Lattice QCD:
"From the 12 GeV to the Exascale & EIC Eras"

Lecture 2

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Temple University

HAMPTON UNIVERSITY GRADUATE STUDIES PROGRAM (HUGS 2019)

THOMAS JEFFERSON NATIONAL ACCELERATOR FACILITY

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OUTLINE OF LECTURE 2

★ **Renormalization within Lattice QCD**
  - perturbatively
  - non-perturbatively

★ **Hadron spectroscopy**

★ **Key points of Lecture 1**

Renormalization

★ Lattice QCD is renormalizable, thus QCD must recover upon continuum limit (removal of regulator)

★ Lattice regularization has a consequence of that (bare) lattice quantities depend on lattice spacing, a

★ However, physical quantities cannot depend on regulator, thus bare quantities must be tuned with a, so that observables are not affected

★ Renormalization:

- UV divergences must be removed prior continuum limit
- Divergences canceled by adjusting the parameters of the action
- physical results are expressed via measurable parameters

(not via parameters in bare Lagrangian)

Renormalization Group Equation

★ **Let $\mathcal{O}$ be a measurable lattice quantity with mass dimension, $d_{\mathcal{O}}$, and in dimensionless form is written as $\hat{\mathcal{O}}$**

★ **Existence of continuum limit:**

$$\mathcal{O}(g_0(a), a) = \frac{\hat{\mathcal{O}}}{a^{d_{\mathcal{O}}}}, \qquad \lim_{a\to 0} \mathcal{O}(g_0(a), a) = \mathcal{O}_{phys}$$

★ **Close to the continuum limit (*a~0*): $\hat{\mathcal{O}} = a^{d_{\mathcal{O}}}\, \mathcal{O}_{phys}$**

and we can determine g_0 as a function of a measurable quantity and *a*

★ **Thus, a global $g_0(a)$ is expected for *a~0,* applicable to all quantities**

★ **Good quantity is quark antiquark static potential, for a pair separated by distance *R* (physical units), and on lattice**

$$V(R, g_0(a), a) = \frac{1}{a}\hat{V}(\frac{R}{a}, g_0(a))$$

Despite the variation of *a*, $V(R, g_0(a), a)$ must be invariant:

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$$a \frac{d}{da} V(R, g_0(a), a) = 0$$

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RGE

★ **RGE gives a definition for the Callan-Symanzik β-function (lattice)**

$$\beta_L(g_0) = -a\frac{\partial g_0}{\partial a}\bigg|_{g,\bar{\mu}}$$

$\bar{\mu}$: renormalization scale

$g\ (g_0)$: renormalized (bare) coupling

★ **β_L-function dictates the relation between g_0 and a**

★ **β-function expanded in terms of g_0 (asymptotic freedom):**

$$\beta_L(g_0) = -b_0 g_0^3 - b_1 g_0^5 - b_2^L g_0^7 + \mathcal{O}(g_0^9)$$

$$\beta(g) = -b_0 g^3 - b_1 g^5 - b_2 g^7 + \mathcal{O}(g^9)$$

$$b_0 = \frac{1}{16\pi^2}\left(11 - \frac{2}{3}N_F\right)$$

$$b_1 = \frac{1}{(16\pi^2)^2}\left(102 - \frac{38}{3}N_F\right)$$

b_0 (LO) and b_1 (NLO) universal, beyond NLO depend on regulator

Renormalization

★ Calculation of physical quantities directly on the lattice does not require renormalization (e.g., hadron masses)

★ Renormalization necessary when one cannot access physical quantities directly (e.g., Form Factors)

★ In most cases renormalization is multiplicative (absence of mixing)

$$\psi^R = Z_\psi^{1/2} \psi^{bare}\,, \quad A^R = Z_A^{1/2} A^{bare}\,, \quad (\bar{\psi}\Gamma\psi)^R = Z_\Gamma^{1/2} (\bar{\psi}\Gamma\psi)^{bare}$$

★ Renormalization procedure not unique:

- Schroedinger functional
- non-perturbatively in numerical simulations
- perturbatively to some order in g_0^2.
- gradient flow
- Ward Identities

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Fermion bilocal operator with Dirac structure Γ

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Non-perturbative Renormalization

★ Preferred for purely non-perturbative estimates (low-energy sector of QCD)

★ Captures the diverging behavior of matrix elements to renormalize

★ Widely used scheme: RI-type (regularization independent)

★ Typically two important choices to make:
- renormalization scale μ
- renormalization scheme

(exceptions include renormalization of vector and axial-vector currents)

★ Results are converted to a common scheme and scale

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I will return to this topic in Lecture 3 (relevant to hadron structure)

Lattice Perturbation Theory

- ★ Lattice formulation extensively used for study of non-perturbative region

- ★ Perturbation theory is also applicable on the lattice
  (small-coupling expansion in the weak-coupling regime)
  Extraction of α_{strong}, β-function, etc

- ★ Lattice pert. theory very useful for computing renormalization functions
  (especially when there is mixing between operators)

- ★ Lattice pert. theory can be used to improve non-perturbative estimates
  (subtraction of finite-a effects)

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I will give selected examples demonstrating the power of lattice pert. theory

What should we first study in Lattice QCD?

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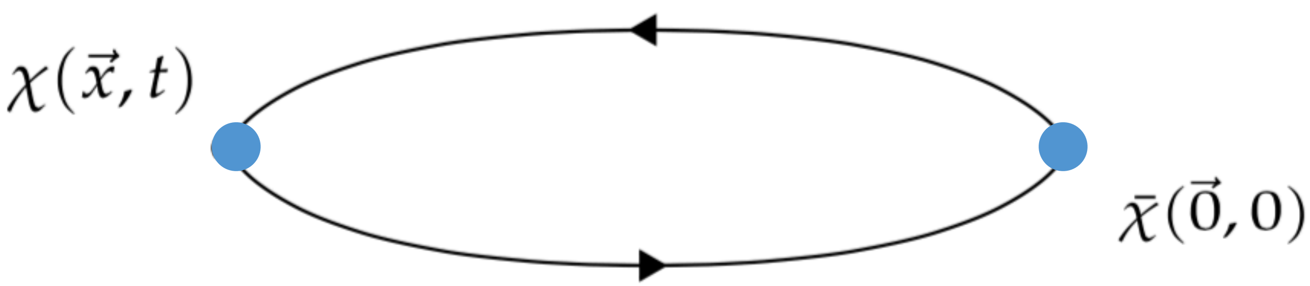
Start from quantities that are (relatively) easy to compute, and can be compared against experimental data

First goals of Lattice QCD

Reproduce the low-lying spectrum

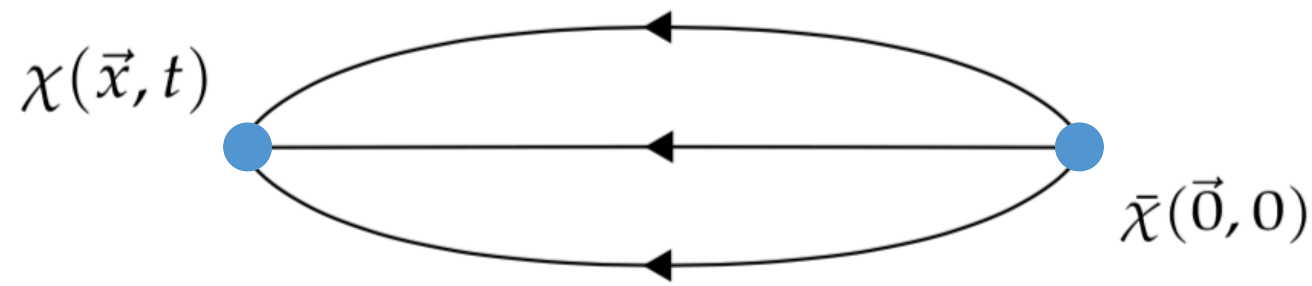
Mesons

e.g. pion, kaon



Baryons

e.g. proton



$$= G(\vec{y},t_y;\vec{x},t_x)$$

Quark propagator

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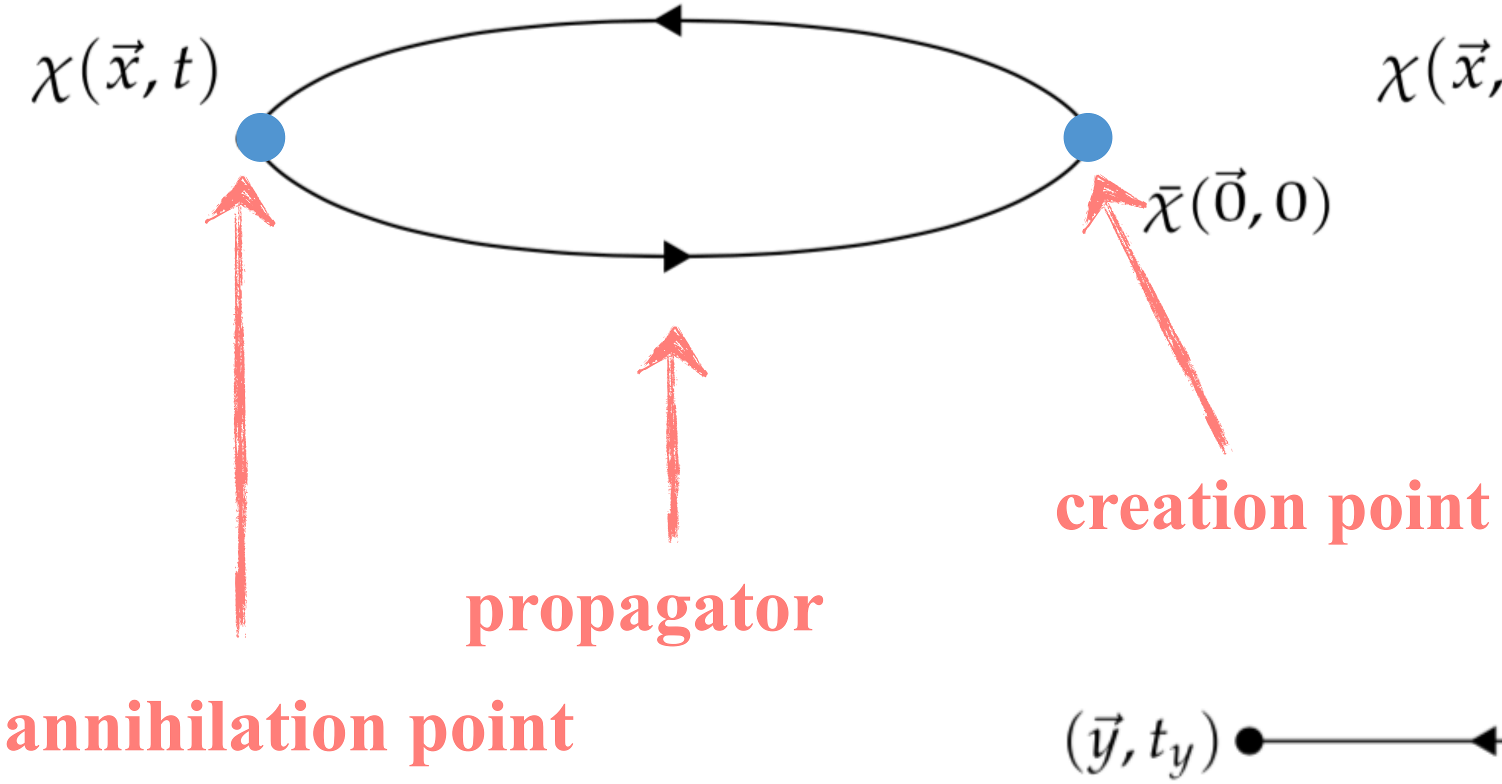

