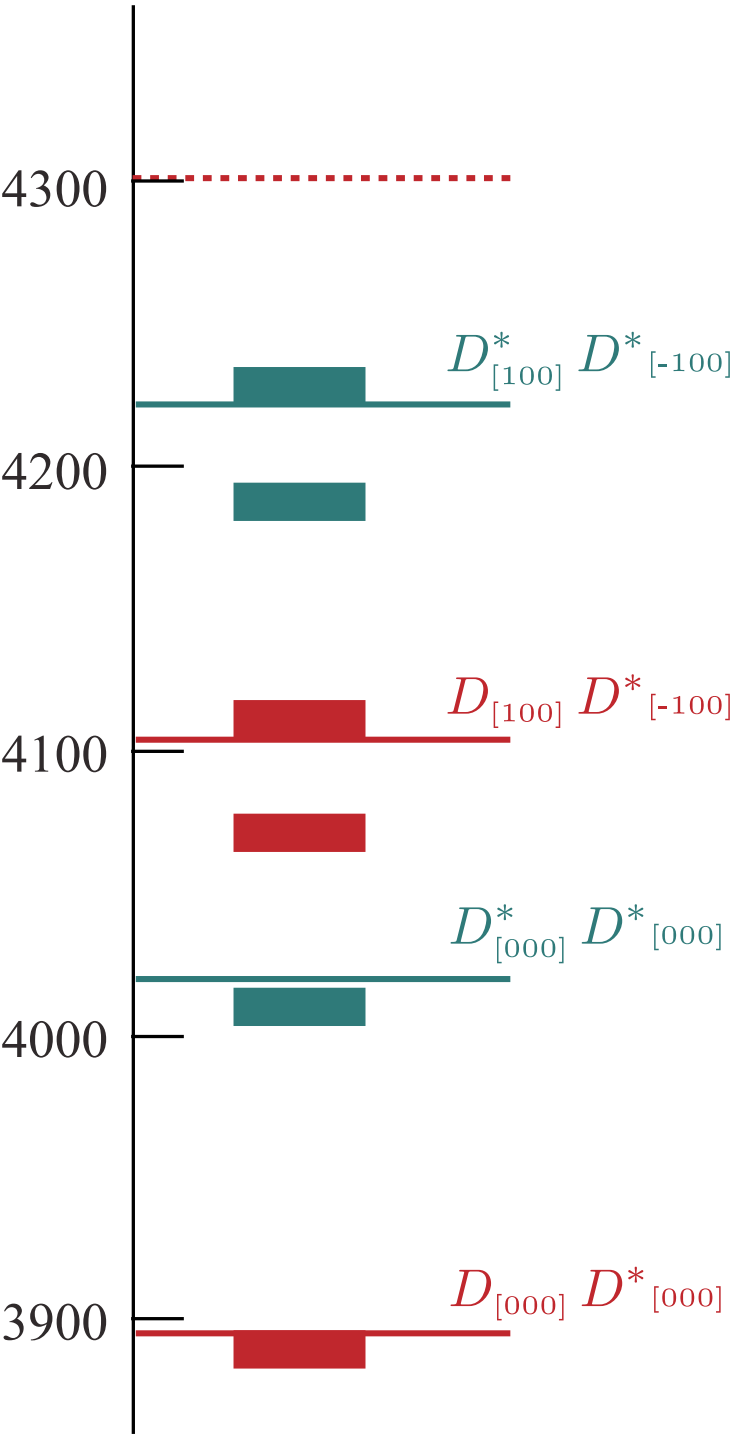
see Eichten & Quigg (2017) for heavy quark symmetry argument





