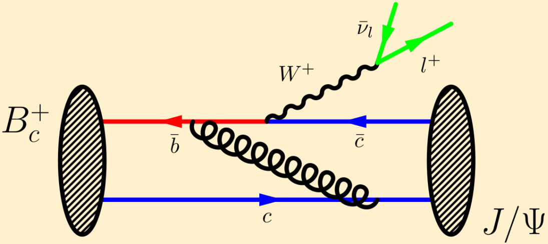
Model-Independent Constraints on Semileptonic decays of B_c^+

Hank Lamm
with Tom Cohen, Rich Lebed, and Anson Berns

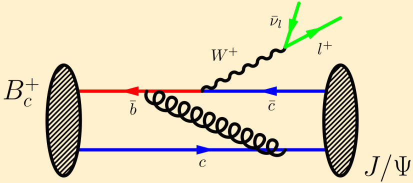


Based on
1807.02730
1808.07360

UNIVERSITY OF MARYLAND

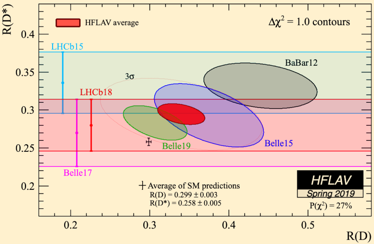
Outline

1. $R_\pi, R_D, R_{D^*}, R_{J/\Psi}$ thats 5 Rs
2. The X Factors
3. Dispersive Approaches
4. Leaps on Bounds



Outline

1. $R_\pi, R_D, R_{D^*}, R_{J/\Psi}$ thats 5 Rs
2. The X Factors
3. Dispersive Approaches
4. Leaps on Bounds

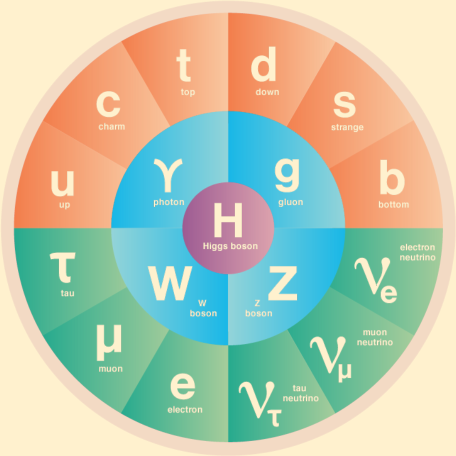


Who ordered that?

Within the Standard Model, *lepton universality* is broken only by the Higgs interaction

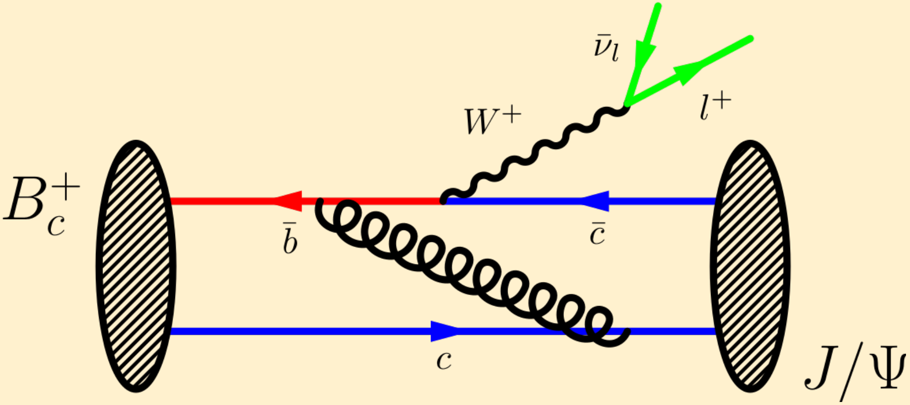
Who ordered that?

Within the Standard Model, *lepton universality* is broken only by the Higgs interaction

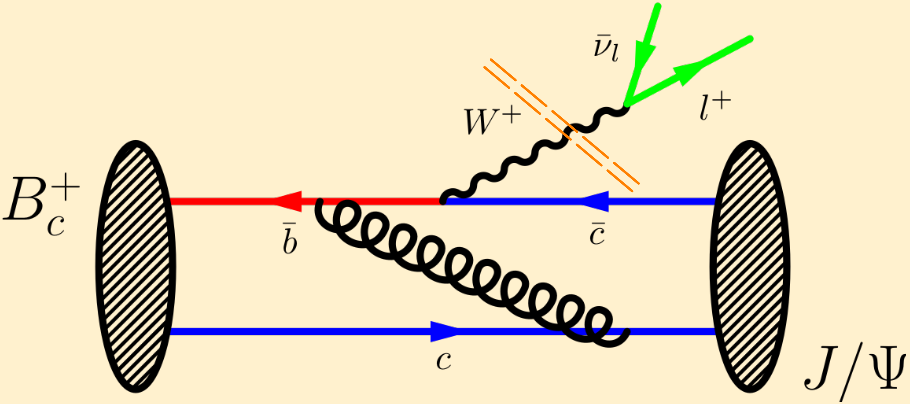


...but m_ν implies this isn't the end of the story

...so let's do some precision physics!