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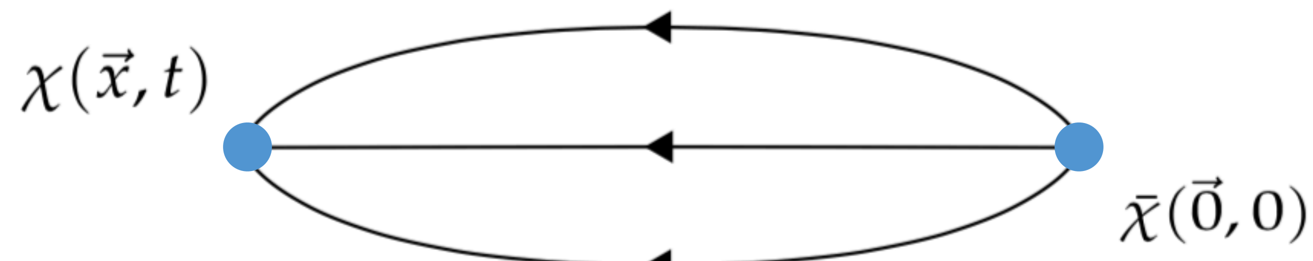

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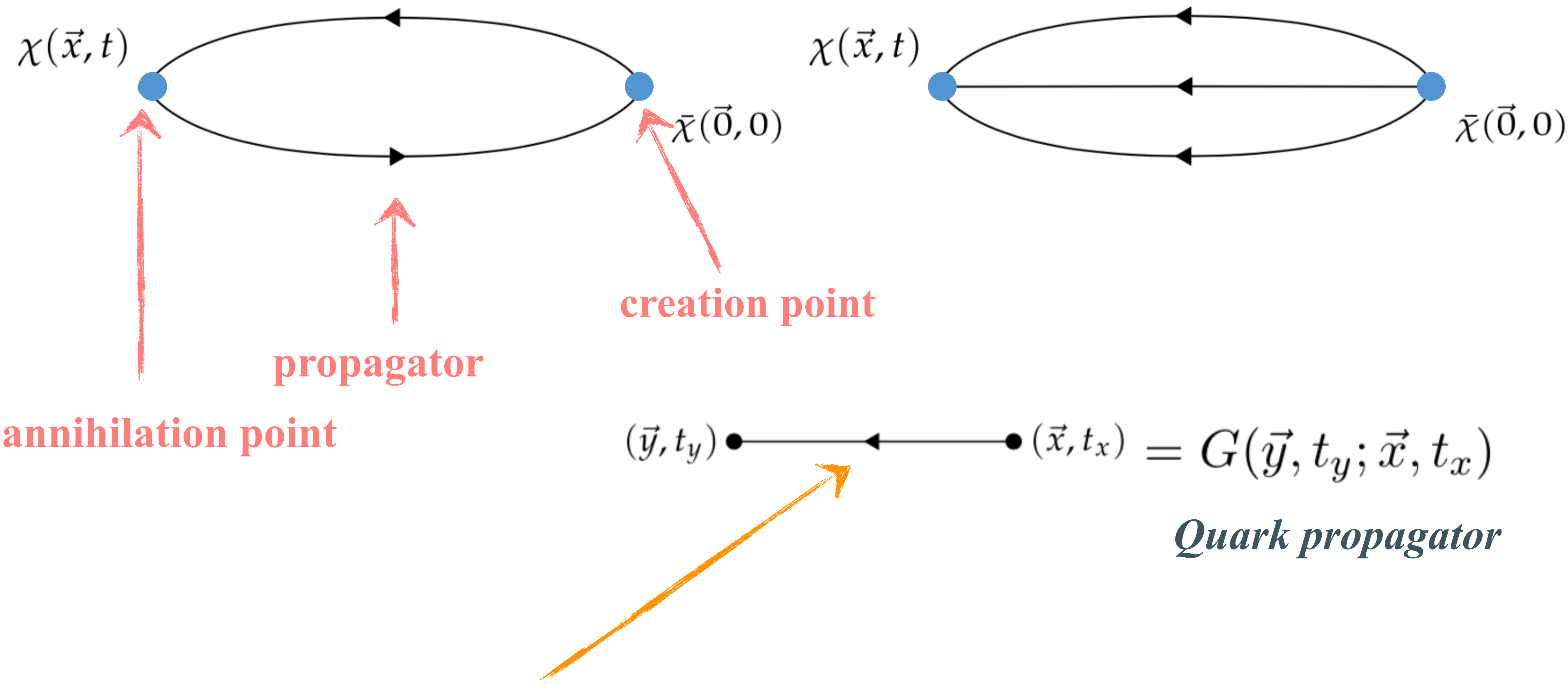

Most costly part of calculation

Calculation of Hadron mass

Extraction of a hadron's mass from its propagator:

★ **Two-point correlator (hadron level, Heisenberg picture):**

$$C(t) = \sum_{\vec{x}} \langle\Omega|\, \chi(\vec{x},t)\bar{\chi}(\vec{0},0)\,|\Omega\rangle = \sum_{\vec{x}} \langle\Omega|\, e^{-i\hat{\vec{p}}\cdot\vec{x}} e^{\hat{H}t} \chi(\vec{0},0) e^{-\hat{H}t} e^{i\hat{\vec{p}}\cdot\vec{x}} \bar{\chi}(\vec{0},0)\,|\Omega\rangle$$

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Insertion of complete set of momentum and energy states:

$$\mathbb{1} = \sum_{\vec{k},n} \frac{1}{2E_n(\vec{k})} |n,\vec{k}\,\rangle\langle\, n,\vec{k}|,$$

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★ **The mass of the hadron appears, for the nth state**

Calculation of Hadron mass

★ **Overlap with ground state, excitations, other hadron states. Thus:**

$$C(t) = \sum_{n'} \frac{1}{2E_n(\vec{k})} |\langle \Omega | \chi(\vec{0},0) | (n',\vec{0}) \rangle|^2 e^{-m_{n'} t}$$

★ **For large enough t the exponential for excited states and multi-hadron states, becomes very small, thus ground-state dominance.**

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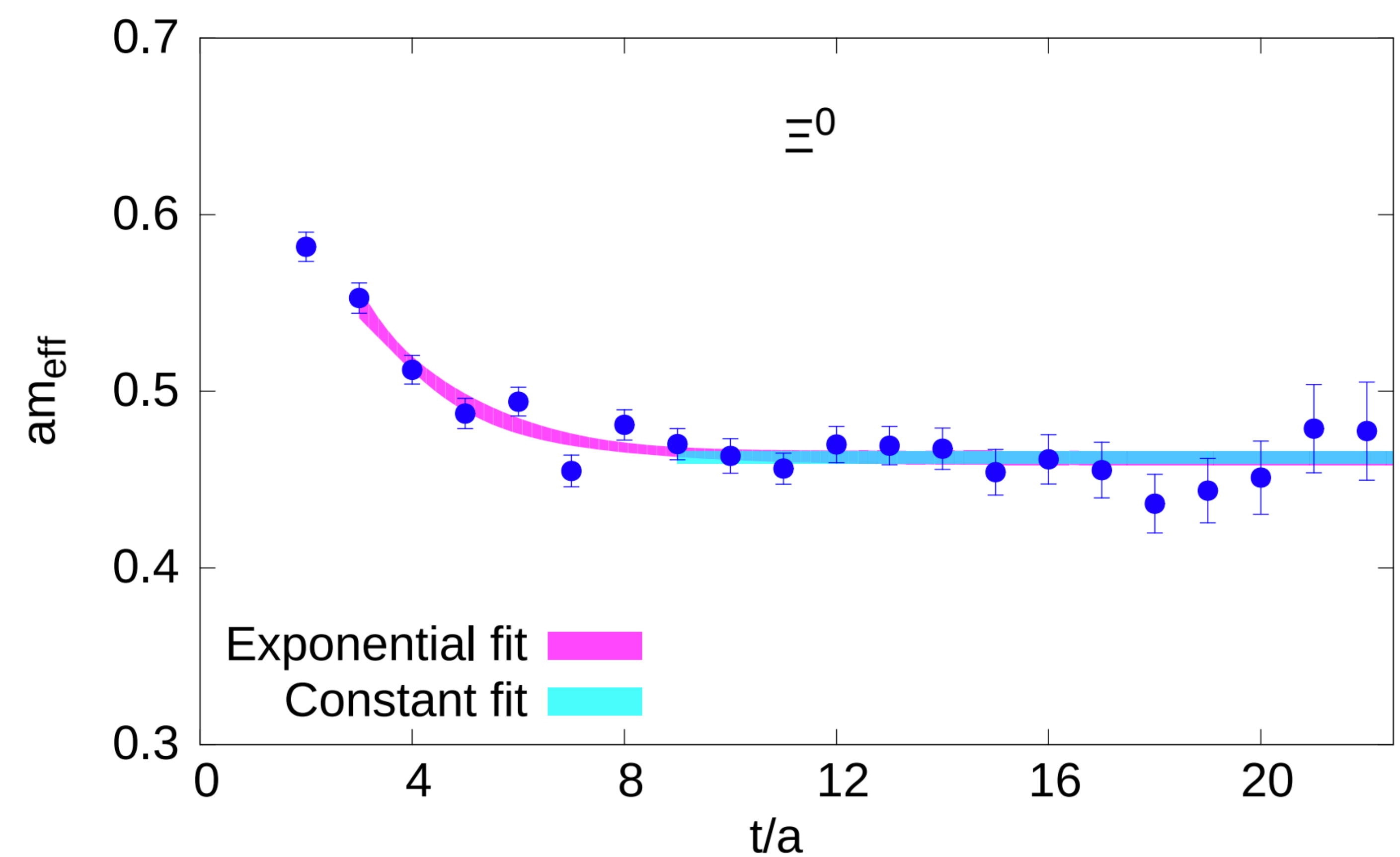

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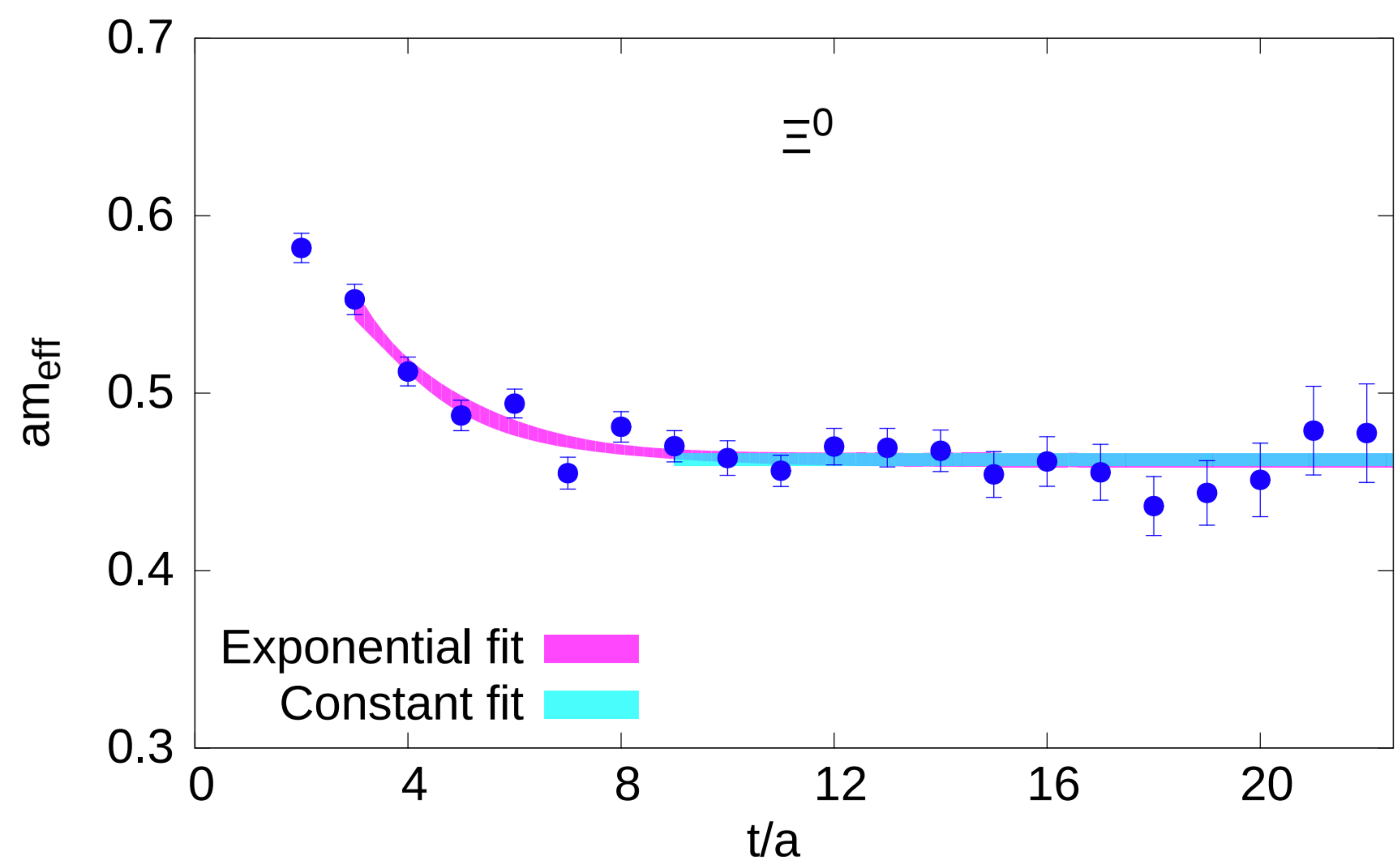