tetraquark operators
& meson-meson operators

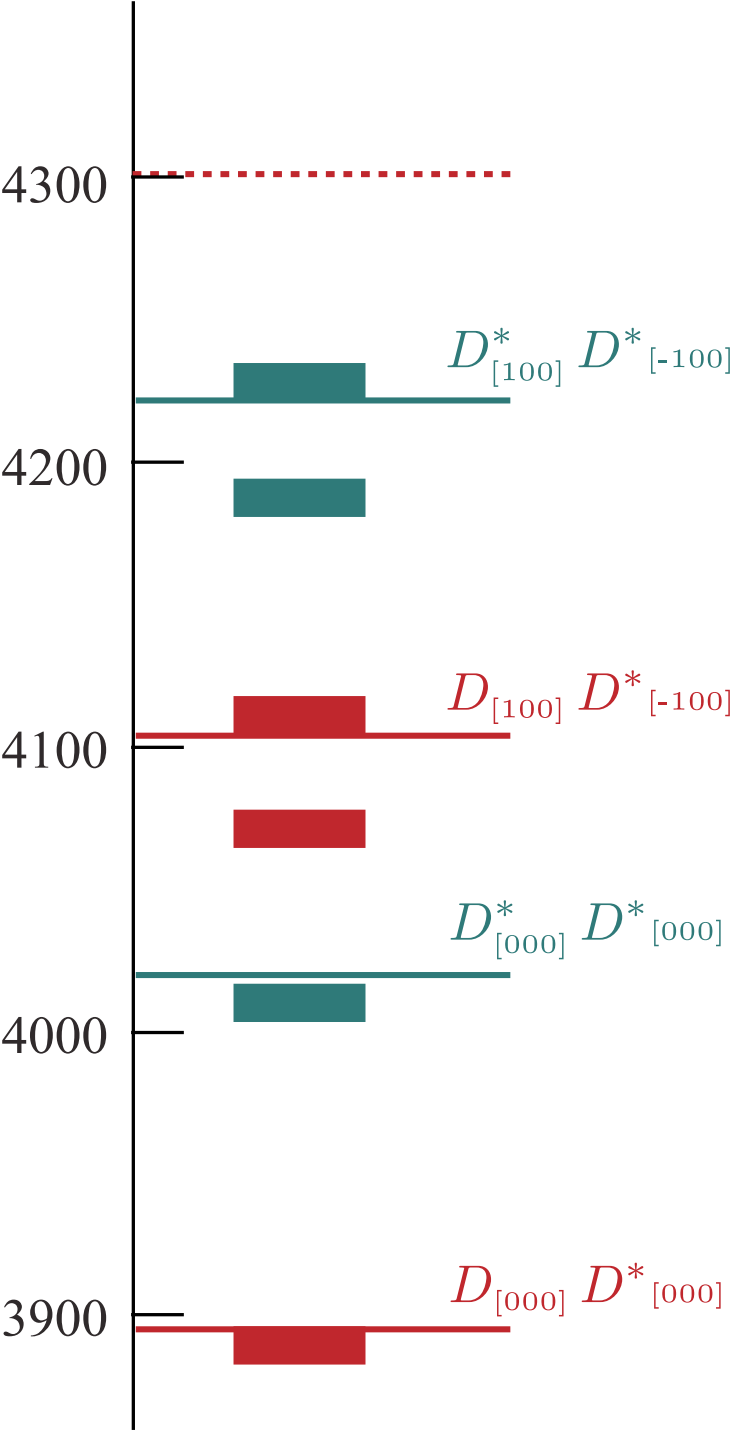
$m_\pi \sim 390$ MeV,
 $L \sim 2$ fm



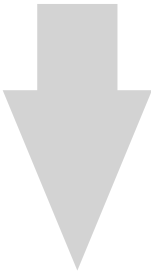
no obvious sign of a narrow resonance ...

tetraquark operators
& meson-meson operators

$m_\pi \sim 390$ MeV,
 $L \sim 2$ fm



just tetraquark ops



completely misleading spectrum

BE CAREFUL WITH SMALL OPERATOR BASES !

technology exists now in lattice QCD to determine properties of excited states

but it needs to be applied carefully

- avoid jumping to conclusions obtained from incomplete calculations
- a systematic approach starts with the simplest resonances and works up to more exciting cases

elastic scattering now well studied, especially $\rho \rightarrow \pi\pi$

a few **coupled-channel cases** have appeared

formalism to **couple resonances to currents** has been applied

$\rho, K^*, \sigma, \kappa, f_0, a_0, f_2, a_2, b_1$
also some D -mesons, ψ' ...

initial suggestions that **double-bottom, isospin=0 channel** might house a **QCD-stable bound state**

looks unlikely that the double-charm analogue is bound or resonant, but more calculation needed

limitation:

formalism to handle three-body decays in development

— needed to go to physical pion mass