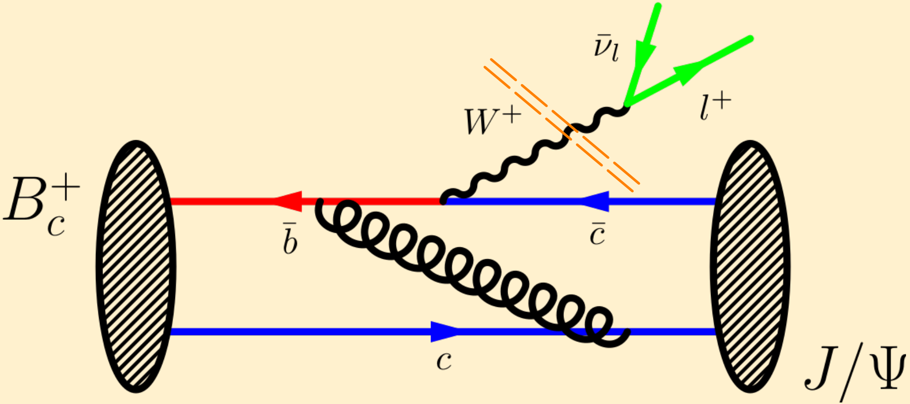


...so let's do some precision physics!



$$\mathcal{M}_{\bar{b}\to\bar{c}l\bar{\nu}} = \frac{L^\mu H_\mu}{q^2+M_W^2} + \mathcal{O}(\alpha_{em}, G_F)$$

...so let's do some precision physics!



$$\mathcal{M}_{\bar{b}\to\bar{c}l\bar{\nu}} = \frac{L^\mu H_\mu}{q^2+M_W^2} + \mathcal{O}(\alpha_{em}, G_F)$$

$$R(h_b \to h_c) \equiv \frac{\mathcal{B}(h_b\to h_c\tau\bar{\nu}_\tau)}{\mathcal{B}(h_b\to h_c l\bar{\nu}_l)} = ???$$

Why not 1 and done?

Why not 1 and done?

After angular integration, $R(h_b \to h_c)$ depends only on the q^2 and m_l in a form:

$$R(h_b \to h_c) = \frac{\int_0^\infty dq^2 \sum_i w_i(m_\tau, q^2) F_i(q^2)}{\int_0^\infty dq^2 \sum_i w_i(m_l, q^2) F_i(q^2)} \quad (1)$$

Why not 1 and done?

After angular integration, $R(h_b \to h_c)$ depends only on the q^2 and m_l in a form:

$$R(h_b \to h_c) = \frac{\int_0^\infty \mathrm{d}q^2 \sum_i w_i(m_\tau, q^2) F_i(q^2)}{\int_0^\infty \mathrm{d}q^2 \sum_i w_i(m_l, q^2) F_i(q^2)} \quad (1)$$

Trivial examples:

$q^2_{min} = m_l^2$ and $q^2_{max} = (m_{h_b} - m_{h_c})^2 \approx (m_b - m_c)^2 \approx (3\ \text{GeV})^2$, then taking simple forms of $w_i F_i$ yield varied $R(h_b \to h_c)$

Why not 1 and done?

After angular integration, $R(h_b \to h_c)$ depends only on the q^2 and m_l in a form:

$$R(h_b \to h_c) = \frac{\int_0^\infty \mathrm{d}q^2 \sum_i w_i(m_\tau, q^2) F_i(q^2)}{\int_0^\infty \mathrm{d}q^2 \sum_i w_i(m_l, q^2) F_i(q^2)} \tag{1}$$

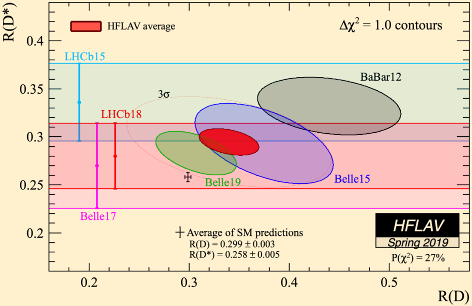
Trivial examples:

$q^2_{min} = m_l^2$ and $q^2_{max} = (m_{h_b} - m_{h_c})^2 \approx (m_b - m_c)^2 \approx (3 \text{ GeV})^2$, then taking simple forms of $w_i F_i$ yield varied $R(h_b \to h_c)$

$w_i F_i = (q^2)^n$	$R(h_b \to h_c)$
$n = -1$	0.16
$n = 0$	0.65
$n = 1$	0.88

Ratios of semileptonic $b-$quark decays, they persisted...

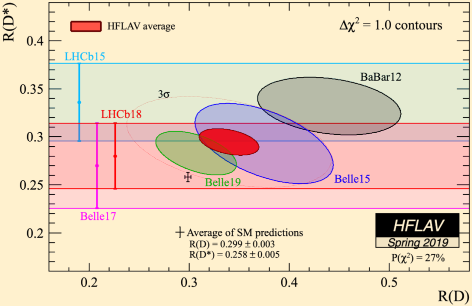
Ratio	Exp	R_{exp}	R_{theory}
$R(B \to \pi^-)$	BELLE	< 1.93 (95% CL)	0.641(17)
$R(B \to D)$	HFLAV	$0.340(27)_{stat}(13)_{syst}$	0.299(3)
$R(B \to D^*)$	HFLAV	$0.295(0.011)_{stat}(0.008)_{syst}$	0.258(5)
$R(B_c^+ \to J/\Psi)$	LHCb[1]	$0.71(0.17)_{stat}(0.18)_{syst}$	



[1] R. Aaij et al. "Measurement of the ratio of branching fractions $\mathcal{B}(B_c^+ \to J/\psi\tau^+\nu_\tau)/\mathcal{B}(B_c^+ \to J/\psi\mu^+\nu_\mu)$". In: (2017). arXiv: 1711.05623 [hep-ex].

Ratios of semileptonic $b-$quark decays, they persisted...

Ratio	Exp	R_{exp}	R_{theory}
$R(B \to \pi^-)$	BELLE	< 1.93 (95% CL)	0.641(17)
$R(B \to D)$	HFLAV	$0.340(27)_{stat}(13)_{syst}$	0.299(3)
$R(B \to D^*)$	HFLAV	$0.295(0.011)_{stat}(0.008)_{syst}$	0.258(5)
$R(B_c^+ \to J/\Psi)$	LHCb[1]	$0.71(0.17)_{stat}(0.18)_{syst}$	



[1] R. Aaij et al. "Measurement of the ratio of branching fractions $\mathcal{B}(B_c^+ \to J/\psi\tau^+\nu_\tau)/\mathcal{B}(B_c^+ \to J/\psi\mu^+\nu_\mu)$". In: (2017). arXiv: 1711.05623 [hep-ex].

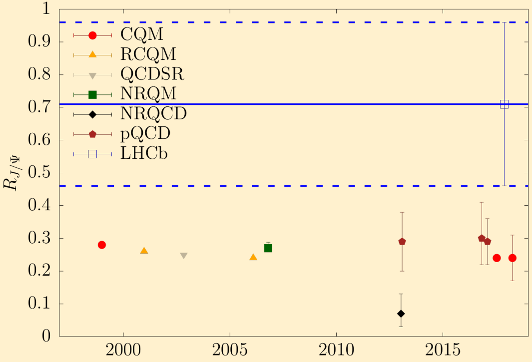
Are we viewing lepton non-universality in $R_{J/\psi}$?

Are we viewing lepton non-universality in $R_{J/\psi}$?

...but what does the Standard Model actually predict?

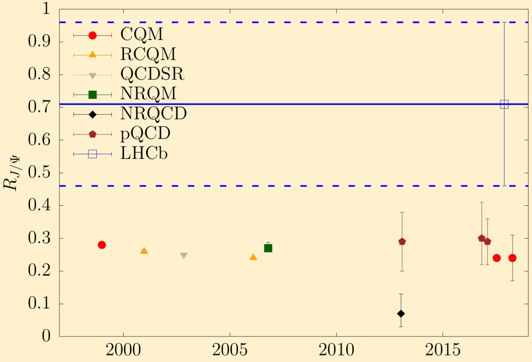
Only model-dependent predictions exist

Model	R_{theory}	Year
CQM	0.28	1998
RCQM	0.26	2000
QCDSR	0.25	2003
RCQM	0.24	2006
NRQM	$0.27^{+0.02}_{-0}$	2006
NRQCD	$0.07^{+0.06}_{-0.04}$	2013
pQCD	$0.29^{+0.09}_{-0.09}$	2013
pQCD	$0.30^{+0.11}_{-0.08}$	2016
pQCD	$0.29^{+0.07}_{-0.07}$	2017
CQM	0.24	2017
CQM	$0.24^{+0.07}_{-0.07}$	2018
Range	**[0,0.55]**	–