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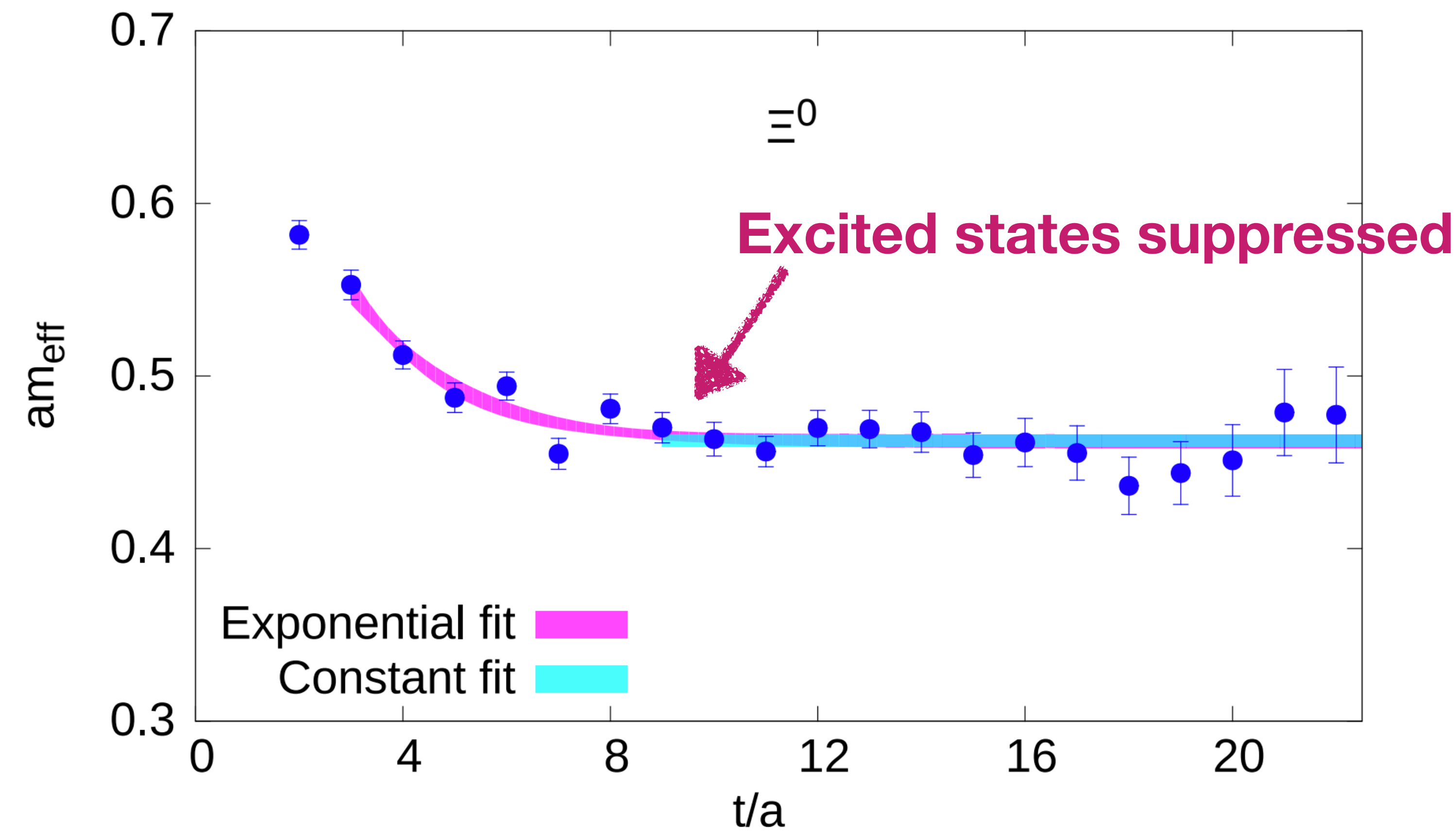

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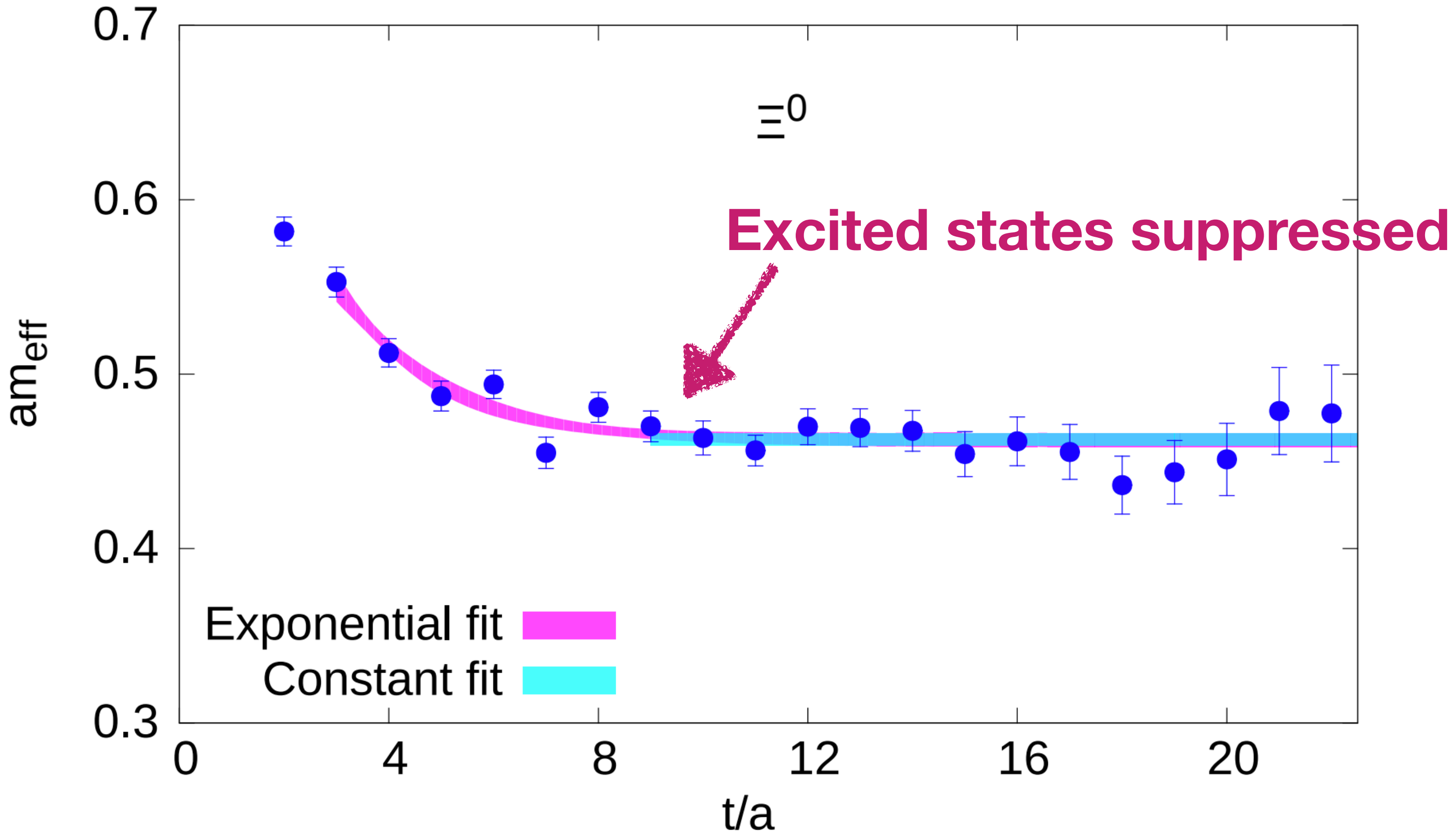
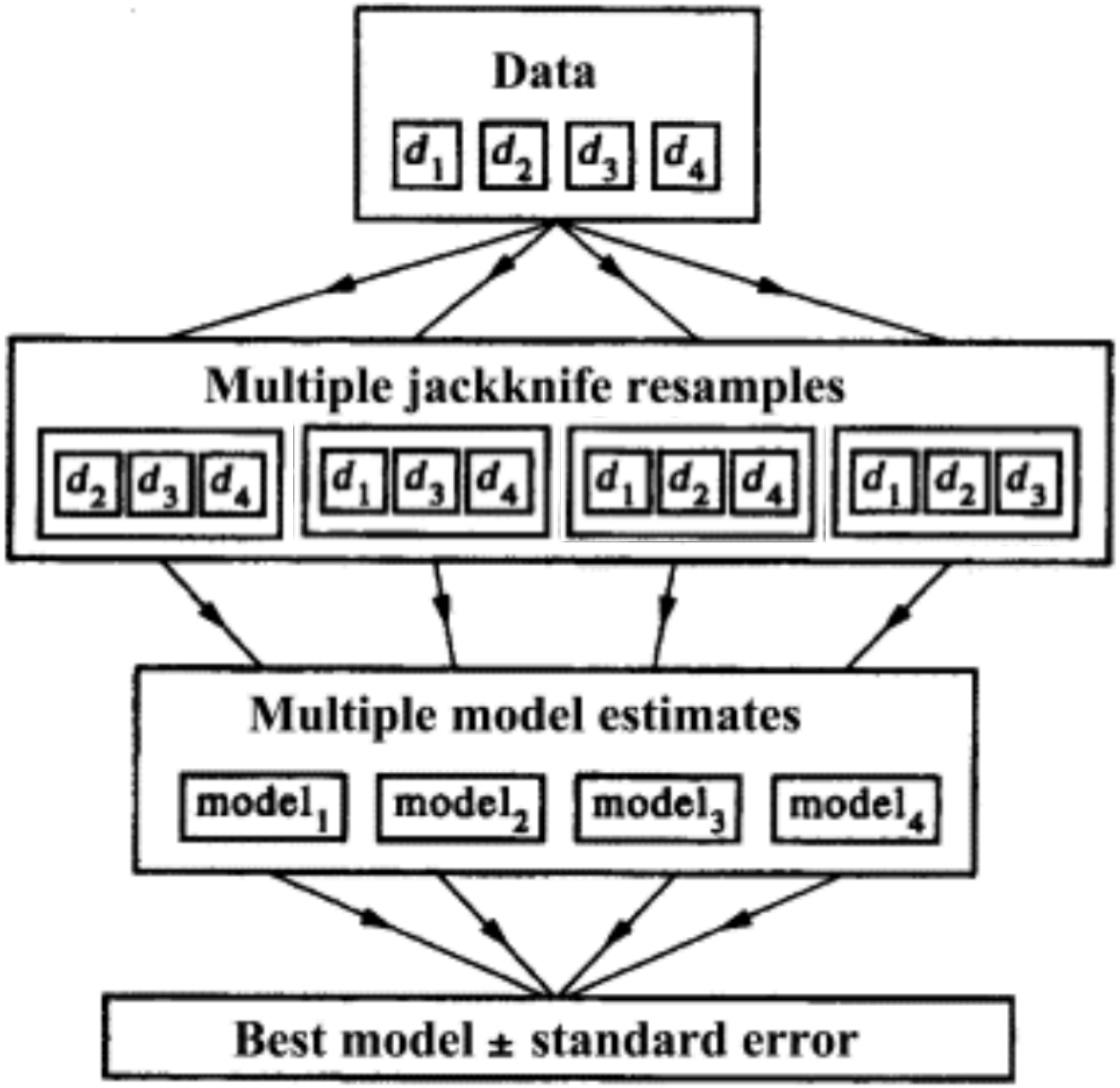
Attend practice session by Luca Leskovec !



