Pentaquarks with Hidden Charm as Hadrocharmonia

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M.E., Victor Petrov, Maxim Polyakov PR D 93, 054039 (2016)
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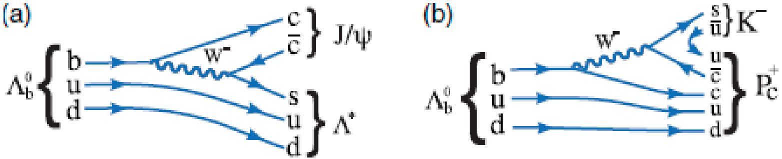
Outline

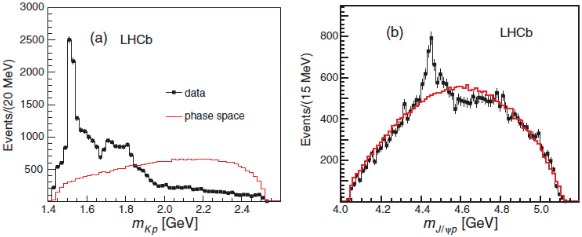
LHCb Collaboration (***b* is for beauty!**)

Summer 2015, LHCb discovered a new beast: pentaquarks

$\Lambda_b^0(udb) \to J/\psi K^- p$ Decays



- Final products: J/ψ, K^-, and p
- $\Lambda_b^0(udb) \to J/\psi \Lambda^*$, $\Lambda^* \to K^- p$
- $\Lambda_b^0(udb) \to K^- P_c^+$, $P_c^+ \to J/\psi p$



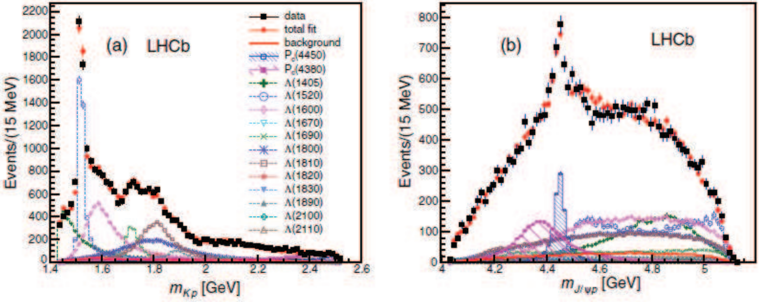
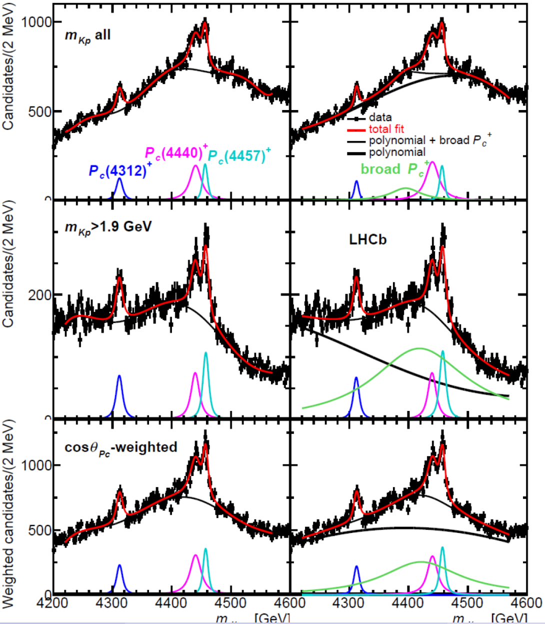


FIG. 3 (color online). Fit projections for (a) m_{Kp} and (b) $m_{J/\psi p}$ for the reduced Λ^* model with two P_c^+ states (see Table I). The data are shown as solid (black) squares, while the solid (red) points show the results of the fit. The solid (red) histogram shows the background distribution. The (blue) open squares with the shaded histogram represent the $P_c(4450)^+$ state, and the shaded histogram topped with (purple) filled squares represents the $P_c(4380)^+$ state. Each Λ^* component is also shown. The error bars on the points showing the fit results are due to simulation statistics.

LHCb resonances

- **Two states with opposite parities:** $J^P = (3/2^+, 5/2^-)$ **or** $J^P = (5/2^+, 3/2^-)$
- **Heavy** $P_c^+(uudc\bar{c})$**:** $M = 4449.8 \pm 1.7 \pm 2.5$ **MeV,** $\Gamma = 39 \pm 5 \pm 19$ **MeV**
- **Light** $P_c^+(uudc\bar{c})$**:** $M = 4380 \pm 8 \pm 29$ **MeV,** $\Gamma = 205 \pm 18 \pm 86$ **MeV**

LHCb, Spring 2019



LHCb, Spring 2019

State	M [MeV]	Γ [MeV]	(95% CL)	$\mathcal{R}$ [%]
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+\ 3.7}_{-\ 4.5}$	(< 27)	$0.30 \pm 0.07^{+0.34}_{-0.09}$
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+\ 8.7}_{-10.1}$	(< 49)	$1.11 \pm 0.33^{+0.22}_{-0.10}$
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+\ 5.7}_{-\ 1.9}$	(< 20)	$0.53 \pm 0.16^{+0.15}_{-0.13}$

What changed?

- $P_c(4450)^+ \Longrightarrow$ **two narrow resonances** $P_c(4440)^+$ **and** $P_c(4457)^+$
- **New narrow resonance** $P_c(4312)^+$
- **No news on** $P_c(4380)^+$
- **No data on spin-parities**

GlueX, May 26, 2019

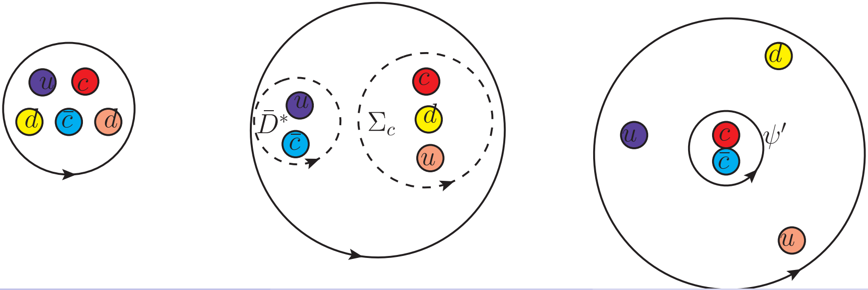
	$\mathcal{B}(P_c^+ \to J/\psi p)$ Upper Limits, %		$\sigma_{\max} \times \mathcal{B}(P_c^+ \to J/\psi p)$ Upper Limits, nb	
	p.t.p. only	total	p.t.p only	total
$P_c^+(4312)$	2.9	4.6	3.7	4.6
$P_c^+(4440)$	1.6	2.3	1.2	1.8
$P_c^+(4457)$	2.7	3.8	2.9	3.9

TABLE V: Summary of the estimated upper limits for the P_c^+ states as discussed in the paper. Separately shown are the results when using the errors of the individual data points (p.t.p.) only and the total ones that include the uncertainties in the model parameters and the overall normalization.

What is Pentaquark?