Calculation of Hadron mass

Results MUST be accompanied by uncertainties

Jackknife resampling for variance and bias estimation



- ★ Choose the number of omitted data in each bin (defines # bins)
- ★ Calculate the average over remaining data in each bin
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- ★ Calculate the statistical error of the above average

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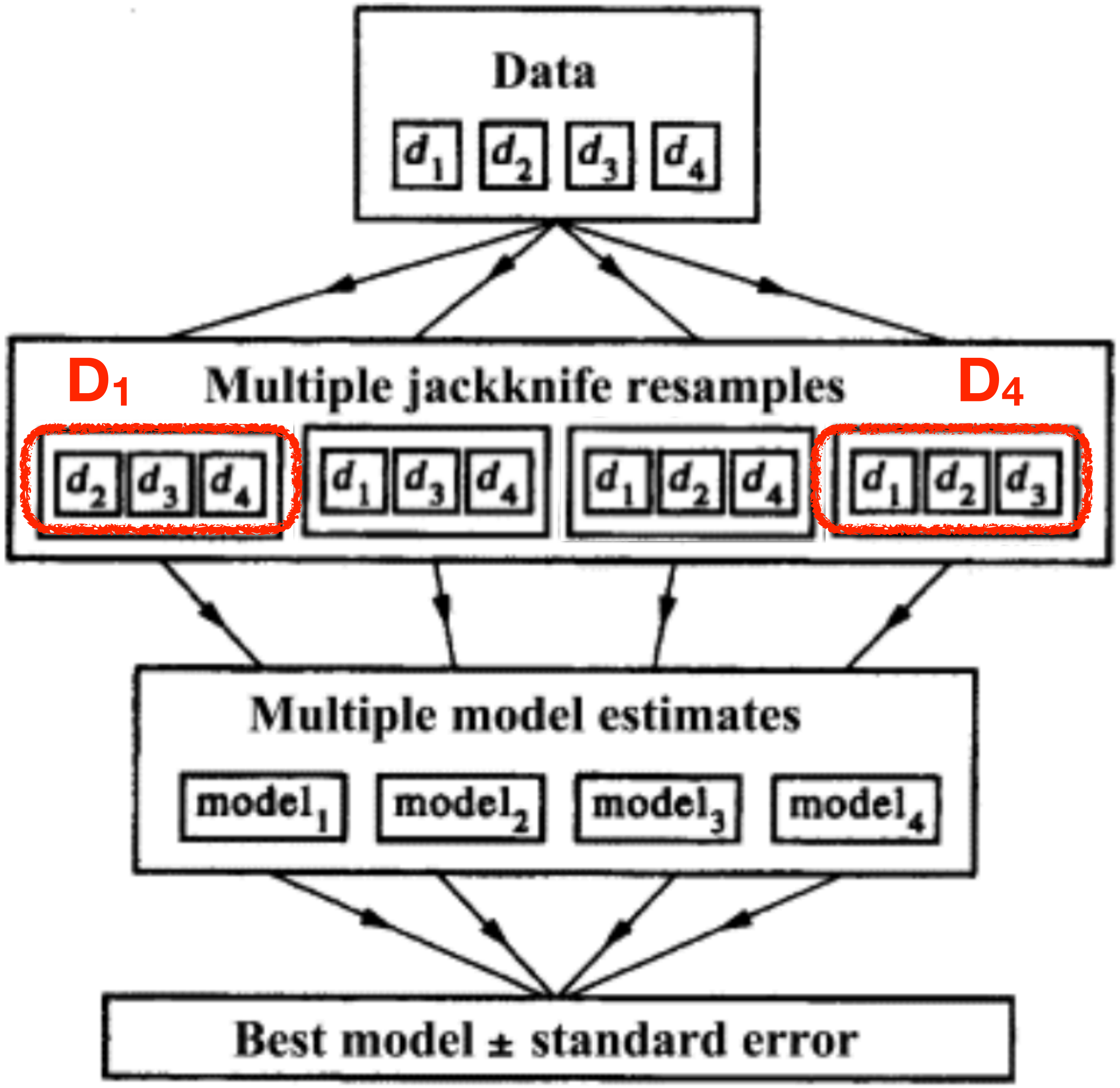

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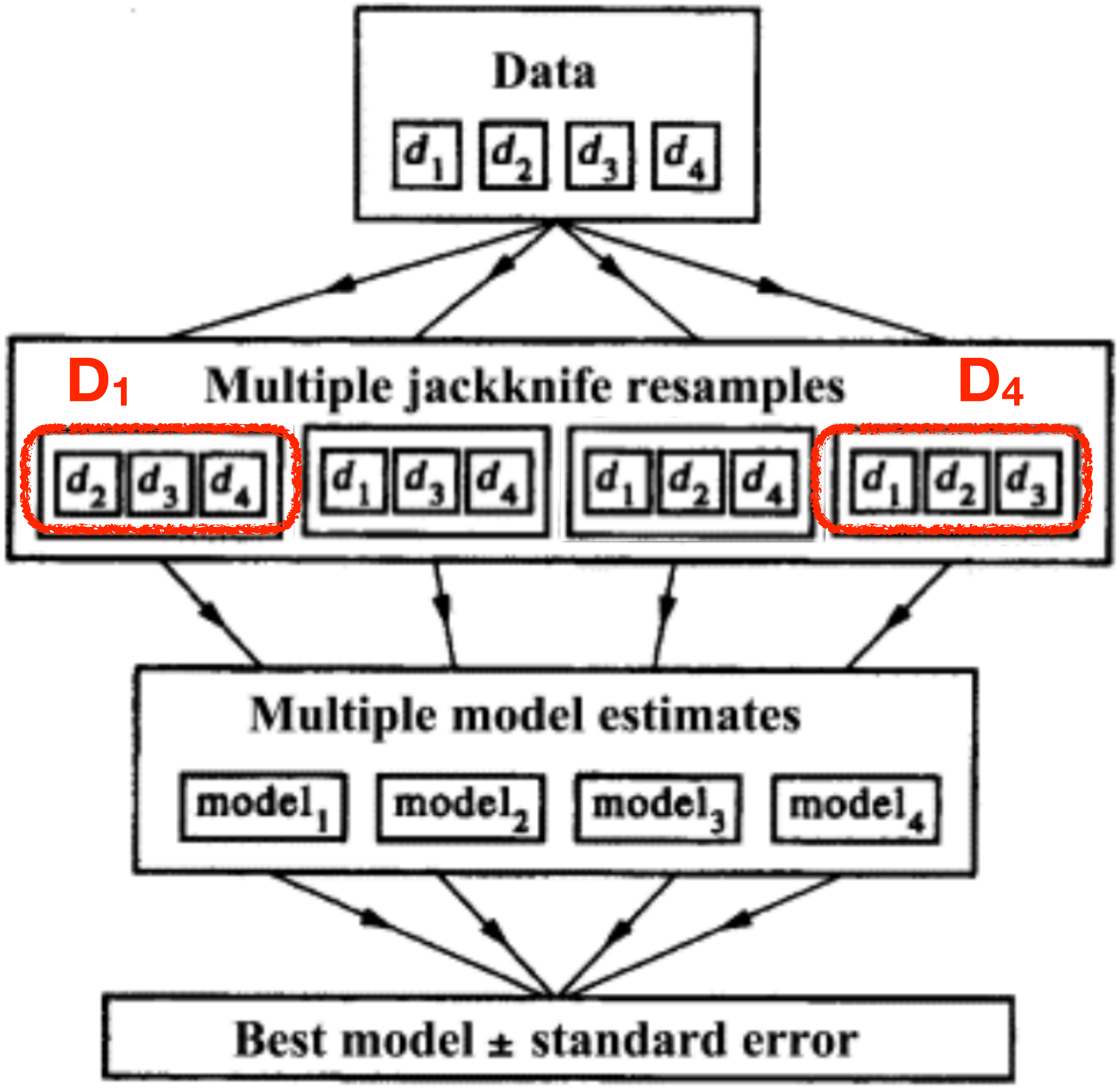

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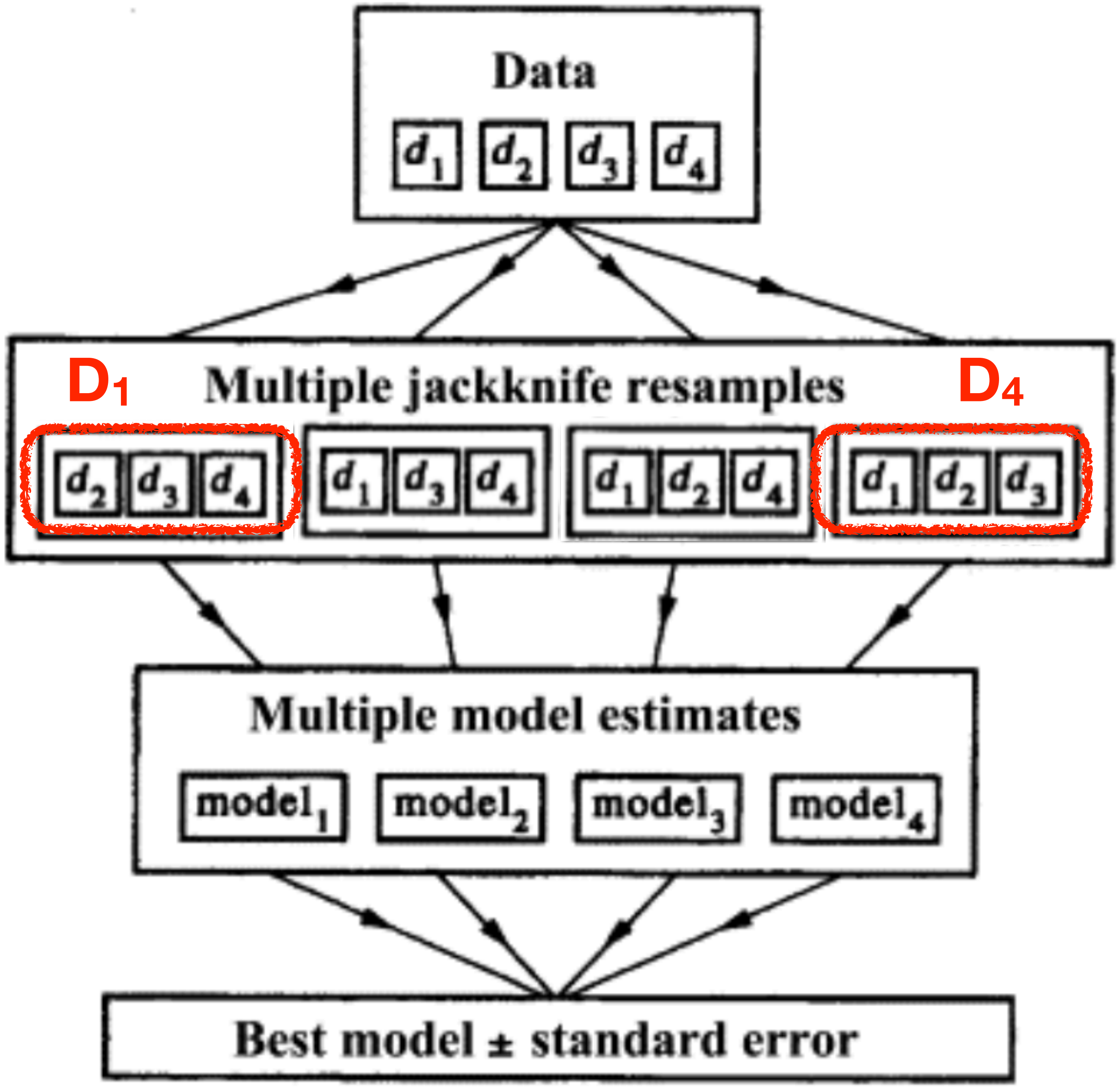

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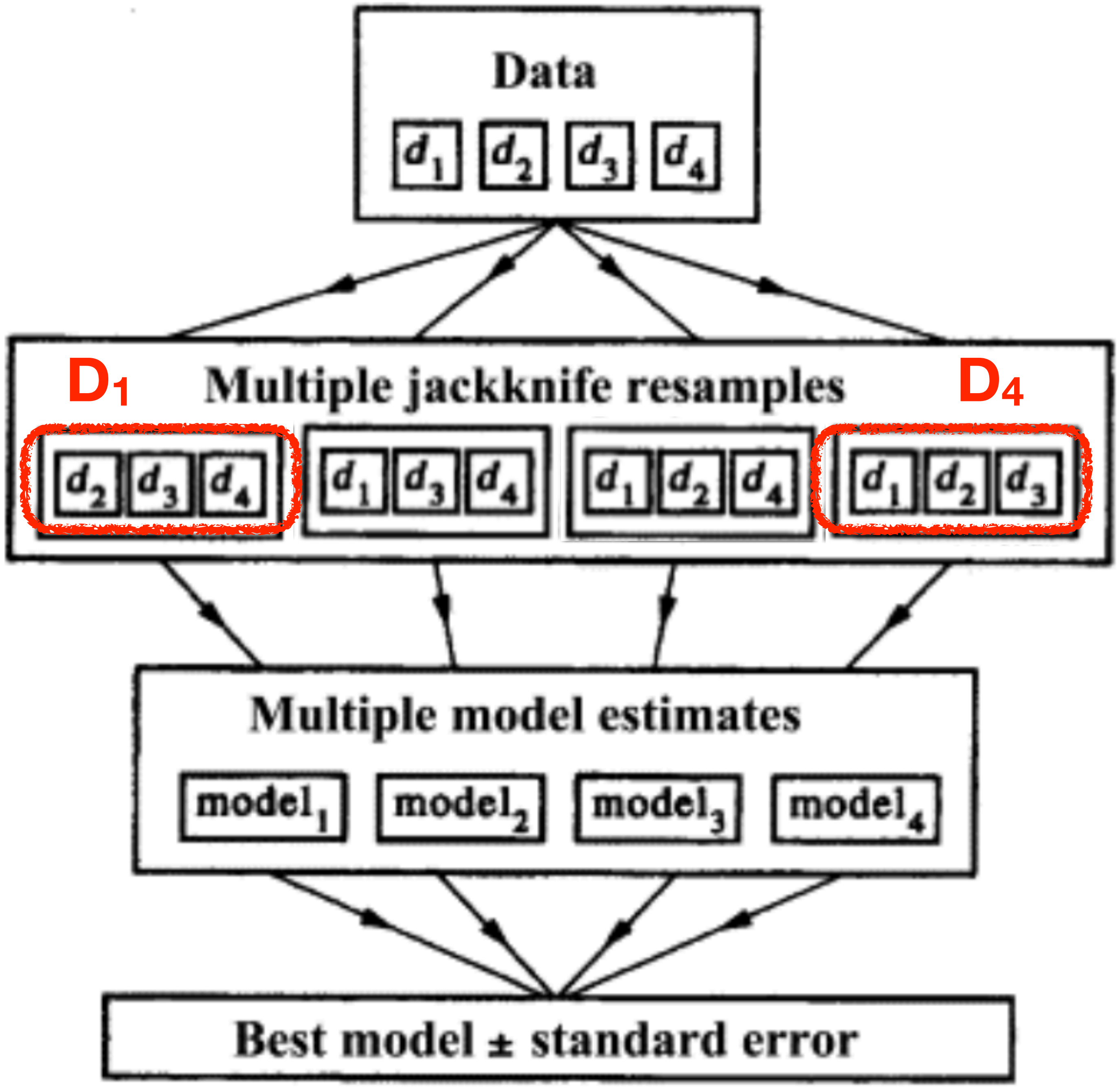

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$$\bar{D} = \sum_{i} \frac{D_{\text{i}}}{N_{\text{bin}}}$$

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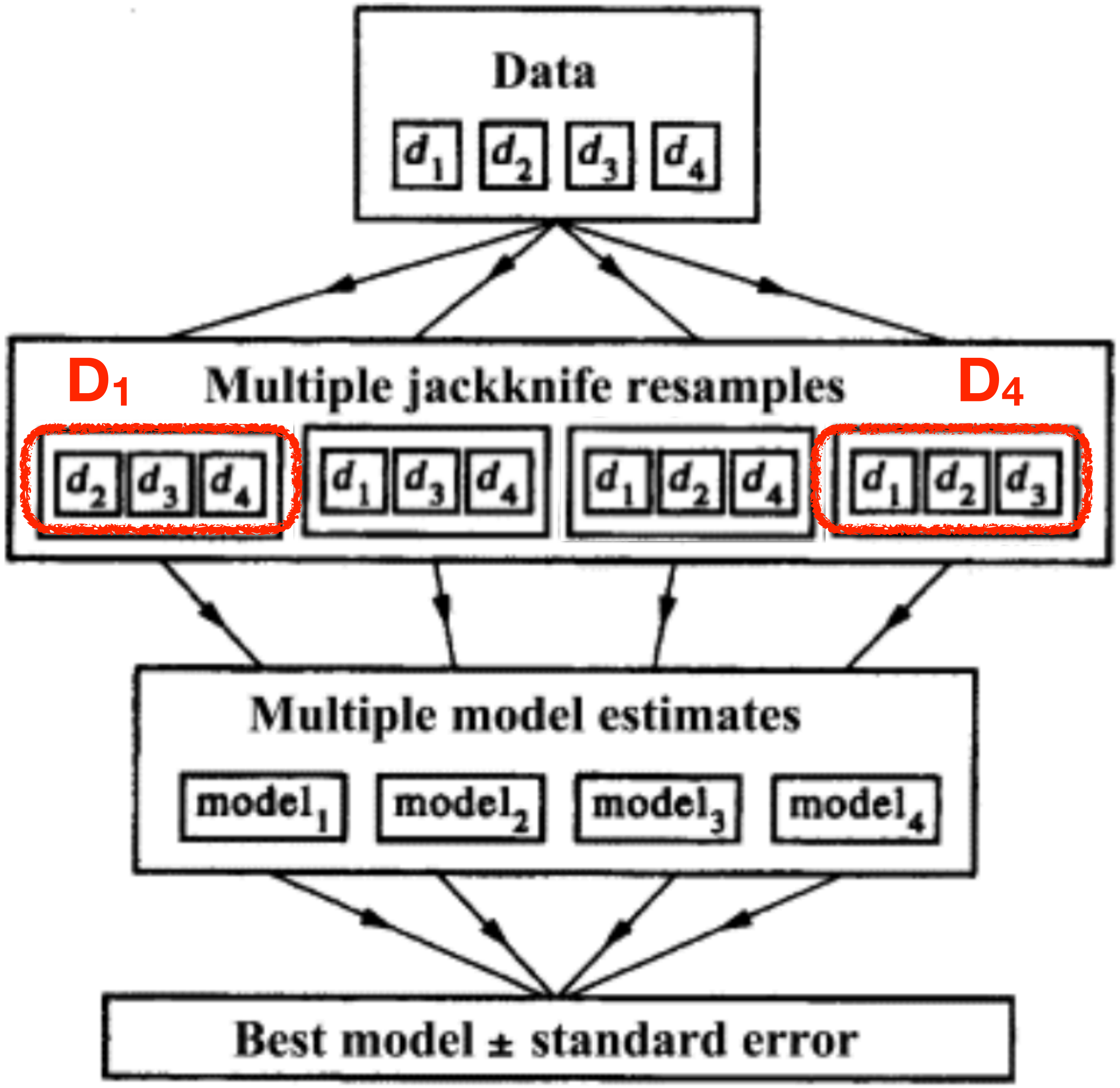

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$$N_{\text{data}} = 4, \quad N_{\text{omit}} = 1\,, \quad N_{\text{bin}} = 4$$

- ★ Calculate the average over remaining data in each bin

$$D_{\text{i}} = \sum_{j \neq i} \frac{d_j}{N_{\text{data}} - N_{\text{omit}}}$$

- ★ Calculate the average of the bins

$$\bar{D} = \sum_i \frac{D_{\text{i}}}{N_{\text{bin}}}$$

- ★ Calculate the statistical error of the above average

$$d\bar{D} = \sqrt{\sum_i (D_i - \bar{D})^2} \sqrt{\frac{N_{\text{bin}} - 1}{N_{\text{bin}}}}$$

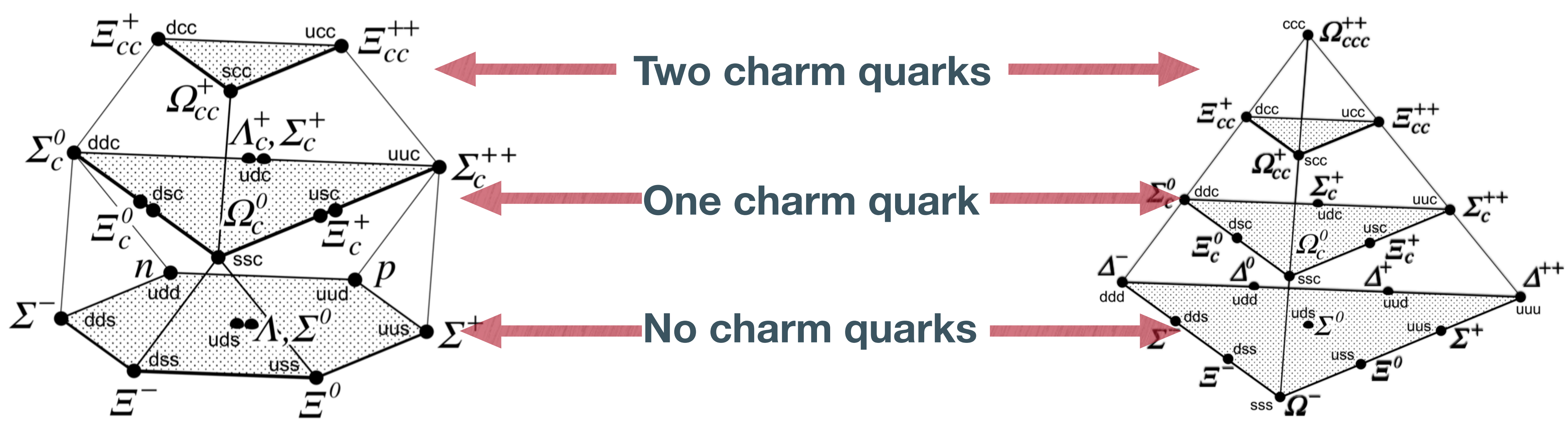
Hadron Spectroscopy

★ **One of main research directions of Lattice QCD with great successes (but beyond the scope of these lectures)**