Theoretical Ideas

- Hadrocharmonium: charmonium-nucleon bound state
- Molecular state, one-pion and/or other light quarks exchanges
- "True" pentaquarks (diquark-diquark-antiquark), other constituent models, tightly bound quarks
- Bound states of colored "baryon" and "meson"
- Kinematical effects



Mesons and Baryons in Large N_c QCD

Tetraquarks

- Meson mass and size are N_c independent, three-meson interaction $\sim 1/\sqrt{N_c}$
- Meson-meson potential $\sim \Lambda_{QCD}/N_c$, size $1/\Lambda_{QCD}$
- Can tetraquark be a loosely bound state of two light mesons?
- Relative momentum $p < \Lambda_{QCD}$ ($1/\Lambda_{QCD}$ - inverse size of potential)
- Kinetic energy is too large: $p^2/\Lambda_{QCD} \sim \Lambda_{QCD} > \Lambda_{QCD}/N_c \Longrightarrow$ no loosely bound molecular tetraquarks made from light quarks
- One heavy & one light meson: no loosely bound tetraquarks
- Can tetraquark be a loosely bound state of two heavy mesons?
- Two mesons with heavy quarks: kinetic energy $p^2/M_Q \sim \Lambda_{QCD}^2/M_Q$
- Kinetic energy $\Lambda_{QCD}^2/M_Q < \Lambda_{QCD}/N_c \Longrightarrow$ loosely bound molecular tetraquarks exist if $N_c\Lambda_{QCD} \ll M_Q$

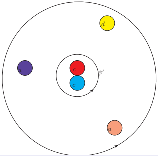
Only molecular tetraquarks exist at large N_c!

Mesons and Baryons in Large N_c QCD

Pentaquarks

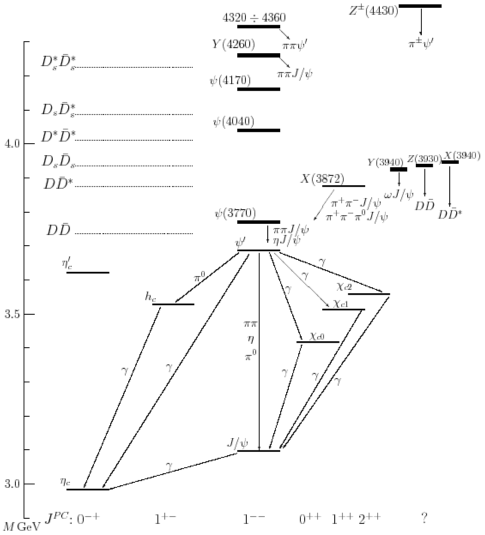
- Baryon size is N_c independent, mass $\sim N_c$ and baryon-baryon-meson vertex $\sim \sqrt{N_c}$
- Baryon-meson potential and its size are N_c independent
- **All kinds of exotic pentaquarks are allowed: molecular, hadrocharmonium, etc.**
- Even in large N_c limit pentaquark structure cannot be determined theoretically
- **True structure of LHCb pentaquarks should be determined experimentally**

Proton - Heavy Quarkonium Interaction in Large N_c QCD



- Baryon size is stable at large N_c
- **Heavy quarkonium at large N_c and large $m_Q \gg \Lambda_{QCD}$ has small size. In the Coulomb approximation Bohr radius:** $a_0 = 4/(N_c \alpha_s m_Q)$
- **Qualitative picture: small charmonium inside a huge proton**
- **Color fields of proton polarize small $\bar{Q}Q$ and hold it inside proton**
- **Why attraction? Induced electric dipole moment is attracted to the external charge**
- **Genesis: heavy quarkonium-nuclei interaction** *(Brodsky et al., 1990; Luke et al., 1992), heavy quarkonium-baryon interaction (Voloshin et al., 2005-2014)*

Charmonium Spectrum

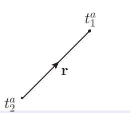


Spectroscopic notation: $(n_r+1)^{(2S+1)}L_J$, or only $(n_r L)$, $P=(-1)^{L+1}$, $C=(-1)^{L+S}$

Proton - Heavy Quarkonium Interaction in Large N_c QCD

QCD Multipole Expansion

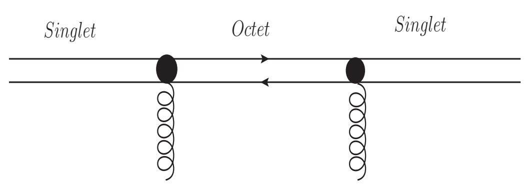
- Gluon field Lagrangian: $\mathcal{L} = -\frac{1}{4g^2}\int d^4x F^{\mu\nu}F_{\mu\nu}$
- Use multipole expansion to calculate small size charmonium interaction with long wavelength gluon field
- Color dipole interaction $H_d = -\boldsymbol{d}\cdot\boldsymbol{E}^a(t_1^a - t_2^a) = -\frac{1}{2}\boldsymbol{r}\cdot\boldsymbol{E}^a(t_1^a - t_2^a)$



- In the color singlet state: $\langle\psi|H_d|\psi\rangle = 0$
- Effective Hamiltonian $H_{eff} = \sum_n \frac{\langle\psi|H_d|n\rangle\langle n|H_d|\psi\rangle}{E-E_n}$
- In color singlet nS state
  $H_{eff} = -\frac{1}{2}\alpha(\psi)\boldsymbol{E}\cdot\boldsymbol{E}$
- $\alpha(\psi) = \frac{1}{3N_c}\langle\psi|\boldsymbol{r}\,G\,\boldsymbol{r}|\psi\rangle$

Chromoelectric Polarizability

- Nonrelativistic approximation
  $\alpha(nS) = \frac{1}{3N_c}\langle nS|\boldsymbol{r}\frac{1}{\frac{-\nabla^2}{m_q}+\frac{1}{2N_c}\frac{\alpha_s}{r}}\boldsymbol{r}|nS\rangle$



- Singlet potential: $-\frac{N_c^2-1}{2N_c}\alpha_s$, octet potential: $\frac{1}{2N_c}\alpha_s$
- **Octet potential is suppressed like $1/N_c$**
- Polarizability α for Coulomb-like system for large N_c (Peskin,1979; Leutwyler, 1981; Voloshin, 1982)
  $\alpha(nS) = \frac{4n^2}{3\alpha_s N_c^2} c_n a_0^3, \quad a_0 = \frac{4}{\alpha_s N_c m_q}, \quad c_1 = \frac{7}{4}, \quad c_2 = \frac{251}{8}, \quad c_n = \frac{5}{16}n^2(7n^2-3)$