★ **Calculation of:**

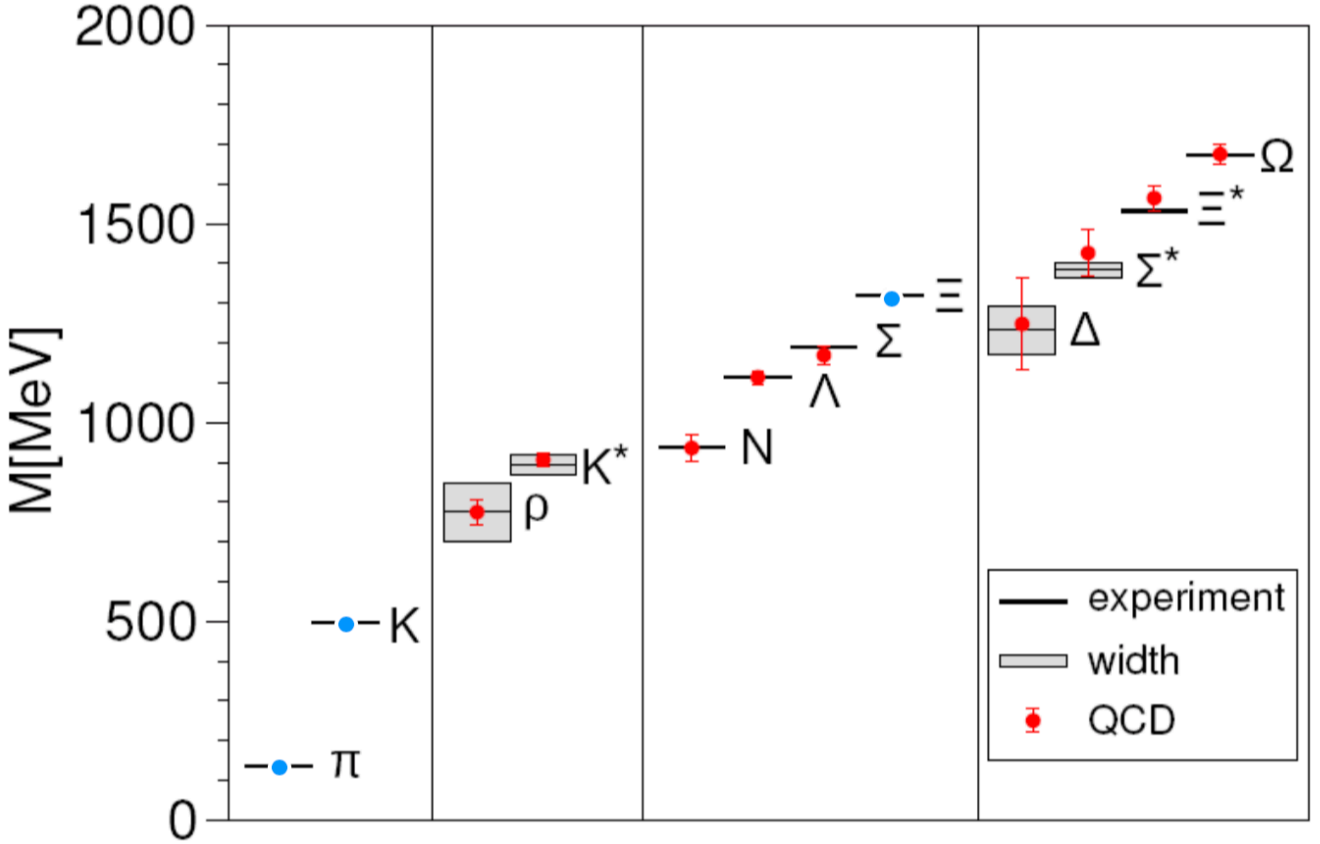
- low-lying baryon and meson states
- Excited and exotic hadrons
- Scattering and resonance states

20-plet of spin-1/2 baryons **20-plet of spin-3/2 baryons**

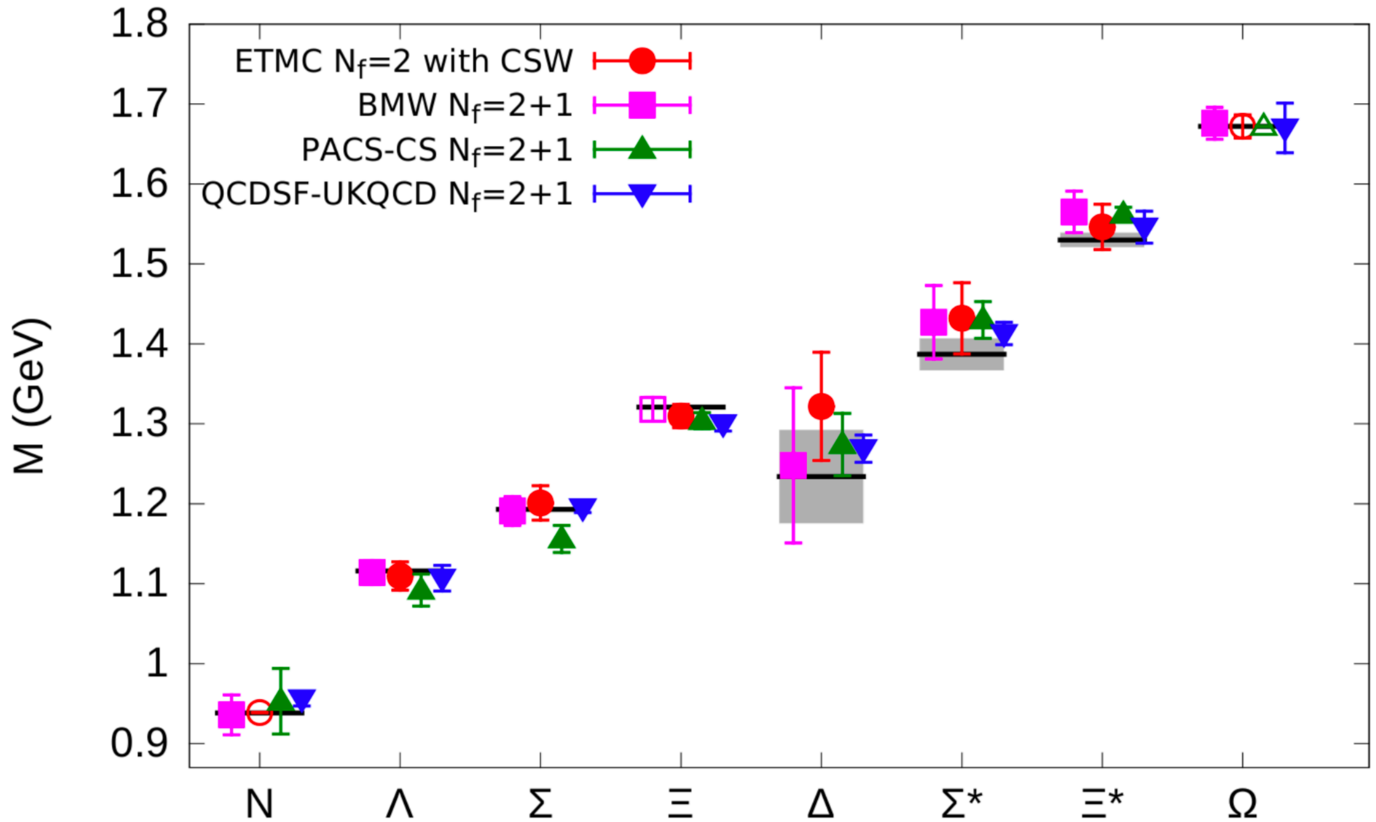


Hadron Spectroscopy

Low-lying meson and baryon states



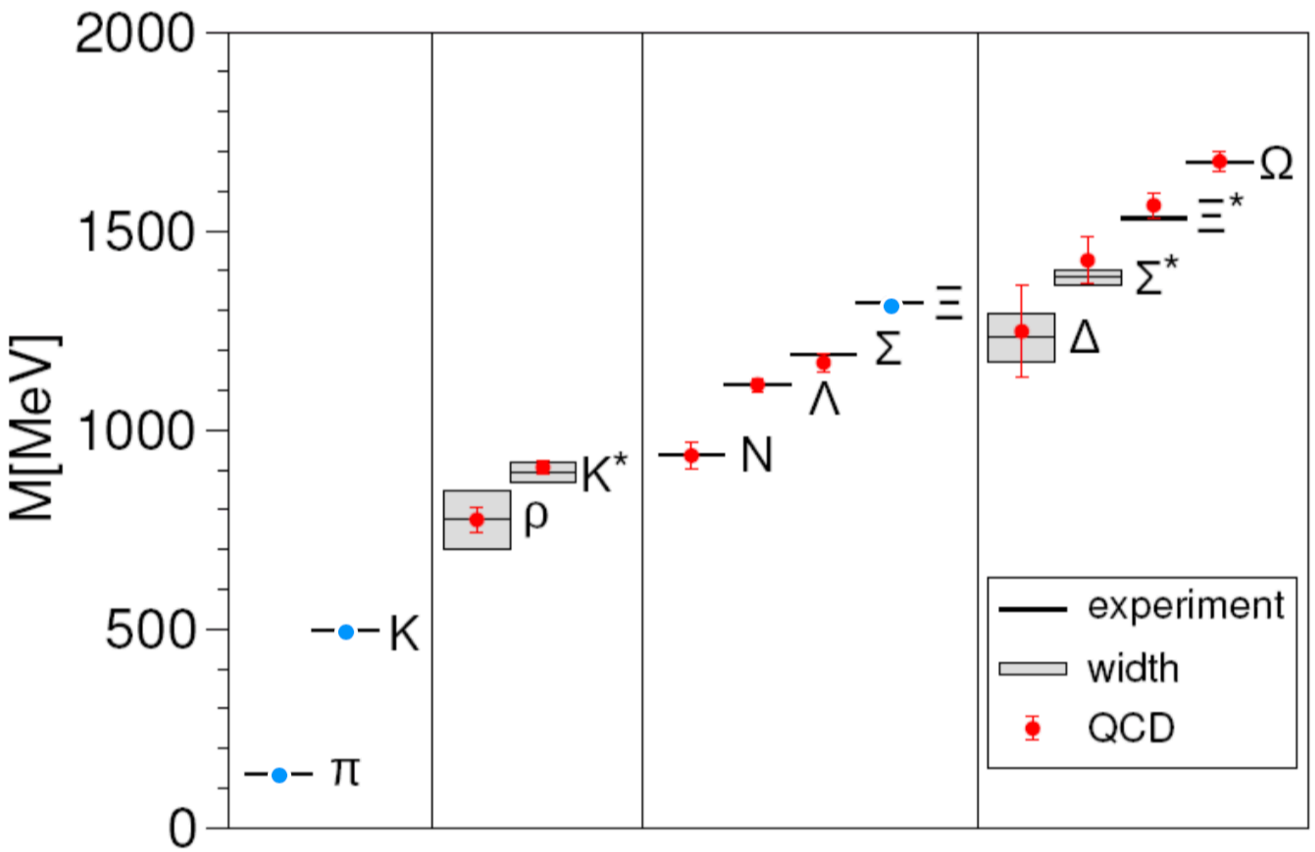
BMW collaboration, Science 322, 1224 (2008)