Numerical polarizabilities

- Coulombic polarizability, $N_c \to \infty$

$$\alpha(1S) \approx 0.2 \ \mathrm{GeV}^{-3}$$

$$\alpha(2S) \approx 12 \ \mathrm{GeV}^{-3}$$

$$\alpha(2S \to 1S) \approx -0.6 \ \mathrm{GeV}^{-3}$$

- Coulombic polarizability without large N_c for $1S$ (*Brambilla et al., 2016*) differs by 5.5% from the $N_c \to \infty$ value
- Phenomenological value from $\psi(2S) \to J/\psi\pi\pi$ decays (*Voloshin, 2008*)

$$|\alpha(2S \to 1S)| \approx 2 \ \mathrm{GeV}^{-3}$$

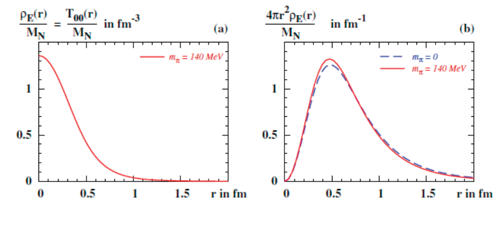
- **Coulombic values are not too reliable, may be used for estimates**

Charmonium-Nucleon Hamiltonian

- Field Hamiltonian
  $H_{eff} = -\frac{1}{2}\alpha(\psi)\boldsymbol{E}\cdot\boldsymbol{E}$
- We need to calculate matrix element in the nucleon state
- Use trace anomaly
  $\boldsymbol{E}^2 = \frac{\boldsymbol{E}^2-\boldsymbol{H}^2}{2} + \frac{\boldsymbol{E}^2+\boldsymbol{H}^2}{2} = g^2\left(\frac{8\pi^2}{bg_s^2}T^{\mu}{}_{\mu} + T^{G}_{00}\right),$
  $b = (11/3)N_c - (2/3)N_f$
- $T_{\mu\nu}$ – QCD energy-momentum tensor, $T^{G}_{\mu\nu}$ – gluon EM tensor
- In the nucleon state (g^2 at the quarkonium radius)
  $V(\boldsymbol{x}) = \langle N| -\frac{1}{2}\alpha\boldsymbol{E}^2(\boldsymbol{x})|N\rangle = -\frac{\alpha}{2}g^2\langle N|\left(\frac{8\pi^2}{bg_s^2}T^{\mu}{}_{\mu} + T^{G}_{00}\right)|N\rangle$
- Matrix elements are model dependent

Charmonium-Nucleon Hamiltonian

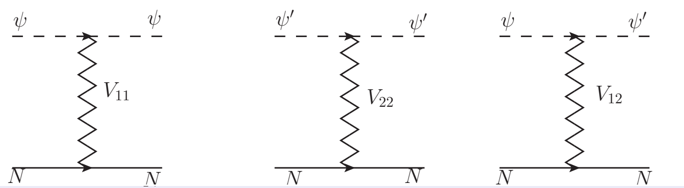
- We make estimates in χQSM (chiral quark soliton) model, but **choice of model is not critical, any model will give similar numbers**
- Assumption: $\langle N|T_{00}^G|N\rangle = \xi\langle N|T_{00}|N\rangle$
- ξ is the fraction of the nucleon energy carried by the gluons
- *diag* $\langle N|T^{\mu\nu}|N\rangle = (\rho_E(\boldsymbol{x}), p(\boldsymbol{x}), p(\boldsymbol{x}), p(\boldsymbol{x}))$, matrix elements were calculated in χQSM (*Polyakov et al., 2007*)
- **Integrals $\int d^3x\rho_E(\boldsymbol{x}) = M_N$, $\int d^3xp(\boldsymbol{x}) = 0$ are model-independent**



Charmonium-Nucleon Hamiltonian

- Interaction potential:
  $$V(\boldsymbol{x}) = -\frac{\alpha}{2} g^2 \langle N| \left(\frac{8\pi^2}{bg_s^2} T^{\mu}{}_{\mu} + \xi T_{00}\right)|N\rangle$$
  $$= -\alpha \frac{4\pi^2}{b}\left(\frac{g^2}{g_s^2}\right)[\nu\rho_E(\boldsymbol{x}) - 3p(\boldsymbol{x})],$$
  where
  $$\nu = \left(1 + \xi\frac{bg_s^2}{8\pi^2}\right) \sim 1.5$$
- For pion $\nu \sim 1.45 - 1.6$ (Novikov, Shifman, 1981). In χQSM model $\nu \sim 1.5$
- Pointlike quarkonium scans energy and pressure inside nucleon

Universal Charmonium-Nucleon Potential



- **Shape of interaction potential between any quarkonium state and nucleons is universal**
- $2S$-quarkonium: $V_{22}(r) \equiv V(r)$
- $1S$-quarkonium: $V_{11}(r) = \frac{\alpha(1S)}{\alpha(2S)} V(r)$
- $1S - 2S$ transition potential : $V_{12}(r) = \frac{\alpha(2S \to 1S)}{\alpha(2S)} V(r)$

Charmonium-Nucleon Bound States

Schrödinger Equation

- **Look for bound states in $J/\psi + N$ and $\psi(2S) + N$ channels**
  $\left(-\frac{\nabla^2}{2\mu} + V(r) - E\right)\Psi_b = 0$, μ is the reduced mass
- **Bound state arises at the critical $\alpha^{cr} = 5.6$ GeV^{-3}**
- $\alpha^{cr} \gg \alpha(1S)_{pert} = 0.2$ **GeV^{-3}** $\Longrightarrow J/\psi$ **does not form bound state with nucleon**
- $\alpha^{cr} \ll \alpha(2S)_{pert} = 12$ **GeV^{-3}** $\Longrightarrow \psi(2S)$ **forms bound state with nucleon**

Bound states

- **At $\alpha(2S) = 17.2$ GeV^{-3} – bound state with mass 4450 MeV, binding energy $E_b = -176$ MeV, $L = 0$**
- This is a narrow states: $\Gamma(P_c(4450) \to J/\psi N) = 11$ MeV
- **Natural hypothesis: $\psi(2S)$-nucleon bound state is LHCb $P_c^+(4450)$**

Hyperfine Splitting

What about degeneracy in spin?

- **Potential does no depend on spin: there are two degenerate states** $J^P = 1/2^-, 3/2^-$
- **Hyperfine effective interaction Hamiltonian is due to interference of the chromoelectric dipole** $E1$ **and the chromomagnetic quadrupole** $M2$ **transitions**
  $H_{\rm eff} = -\frac{\alpha}{2m_Q} S_j \langle N(p')|E_i^a (D_i B_j)^a|N(p)\rangle$
- **Suppressed by** $1/m_Q$ **in comparison with the binding potential**
- Can be simplified to the effective potential
  $V_{\rm hfs}(\mathbf{r}) = \frac{g_A^{(0)}\alpha}{m_Q}\frac{\pi M_A^4}{18N_f}\;\frac{e^{-M_A r}}{r}\,(2 - M_A r)\;(\mathbf{S}\cdot\mathbf{s_N})$
  where $g_A^{(0)}$ - singlet axial form factor at $q^2 = 0$, M_A- dipole mass parameter, $\boldsymbol{S}$ - charmonium spin, $\boldsymbol{s}_N$ - nucleon spin

Hyperfine splitting

Hyperfine mass splitting between $J^P = 1/2^-$ and $3/2^-$ hadrocharmonium pentaquarks as a function of the dipole mass parameter M_A

M_A [GeV]	0.8	0.9	1.0	1.1
$\Delta E_{\rm hfs}$ [MeV]	21.1	27.7	34.9	42.5

Compatible with experimental splitting between $P_c(4457)^+$ and $P_c(4440)^+$

More Pentaquark states

Flavor Symmetry

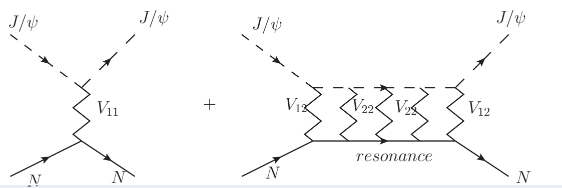
- Nucleon is a member of the baryon octet
- In the linear approximation $\psi(2S)B$ binding energy is proportional to the baryon octet splitting ΔM
$\Delta E = -\frac{\mu_1}{m_N^2}\left\langle N\left|-\frac{\boldsymbol{\nabla}^2}{2\mu_1}\right|N\right\rangle \Delta M$
- **We predict octet of pentaquarks!**
- **Gell-Mann-Okubo formula for pentaoctet:** $\frac{m_{P_N}+m_{P_\Xi}}{2}=\frac{m_{P_\Sigma}+3m_{P_\Lambda}}{4}$

TABLE I. Penta Octet ($J^P = 3/2^-$): Masses and Widths

P_B[a]	Mass (MeV)	$M_P - M_{P_c}$ (MeV)[b]	$M_B - M_N$ (MeV)[c]	Width (MeV)[d]
P_N ($P_c(4450)$)	4449	0	0	11
P_Σ	4665	217	253	14
P_Λ	4598	150	176	13
P_Ξ	4776	327	378	15

Resonance Width

How to calculate partial width $P_c \to J/\psi + N$?



- **Resonance in $J/\psi N$ scattering in two-channel problem: $J/\psi + N$ and $\psi(2S) + N$**

Two-Channel Scattering Problem

- **Scattering problem (E - nonrelativistic $J/\psi N$ energy in the CM frame)**

$$H\Psi = E\Psi, \qquad \Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$H = \left(\begin{array}{c|c} -\frac{\nabla^2}{2\mu_1} + V_{11} & V_{12} \\ \hline V_{12} & -\frac{\nabla^2}{2\mu_2} + V_{22} + \Delta \end{array}\right)$$

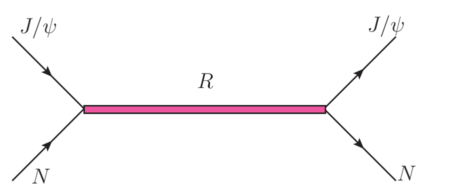
μ_1 – J/ψ-N **reduced mass,** μ_2 – $\psi(2S)$-N **reduced mass,** $\Delta = m_{\psi(2S)} - m_{J/\psi}$

- $\psi_1(\boldsymbol{x}) = e^{i\boldsymbol{q}\cdot\boldsymbol{x}}$ **– incoming plane wave in $J/\psi + N$ channel**
- **Perturbation theory solution**

$$\Psi_2(\boldsymbol{x}) = -\int d^3x' G_2(\boldsymbol{x}, \boldsymbol{x}') V_{12}(\boldsymbol{x}') e^{i\boldsymbol{q}\cdot\boldsymbol{x}'}$$

$$G_2(\boldsymbol{x}, \boldsymbol{x}') = \left\langle \boldsymbol{x} \left| \frac{1}{-\frac{\nabla^2}{2\mu_2} - E + \Delta + V - i0} \right| \boldsymbol{x}' \right\rangle$$

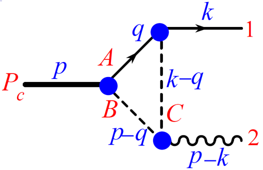
Resonance Scattering



- **Near the resonance**
  $G_2(\boldsymbol{x}, \boldsymbol{x}') = \frac{\psi_R(\boldsymbol{x})\psi_R^*(\boldsymbol{x}')}{E_R - E}$

- **Outgoing wave**
  $\delta\Psi_1(x) = \int d^3x' G_1(\boldsymbol{x}, \boldsymbol{x}') V_{12}(\boldsymbol{x}') \psi_R^*(\boldsymbol{x}') \frac{\int d^3x'' V_{12}(\boldsymbol{x}'')\psi_R(\boldsymbol{x}'') e^{i\boldsymbol{q}\cdot\boldsymbol{x}''}}{E_R - E}$
- **Wave function at large distances in $J/\psi N$ channel**
  $\Psi_1(\boldsymbol{x}) + \delta\Psi_1(x) = e^{i\boldsymbol{q}\cdot\boldsymbol{x}} + f(\theta)\frac{e^{iqr}}{r}$
- **Breit-Wigner resonance formula**
  $$f(\theta) = -\frac{2l+1}{q}\frac{\Gamma/2}{E - E_R} P_l(\cos\theta)$$
- **Partial width Γ in $J/\psi N$ channel**
  $\Gamma = \left(\frac{\alpha(2S\to 1S)}{\alpha(2S)}\right)^2 (4\mu_1 q) \left|\int_0^\infty dr r^2 R_l(r) V(r) j_l(qr)\right|^2$
- $\Gamma(P_c \to J/\psi + N) = 11$ **MeV at** $\alpha(2S \to 1S) = 2$ **GeV**$^{-3}$ **– phenomenological value**

Decays into channels with open charm



Semirelativistic approximation

- **Nonrelativistic constituents but relativistic kinematics for final particles**
- $\Gamma = g_1^2 g_2^2 \frac{k}{4\pi^2} \frac{E_1 E_2}{M_{P_c}} \int d\Omega_k \left| \int d^3 r e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} V(\boldsymbol{r},\boldsymbol{k}) \psi(\boldsymbol{r}) \right|^2$
- $V(\boldsymbol{r},\boldsymbol{k}) = \int \frac{d^3 q}{(2\pi)^3} e^{i\boldsymbol{q}\cdot\boldsymbol{r}} V(\boldsymbol{q},\boldsymbol{k})$
- $V(\boldsymbol{q},\boldsymbol{k})$ – **from relativistic scattering amplitude $\mathcal{A}_{A+B\to 1+2}(\boldsymbol{q},\boldsymbol{k})$ with the nonrelativistic initial particles**
- For spinless particles $V(\boldsymbol{q},\boldsymbol{k}) = 1/[M_*^2(C) + (\boldsymbol{k}-\boldsymbol{q})^2]$, $M_*(C) = \{M_C^2 - [M_A - (M_1^2 + \boldsymbol{k}^2)^{1/2}]^2\}^{1/2}$
- **Account for spin and orbital momenta**

Decays of $P_c(4440)$ and $P_c(4457)$

Decay mode	$\Gamma\left(\frac{1}{2}^-\right)$ MeV	$\Gamma\left(\frac{3}{2}^-\right)$ MeV
$P_c \to J/\psi N$	11	11
$P_c \to \Lambda_c \bar{D}$	18.7	0.6
$P_c \to \Sigma_c \bar{D}$	1.4	0.04
$P_c \to \Lambda_c \bar{D}^*$	13.7	4.2
$P_c \to \Sigma_c^* \bar{D}$	0.004	0.4
Total width	44.8	16.2