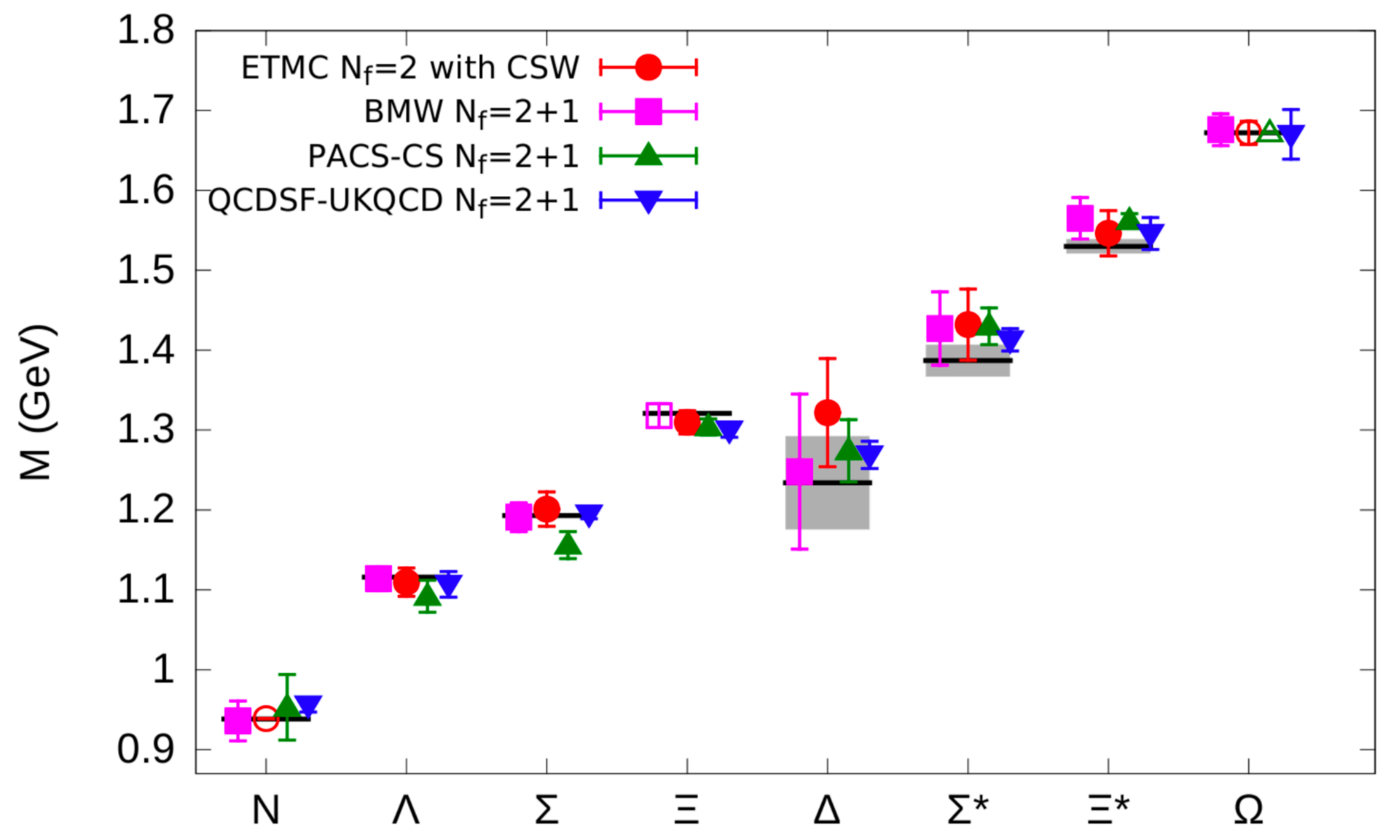
[C. Alexandrou et al.,Phys. Rev. D 90, 074501 (2014)]

Hadron Spectroscopy

Low-lying meson and baryon states



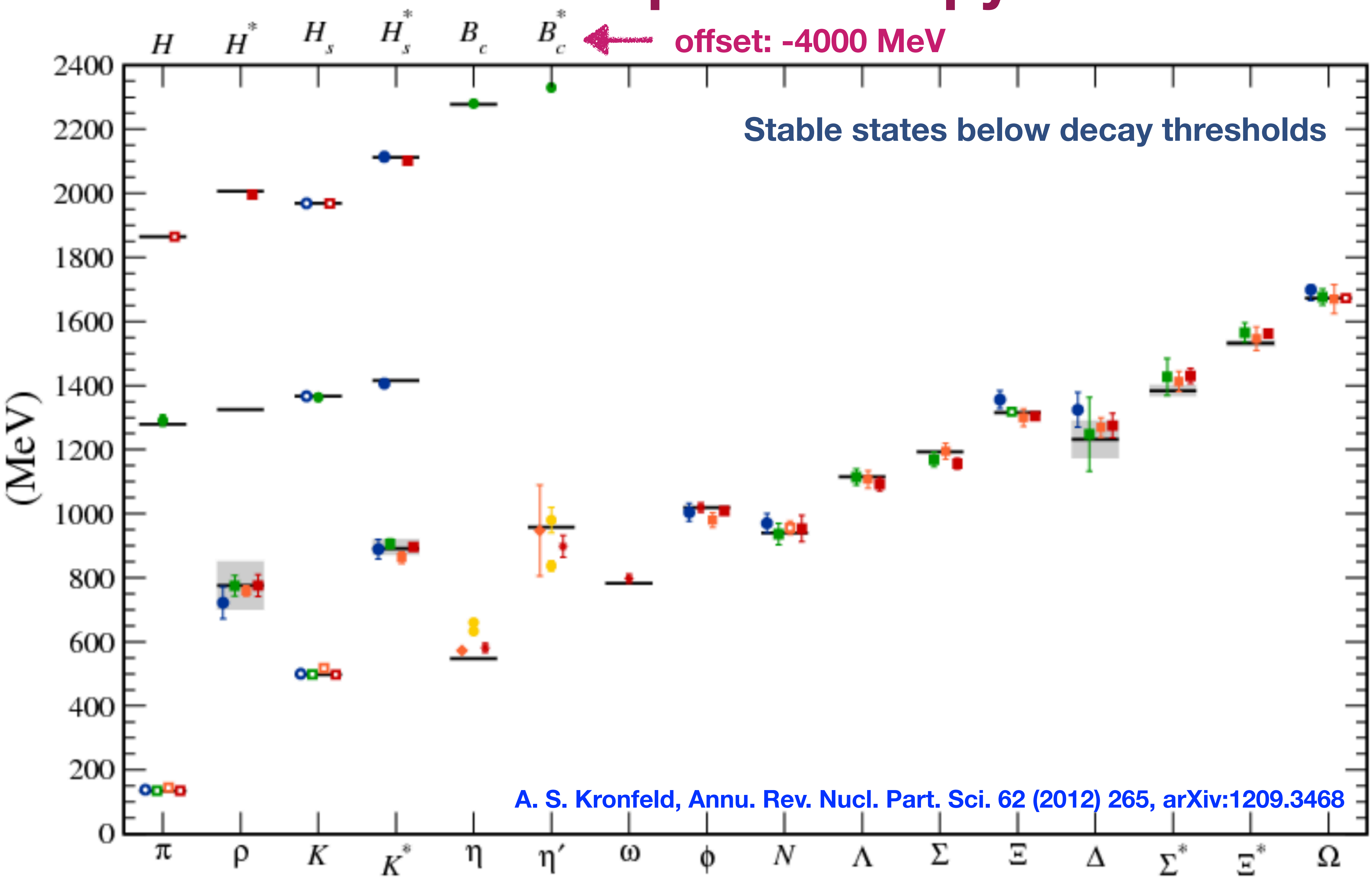
BMW collaboration, Science 322, 1224 (2008)



[C. Alexandrou et al.,Phys. Rev. D 90, 074501 (2014)]

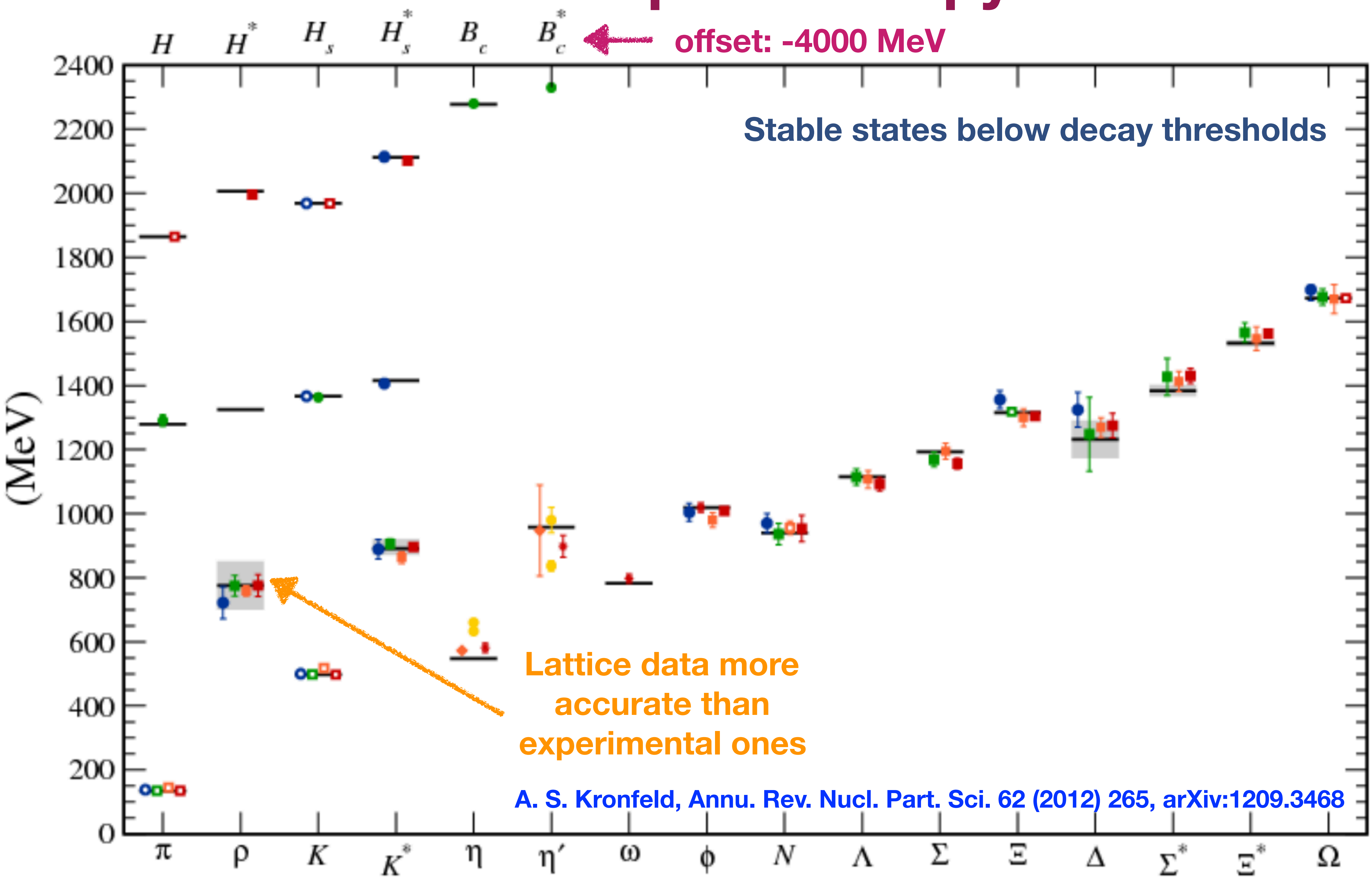
Lattice results reproduce experimental values