- Decays into states with open charm go by heavy meson exchanges
- Three-particle decays with extra pions are either banned kinematically or suppressed due to Goldstone nature of pion
- Decays to open charm of $J^P = 1/2^-$ hadrocharmonium are enhanced
  - $L = 0$ due to central potential is allowed in $J^P = 1/2^-$ decays
  - Only $L = 2$ due to tensor potential is allowed in $J^P = 3/2^-$ decays
  - Central potential is stronger than tensor potential
  - Also larger Clebsch-Gordon coefficients
- LHCb data: $\Gamma_{tot}(P_c(4440)) \simeq 3\Gamma_{tot}(P_c(4457)) \Longrightarrow$ **$P_c(4440)$ – $1/2^-$, $P_c(4457)$ – $3/2^-$**

Contradiction with GlueX: $\mathcal{B}(P_c^+ \to J/\psi p) \leq 3\%$

Decay mode	$\Gamma\left(\frac{1}{2}^-\right)$ MeV	$\Gamma\left(\frac{3}{2}^-\right)$ MeV
$P_c \to J/\psi N$	11	11
$P_c \to \Lambda_c \bar{D}$	18.7	0.6
$P_c \to \Sigma_c \bar{D}$	1.4	0.04
$P_c \to \Lambda_c \bar{D}^*$	13.7	4.2
$P_c \to \Sigma_c^* \bar{D}$	0.004	0.4
Total width	44.8	16.2

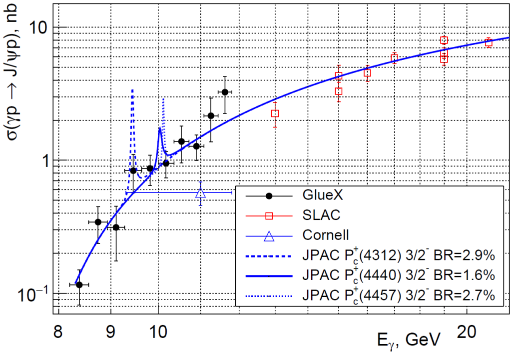


FIG. 5: GlueX results for the J/ψ total cross-section vs beam energy, Cornell [15], and SLAC [16] data compared to the JPAC model [6] corresponding to $\mathcal{B}(P_c^+(4312) \to J/\psi p) = 2.9\%$, $\mathcal{B}(P_c^+(4440) \to J/\psi p) = 1.6\%$, and $\mathcal{B}(P_c^+(4457) \to J/\psi p) = 2.7\%$, for the $J^P = 3/2^-$ case as discussed in the paper.

Narrow resonance $P_c(4312)$

- Bound state of $\chi_{c0}(1P)$ ($J^P = 0^+$) and the nucleon
- Interaction potential is proportional to chromoelectric polarizability tensor $\chi_{c0}(1P)$
  $\alpha_{ik} = \alpha_1(J,S)\delta_{ik} + \alpha_2(J,S)J_iJ_k$
- Effective interaction is a sum of the central and tensor potentials
  $H = V_c(r) + V_t\left[(\boldsymbol{n}\cdot\boldsymbol{J})(\boldsymbol{n}\cdot\boldsymbol{J}) - \frac{\boldsymbol{J}^2}{3}\right]$
- $V_c(r)$ differs from $\psi(2S)N$ interaction by the substitution
  $\alpha = (1/3)\sum_i \alpha_{ii}$

Perturbative polarizabilities of heavy quarkonium $1P$ states in units of $\frac{a_B^4 m_Q}{2N_c}$ (a_B is the Bohr radius)

S	J	α_1	α_2	α
0	1	105	−78	53
1	2	79	−13	53
1	1	27	39	53
1	0	53	0	53

Narrow resonance $P_c(4312)$

- We expect that ratios of perturbative polarizabilities are closer to the real world than their absolute values.
- In perturbation theory $\alpha(1P)/\alpha(2S) \approx 0.63$
- **Schrödinger equation with $0.63 \to 0.58$ has solution with mass 4312 MeV and binding energy 42 MeV!**
- **Prediction: $J^P = 1/2^+$ is spin-parity of $P_c(4312)$**
- **No hyperfine partner with approximately the same mass**
- Experimentally (LHCb) $\Gamma_{tot}(P_c(4312) \sim 10$ MeV
- Decays into state with open charm are suppressed in comparison with the decays of $P_c(4457)$ and $P_c(440)$ due to smaller binging energy, compare 42 MeV and ~ 170 MeV
- Total decay width of $\Gamma_{tot}(\chi_{c0}) \approx 10.8$ MeV. Decays into light hadrons dominate
- Decays of loosely bound χ_{c0} explain total width of $P_c(4312)$

Narrow resonance $P_c(4312)$

- **What about the decay $P_c(4312) \to J/\psi + N$, where $P_c(4312)$ was observed?**
- **Transition $\chi_{c0} \to J/\psi$ can be due to three-gluon exchange, estimate is a challenging problem!**

More hadrocharmonium states

- **Trace of polarizability tensor is one and the same for all $1P$ states$\Longrightarrow$ $\chi_{c1}(1P)$, $\chi_{c2}(1P)$, and $h_c(1P)$ should bind with the nucleon**
- **Polarizability of $\eta_c(2S)$ coincides with polarizability of $\psi(2S)$ – also should bind!**

Expected hadrocharmonium pentaquarks

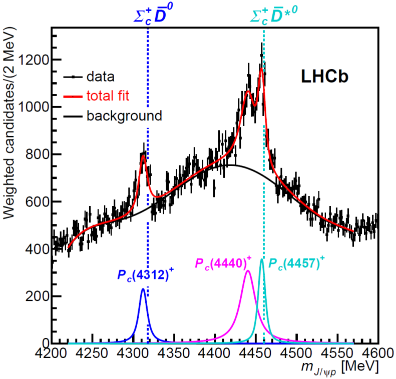
Constituents	Binding energy [MeV]	Mass [MeV]	Spin-parity
$\eta_c(2S)N$	**176.1**	**4401**	$1/2^-$
$\chi_{c1}(1P)N$	**44.2**	**4406**	$3/2^+, 1/2^+$
$h_c(1P)N$	**43.9**	**4421**	$1/2^+, 3/2^+$
$\chi_{c2}(1P)N$	**43.7**	**4452**	$5/2^+, 3/2^+$

- Degeneracy between states with different spins is lifted by hyperfine interaction
- Magnitude of HFS is roughly the same as splitting between $P_c(4440)$ and $P_c(4457)$
- Natural width of all constituents $\sim 1-2$ MeV
- Pentaquarks decays like $(\chi_{c2}(1P)N) \to \chi_{c1}(1P) + N$ go due to nonzero transitional polarizabilities $\alpha_{ik}(\chi_{c2}(1P) \to \chi_{c1}(1P)$
- Hidden charm partial widths at the level of 10-20 MeV
- Comparable widths are expected for charm decaysh
- **In the interval 4380-4430 MeV we expect a grid of hadrocharmonium states with the step 10-15 MeV and widths 10-30 MeV**
- **Speculation: wide LHCb $P_c(4380)$ could be resolved in a series of narrow overlapping resonances**
- $(\chi_{c2}(1P)N)$ **hadrocharmonium with the mass 4452 MeV is between** $P_c(4440)$ **and** $P_c(4457)$**. Interpretation?**

What about Bottomonium-Nucleon Bound States?

- Perturbative polarizability depends on Bohr radius and quark mass: $\alpha(1S) = \frac{78}{425} a_0^4 m_Q$, $\alpha(2S) = \frac{67264}{663} a_0^4 m_Q$
- Bohr radius is smaller than for charmonium, quark mass is larger: $\alpha(1S) \approx 0.07\ \text{GeV}^{-3}$, $\alpha(2S) \approx 5\ \text{GeV}^{-3}$
- **No $\Upsilon(1S)N$ bound state**
- **Inconclusive results for $\Upsilon(1S)N$ bound states**
- **Better handle on polarizability is needed!**

Molecular Pentaquarks?

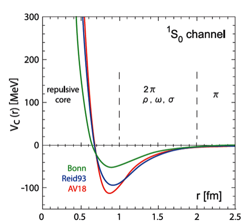


- $P_c(4312)^+$ is 5 MeV below $\Sigma_c^+ \bar{D}^0$ threshold
- $P_c(4457)^+$ is 2 MeV below $\Sigma_c^+ \bar{D}^{*0}$ threshold
- **Are pentaquarks loosely bound molecular states?**

Reminder: Meson Exchanges and Nuclear Potential

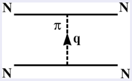
Meson Exchanges, Nuclei, $E_b \sim 8-9$ MeV/N

- Nuclear forces: π, η, σ, ρ,and ω exchanges
- π: Large distance (> 1.5 fm) attraction
- σ (or two-pion): intermediate distance attraction
- ρ, ω: short distance repulsion
- A strong repulsion core at short ($0.3-0.7$ fm) distances is required
- Typical internucleon distance is 0.7-1 fm: pion exchange is strongly suppressed, σ, ρ, ω are dominant
- Sophisticated potentials exist: scores of parameters, successful for description, poor at predictions



Loosely bound Deuteron

- Binding energy 2.2 MeV, $r \sim 2$ fm: very loosely bound system
- Light pion exchange dominates. Törnkvist, 1991: Can one describe deuteron as a result of one-pion exchange?



- One-pion nucleon-nucleon potential in momentum space:

$$V(\boldsymbol{q}) = -\frac{4g_{\pi NN}^2}{M_N^2}(\boldsymbol{T}_1 \cdot \boldsymbol{T}_2)\frac{(\boldsymbol{S}_1 \cdot \boldsymbol{q})(\boldsymbol{S}_2 \cdot \boldsymbol{q})}{\boldsymbol{q}^2 + m_\pi^2}$$

- Coordinate space potential is a sum of spin-spin and tensor potentials:

$$V(\boldsymbol{r}) = V_C(\boldsymbol{r}) + S_{12}(\boldsymbol{S}_1, \boldsymbol{S}_2, \boldsymbol{n})V_T(\boldsymbol{r})$$

$$S_{12}(\boldsymbol{S}_1, \boldsymbol{S}_2, \boldsymbol{n}) = 3(\boldsymbol{S}_1 \cdot \boldsymbol{n})(\boldsymbol{S}_2 \cdot \boldsymbol{n}) - (\boldsymbol{S}_1 \cdot \boldsymbol{S}_2)$$

$$V_C = \frac{g_{\pi NN}^2}{M_N^2}(\boldsymbol{T}_1 \cdot \boldsymbol{T}_2)(\boldsymbol{S}_1 \cdot \boldsymbol{S}_2)\left(m_\pi^2 \frac{e^{-m_\pi r}}{3\pi r} - \frac{4}{3}\delta^{(3)}(\boldsymbol{r})\right)$$

$$V_T = \frac{g_{\pi NN}^2}{M_N^2}(\boldsymbol{T}_1 \cdot \boldsymbol{T}_2)\left(m_\pi^2 r^2 + 3m_\pi r + 3\right)\frac{e^{-m_\pi r}}{3\pi r^3}$$

Molecular Deuteron

One-Pion Exchange Potential

- **Needs improvement at short distances: δ-function and $1/r^3$ are unphysical**
- **Recipe: subtract δ-function, use dipole form factor $[(\Lambda^2 - m_\pi^2)/(\Lambda^2 + \boldsymbol{q}^2)]^2$ or hard core (wall) at small ($\sim 0.4 - 0.5$ fm) distances**
- **Total spin $S = 0, 1$ and isospin $T = 0, 1$ are conserved separately**
- **Central potential vanishes in the chiral limit! Not strong enough to bind nucleons**
- **Binding arises only due to interference of S- and D- waves**
- **Only bound state with the deuteron quantum numbers $S = 1$, $T = 0$ exists for realistic small distance cutoff**
- $E_b = 2.2$ MeV for hard wall at $r_0 = 0.485$ fm. RMS $r = 1.98$ fm, D-wave $\sim 7\%$
- $E_b = 2.2$ MeV for $\Lambda = 800$ MeV. RMS $r = 1.92$ fm, D-wave $\sim 5\%$

Deuteron Hamiltonian

$$H = \left(\begin{array}{c|c} -\frac{\nabla^2}{2\mu} + V_C & \sqrt{8} V_T \\ \hline \sqrt{8} V_T & -\frac{\nabla^2}{2\mu} + \frac{3}{\mu r^2} + V_C - 2V_T \end{array} \right)$$

Nonregularized potentials

$$V_c = -\frac{g^2_{\pi NN} m^2_\pi}{M_N^2} \frac{e^{-m_\pi r}}{16\pi r}, \; V_T = -\frac{g^2_{\pi NN}}{M_N^2} \frac{e^{-m_\pi r}}{16\pi r^3} (3 + 3m_\pi r + m_\pi^2 r^2)$$

Regularized potentials

$$V_{c,reg} = -\frac{g^2_{\pi NN} m^2_\pi}{M_N^2} Y(\Lambda, m_\pi, r), \; V_T = -\frac{g^2_{\pi NN}}{M_N^2} \frac{e^{-m_\pi r}}{16\pi} Z(\Lambda, m_\pi, r)$$

$$Y(\Lambda, m_\pi, r) = \frac{e^{-m_\pi r} - e^{-\Lambda r}}{r} - \frac{\Lambda^2 - m_\pi^2}{2\Lambda} e^{-\Lambda r}, \; Z(\Lambda, m_\pi, r) = r \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} Y(\Lambda, m_\pi, r) \right)$$

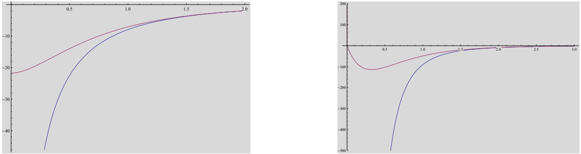


Figure: **Central (left) and tensor (right) potentials**

Loosely bound Pentaquarks?