Hadron Spectroscopy



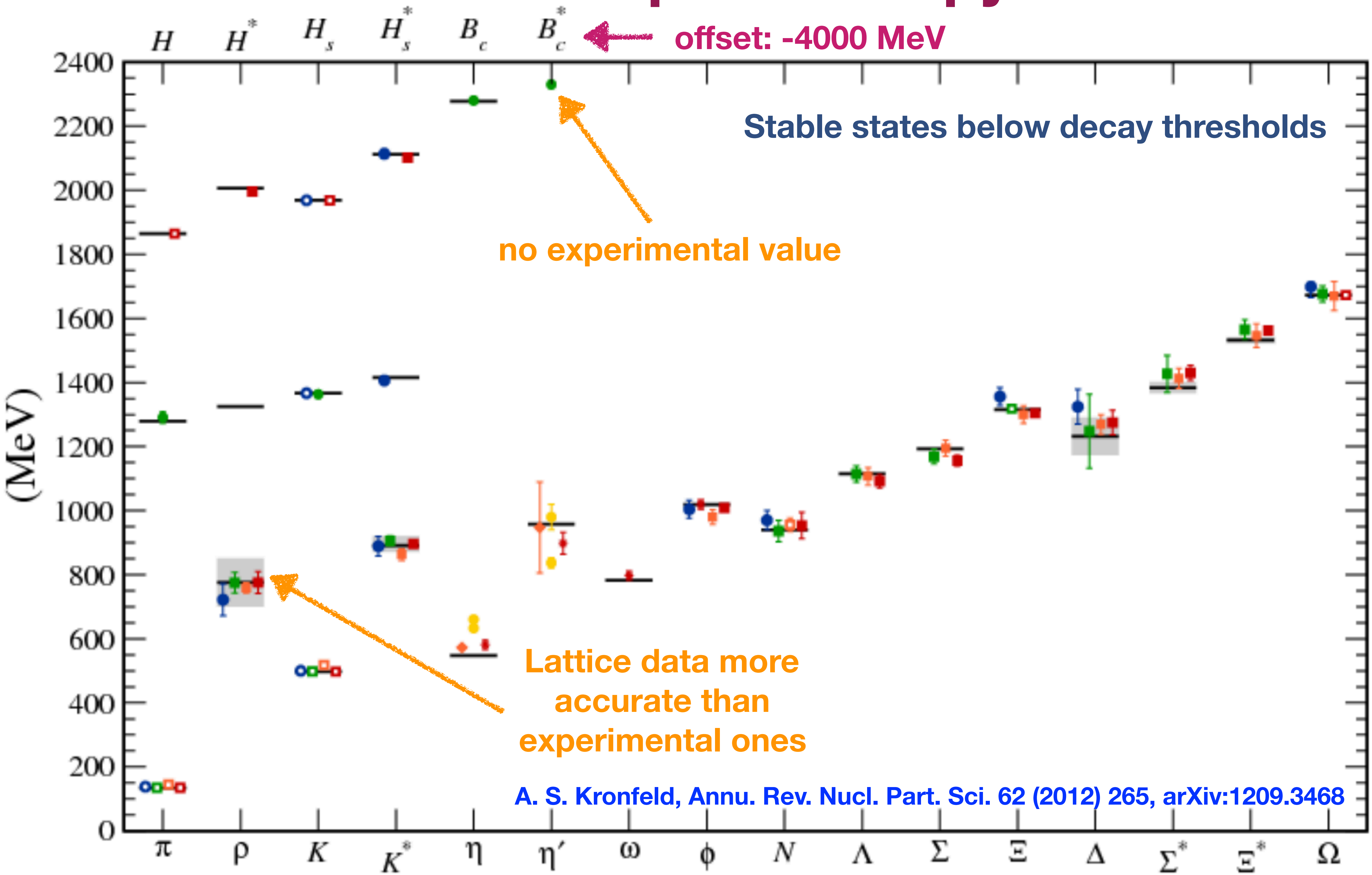
Lattice results reproduce experimental values

Hadron Spectroscopy



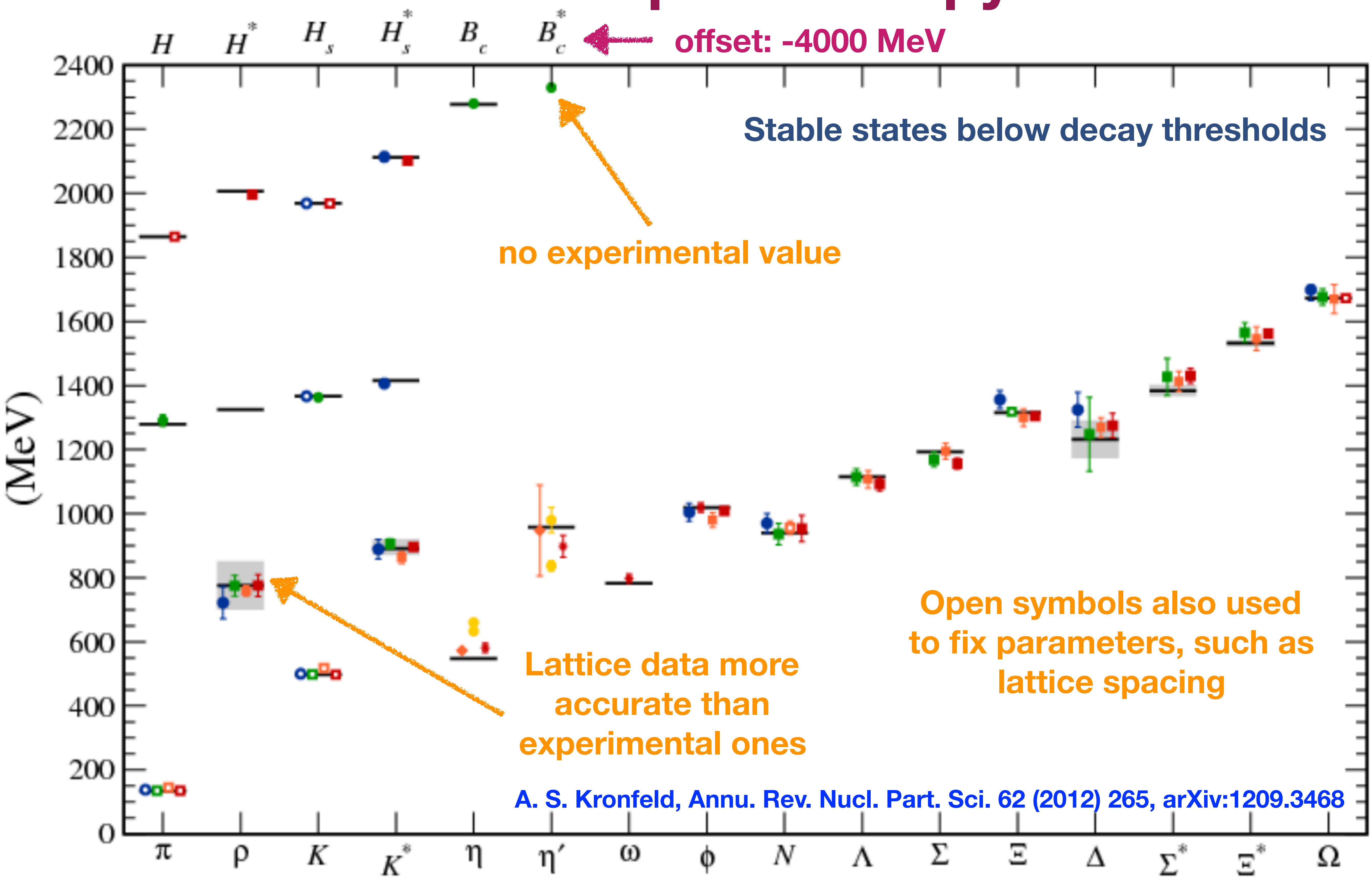
Lattice results reproduce experimental values

Hadron Spectroscopy



Lattice results reproduce experimental values

Hadron Spectroscopy

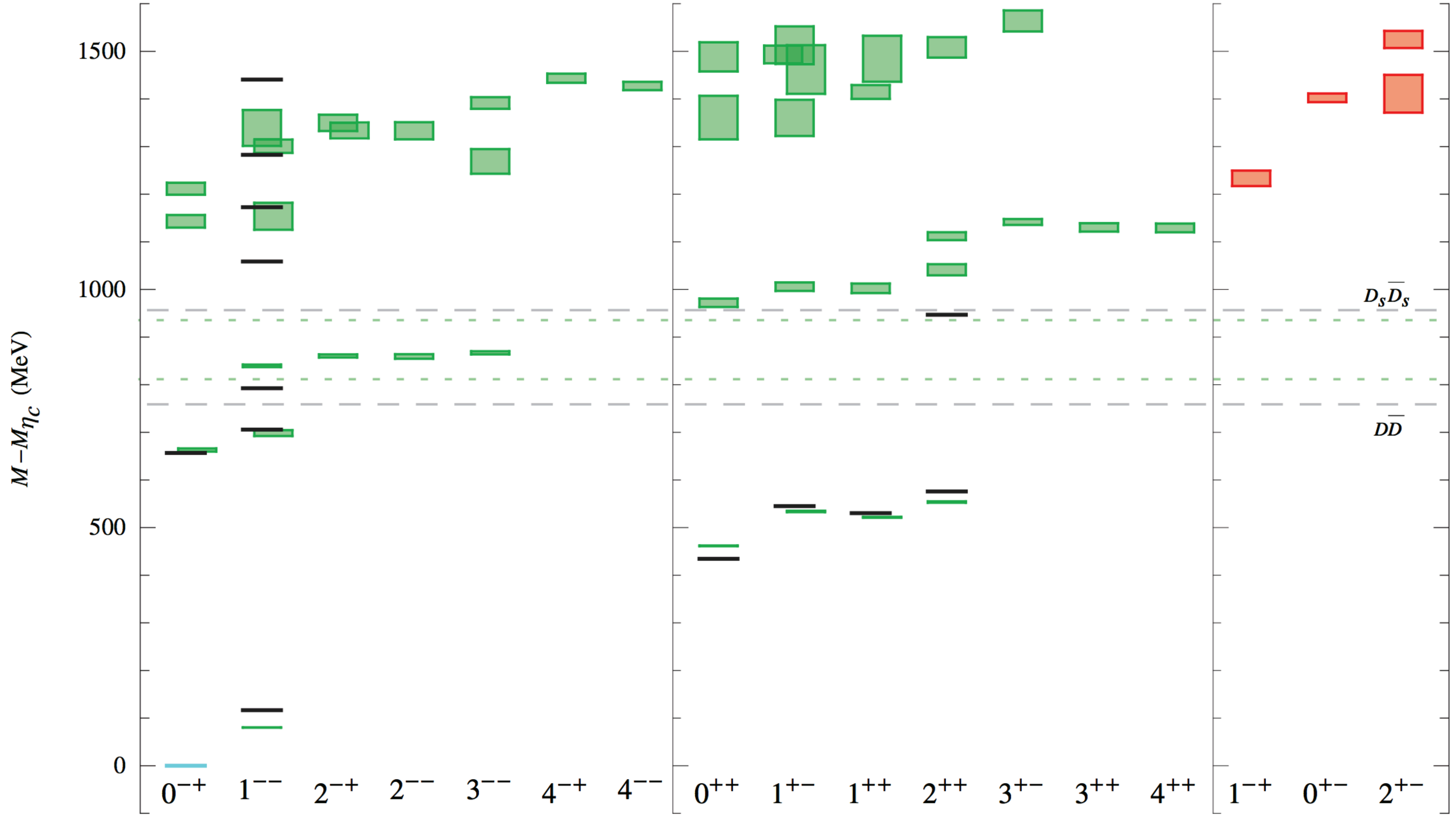


Lattice results reproduce experimental values

Hadron Spectroscopy

Excited and exotic hadrons

charmonium spectrum (< 4.5 GeV)



L. Liu et al. (Hadron Spectrum Collaboration), JHEP 07, 126 (2012), arXiv:1204.5425

Summary of Lecture 2

Key points of Lecture 2

★ Renormalization is an indispensable part of lattice calculations

★ Well-defined perturbative and non-perturbative renormalization procedures (see also Lecture 3)

★ Calculation of nucleon and pion mass has been an important starting point for lattice QCD

★ Hadron Spectroscopy has advanced tremendous and can provide predictions and input for experiments

Thank you