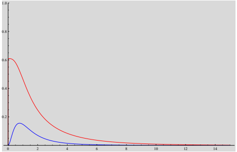


Figure: Deuteron S (red) and D (blue) waves

Hadrons with open charm

- **Charmed baryons:**
  - $\Lambda' s(udc)$: $\Lambda_c^+(2286)(I=0, J=\frac{1}{2}^+)$, $\Lambda_c^+(2595)(I=0, J=\frac{1}{2}^-)$, $\Lambda_c^+(2625)(I=0, J=\frac{3}{2}^-)$, $\Lambda_c^+(2880)(I=0, J=\frac{5}{2}^+)$
  - $\Sigma' s(uuc, udc, ddc)$: $\Sigma_c(2455)(I=1, J^P=\frac{1}{2}^+)$, $\Sigma_c^*(2520)(I=1, J^P=\frac{3}{2}^+)$, $\Sigma_c(2800)(I=1, J^P=?)$
- **Charmed Mesons ($D^+(c\bar{d})$, $D^0(c\bar{u})$, $D^-(\bar{c}d)$):**
  - $D^0(1865)(I=\frac{1}{2}, J^P=0^-)$, $D^\pm(1870)(I=\frac{1}{2}, J^P=0^-)$, $D^{*0}(2007)(I=\frac{1}{2}, J^P=1^-)$, $D^{*+}(2010)(I=\frac{1}{2}, J^P=1^-)$

Loosely bound Pentaquarks?

One-Pion Exchange Potential for Pentaquarks

- **Törnkvist (1991,1994): one-pion mechanism with interplay of waves with different orbital momenta for hidden charm tetraquarks.**
- **Can such mechanism work for hidden charm pentaquarks?**
- **Can** $\Sigma_c^*(2520)$ **and** $\bar{D}(1870)$ **form** $P_c(4380)$**?** $(M_{\Sigma_c^*} + M_{\bar{D}}) - M_{P_c(4380)} \approx 10 MeV$
- **One-pion exchange cannot bind** $\Sigma_c^* \bar{D}(1870)$**,** $\pi\pi D$ **vertex is banned by parity**
- **Can** $\Sigma_c(2455)$ **and** $\bar{D}^*(2010)$ **form** $P_c(4450)$**?** $(M_{\Sigma_c} + M_{\bar{D}^*}) - M_{P_c(4450)} \approx 15 MeV$
- **Unlike deuteron, total spin does not commute with one-pion potential. Only total angular momentum is conserved**
- $\Sigma_c \bar{D}^*(2010)$ **with** $J = 3/2$ **is a superposition of three states** $|L = 0, S = 3/2\rangle$**,** $|L = 2, S = 1/2\rangle$**, and** $|L = 2, S = 3/2\rangle$

One-Pion Exchange Potential for Pentaquarks

Hamiltonian in $J=3/2,\ T=1/2$ subspace

$$H=\left(\begin{array}{c|c|c} -\frac{\nabla^2}{2\mu}+V_C & -\frac{1}{2}V_T & V_T \\ \hline -\frac{1}{2}V_T & -\frac{\nabla^2}{2\mu}+\frac{3}{\mu r^2}-2V_C & \frac{1}{2}V_T \\ \hline V_T & \frac{1}{2}V_T & -\frac{\nabla^2}{2\mu}+\frac{3}{\mu r^2}+V_C \end{array}\right)$$

Nonregularized potentials

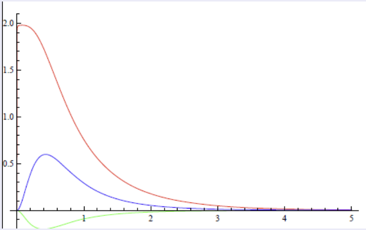
$$V_C(r)=-\frac{m_\pi^2}{F_\pi^2}\frac{e^{-m_\pi r}}{18\pi r},\ V_T(r)=-\frac{1}{F_\pi^2}(3+3m_\pi r+m_\pi^2 r^2)\frac{e^{-m_\pi r}}{9\pi r^3}$$

Regularized potentials

$$V_{C,reg}(r)=-\frac{m_\pi^2}{F_\pi^2}\frac{1}{18\pi}Y(\Lambda,m_\pi,r),\ V_{T,reg}(r)=-\frac{1}{F_\pi^2}\frac{1}{9\pi}Z(\Lambda,m_\pi,r)$$

$\Sigma_c \bar{D}^*$ bound state as $P_c(4450)$

- **Nonregularized potential and hard core at $r_0 = 0.33$ fm: $E_b = 14.7$ MeV, $J^P = 3/2^-$, $T = 1/2$, RMS $r = 1.6$ fm. D-wave fraction 18%. Steep dependence on r_0**
- **Regularized potential, $\Lambda = 1430$ MeV: $E_b = 14.7$ MeV, $J^P = 3/2^-$, $T = 1/2$, RMS $r = 1.24$ fm. D-wave fraction 12%. Steep dependence on Λ**
- **No $\Sigma_c \bar{D}^*$ bound state with other quantum numbers**



- **Drawback: fine tuning needed!**

Are there other deuteronlike pentaquarks?

$\Sigma_c^*\bar{D}^*$ bound state

- **Only total angular momentum is conserved**
- $\Sigma_c^*\bar{D}^*(2010)$ **with** $J = 5/2$ **is a superposition of four states** $|L=0, S=5/2\rangle$, $|L=2, S=1/2\rangle$, $|L=2, S=3/2\rangle$, **and** $|L=2, S=5/2\rangle$
- Hamiltonian

$$\begin{pmatrix} -\frac{\nabla^2}{2\mu}+V_C & -\sqrt{\frac{3}{5}}V_T & \frac{\sqrt{21}}{10}V_T & \frac{3}{5}\sqrt{14}V_T \\ -\sqrt{\frac{3}{5}}V_T & -\frac{\nabla^2}{2\mu}+\frac{3}{\mu r^2}-\frac{5}{3}V_C & \frac{1}{2}\sqrt{\frac{7}{5}}V_T & 2\sqrt{\frac{6}{35}}V_T \\ \frac{\sqrt{21}}{10}V_T & \frac{1}{2}\sqrt{\frac{7}{5}}V_T & -\frac{\nabla^2}{2\mu}+\frac{3}{\mu r^2}-\frac{2}{3}V_C+\frac{8}{7}V_T & -\frac{1}{7}\sqrt{\frac{3}{2}}V_T \\ \frac{3}{5}\sqrt{14}V_T & 2\sqrt{\frac{6}{35}}V_T & -\frac{1}{7}\sqrt{\frac{3}{2}}V_T & -\frac{\nabla^2}{2\mu}+\frac{3}{\mu r^2}+V_C+\frac{6}{7}V_T \end{pmatrix}$$

- **Nonregularized potentials** $V_C(r) = -\frac{m_\pi^2}{F_\pi^2}\frac{e^{-m_\pi r}}{12\pi r}$, $V_T(r) = -\frac{1}{F_\pi^2}(3+3m_\pi r+m_\pi^2 r^2)\frac{e^{-m_\pi r}}{18\pi r^3}$
- **Regularized potentials** $V_{C,reg}(r) = -\frac{m_\pi^2}{F_\pi^2}\frac{1}{12\pi}Y(\Lambda, m_\pi, r)$, $V_{T,reg}(r) = -\frac{1}{F_\pi^2}\frac{1}{18\pi}Z(\Lambda, m_\pi, r)$

$\Sigma_c^* \bar{D}^*$ bound state

- $M_{\Sigma_c^*} + M_{\bar{D}^*} = 4530$ MeV
- **No loosely bound states with $E_b \sim 10 - 20$ MeV!**
- **Bound state at the position of $P_c(4450)$, $E_b = 82$ MeV, $J^P = 5/2^-$, $T = 1/2$, $\Lambda = 1400$ MeV**
- **Could be better match to $P_c(4450)$ than $\Sigma_c \bar{D}^*$ bound state at $\Lambda = 2000$ MeV and phenomenological constants**
- **But RMS radius $r = 0.78$ fm (too small), fraction of D wave 25% (too large), steep dependence on Λ**
- **More $\Sigma_c^* \bar{D}^*$ bound states:**
  - $M = 4526$ **MeV,** $J = 1/2$, $T = 3/2$, $E_b = 1.4$ **MeV, RMS** $r = 3$ **fm, fraction of** D **wave 6%,** $\Lambda = 1400$ **MeV**
  - **Almost ideal deuteronlike state, but steep dependence on Λ makes it unreliable!**

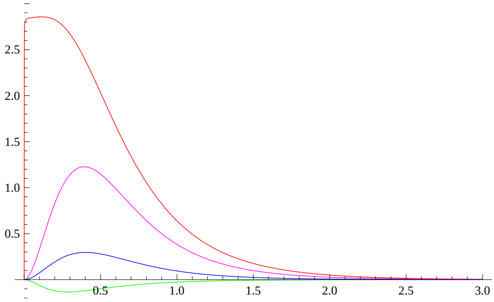


Figure: **Normalized wave functions of the $\Sigma_c^*\bar{D}^*$ bound state with $J^P = 5/2^-$, $T = 1/2$ and the binding energy 74.8 MeV. The $|L=0, S=5/2\rangle$, $|L=2, S=1/2\rangle$, $|L=2, S=3/2\rangle$, and $|L=2, S=5/2\rangle$ wave functions are red, green, blue, and respectively. Axis x is in fermi and $\Lambda = 1400$ MeV**

Decay widths in Molecular and Hadrocharmonium Pictures

TABLE I. Pentaquark $P_c(4450)$ decay widths in the molecular picture.

Decay mode	L[a]	k[b] (MeV)	m_*[c] (MeV)	Γ[d] (MeV)
$P_c \to \Lambda_c \bar{D}$	2	798	136	6.8
$P_c \to \Sigma_c \bar{D}$	2	529	128	1.4
$P_c \to \Lambda_c \bar{D}^*$	0, 2	579	101	13.3
$P_c \to \Sigma_c^* \bar{D}$	0, 2	360	107	0.2
$P_c \to J/\psi N$	0	820	1421	0.03
Total width				21.7

[a]Lowest allowed orbital momentum.
[b]Final momentum.
[c]Effective exchanged mass.
[d]Decay width.

TABLE IV. Pentaquark $P_c(4450)$ decay widths in the hadrocharmonium picture.

Decay mode	L[a]	k[b] (MeV)	$M_*(D)$[c] (MeV)	Γ[d] (MeV)
$P_c \to J/\psi N$	0	820		11
$P_c \to \Lambda_c \bar{D}$	2	798	1133	0.6
$P_c \to \Sigma_c \bar{D}$	2	529	1005	0.04
$P_c \to \Lambda_c \bar{D}^*$	0,2	579	1218	4.2
$P_c \to \Sigma_c^* \bar{D}$	0,2	360	959	0.4
Total width				16.2

[a]Lowest allowed orbital momentum.
[b]Final momentum.
[c]Effective exchanged mass.
[d]Decay width.

Molecular $\Gamma(P_c \to J/\psi p)$ is compatible with the GlueX branching $\mathcal{B} \leq 3\%$!

Summary on Molecular Pentaquarks

- **Include exchanges by other light mesons: σ, ρ, ω, and η**
- **The one-pion exchange scenario is reliable if the characteristics of the bound states (RMS radius, fraction of the D-wave squared, parameter Λ) remain stable**
- **Rather substantial changes of parameters are observed!**
- **One-pion exchange scenario is not stable with respect to inclusion of other light meson exchanges. Hard to insist that it is the dominant binding mechanism even for loosely bound pentaquarks**
- **Both the one-pion exchange and nuclear type scenarios for pentaquarks suffer from steep dependence on the short distance regularization parameter Λ (or position of the hard wall at small distances)**
- Almost any experimental data on pentaquarks can be described by nuclear type and/or one-pion exchange potentials

Summary on Molecular Pentaquarks Pentaquarks

- **Dependence on short distance regularization and lack of theoretical handles on the magnitude of Λ deprive both molecular scenarios of predictive power**
- No natural explanation for $P_c(440)$ and $P_c(4457)$ splitting
- For $P_c(4312)$ no $\pi\pi D$ vertex
- **New LHCb pentaquarks are close to open charm meson-baryon thresholds**
- **Branching to j/ψ is compatible with the GlueX data**
- But LHCb discovered pentaquarks in $J/\psi p$ channel

Summary on Hadrocharmonium Pentaquarks

- **Effective interaction potential between small quarkonium and ordinary baryons is due to chromoelectric dipole interaction**
- $P_c(4440)$, $J^P = 1/2^-$, $\Gamma_{tot} \sim 45$ **MeV and** $P_c(4457)$, $J = 3/2^-$, $\Gamma \sim 16$ **MeV are** $\psi(2S)N$ **bound states**
- **Hyperfine splitting** $P_c(4440)$-$P_c(4457)$ **arises naturally and is explained quantitatively**
- $P_c(4312)$, $J^P = 1/2^+$, $\Gamma \sim 11$ MeV is a $\chi_{c0}N$ bound state
- **Narrow bound states** $\eta_c(2S)N$, $\chi_{c1}(1P)N$, $h_c(1P)N$, $\chi_{c2}(1P)N$ **with known masses and spin-parities in the same mass region as** $P_c(4380)$ **are predicted**
- **Spectrum of pentaquarks mimics spectrum of low lying baryons. Pentaquarks come in flavor octets with mass splittings subject to Gell-Mann-Okubo splittings roughly the same as in the baryon octet**

Summary on Hadrocharmonium Pentaquarks

- Spin degeneracy between states with identical quantum numbers but different spins is lifted in QCD multipole expansion. Splittings are roughly an order of magnitude smaller than binding energies
- Possible existence of new pentaquarks that are bound states of bottomonia and ordinary baryons should be explored
- Apparent inconsistency with GlueX data

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More experimental and theoretical work is needed!

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Thank you!