Only model-dependent predictions exist

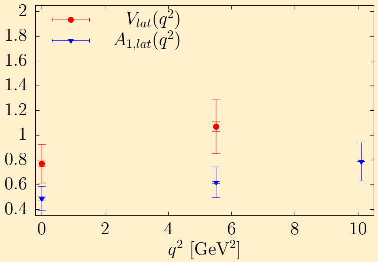
Model	R_{theory}	Year
CQM	0.28	1998
RCQM	0.26	2000
QCDSR	0.25	2003
RCQM	0.24	2006
NRQM	$0.27^{+0.02}_{-0}$	2006
NRQCD	$0.07^{+0.06}_{-0.04}$	2013
pQCD	$0.29^{+0.09}_{-0.09}$	2013
pQCD	$0.30^{+0.11}_{-0.08}$	2016
pQCD	$0.29^{+0.07}_{-0.07}$	2017
CQM	0.24	2017
CQM	$0.24^{+0.07}_{-0.07}$	2018
Range	**[0,0.55]**	–



Taking the largest/smallest $\mathcal{B}(B_c^+ \to J/\psi \tau^+ \bar{\nu}_\tau)$ and $\mathcal{B}(B_c^+ \to J/\psi l^+ \bar{\nu}_l)$ and compute a **worst-case** scenario $R_{J/\psi} = [0, 3]$

Outline

1. $R_\pi, R_D, R_{D^*}, R_{J/\Psi}$ thats 5 Rs
2. The X Factors
3. Dispersive Approaches
4. Leaps on Bounds



What's the worst that can happen?

The structure of the **Standard Model** puts **restrictions** on how the hadronic matrix element can vary

What's the worst that can happen?

The structure of the **Standard Model** puts **restrictions** on how the hadronic matrix element can vary

$$\langle V(p',\epsilon)|V^\mu - A^\mu|P(p)\rangle = \frac{2i\epsilon^{\mu\nu\rho\sigma}}{M+m}\epsilon^*_\nu p'_\rho p_\sigma V(q^2) - (M+m)\epsilon^{*\mu}A_1(q^2) + \frac{\epsilon^*\cdot q}{M+m}(p+p')^\mu A_2(q^2) + 2m\frac{\epsilon^*\cdot q}{q^2}q^\mu A_3(q^2) - 2m\frac{\epsilon^*\cdot q}{q^2}q^\mu A_0(q^2) \quad (2)$$

$$A_3(q^2) = \frac{M+m}{2m}A_1(q^2) - \frac{M-m}{2m}A_2(q^2) \quad (3)$$

where $A_3(0) = A_0(0)$ and the masses are given by $M = m_P, m = m_V$

Factors on Factors on Factors....

Factors on Factors on Factors....

$$
\begin{aligned}
g &= \frac{2V}{M+m} \qquad f = (M+m)A_1 \\
a_+ &= -\frac{A_2}{M+m} \\
a_- &= -\frac{2m}{t}\left(\frac{M+m}{2m}A_1 - \frac{M-m}{2m}A_2 - A_0\right)
\end{aligned} \tag{4}
$$

Factors on Factors on Factors....

$$g = \frac{2V}{M+m} \qquad f = (M+m)A_1$$
$$a_+ = -\frac{A_2}{M+m}$$
$$a_- = -\frac{2m}{t}\left(\frac{M+m}{2m}A_1 - \frac{M-m}{2m}A_2 - A_0\right) \tag{4}$$

$$\mathcal{F}_1 = \frac{1}{m}\left[2k^2 t a_+ - \frac{1}{2}(t - M^2 + m^2)f\right]$$
$$\mathcal{F}_2 = \frac{1}{m}\left[f + (M^2 - m^2)a_+ + t a_-\right] \tag{5}$$

where $t = q^2$, $k^2 = \frac{(t_+ - t)(t_- - t)}{4t}$, $t_\pm = (M \pm m)^2$.

Factors on Factors on Factors....

$$g = \frac{2V}{M+m} \qquad f = (M+m)A_1$$
$$a_+ = -\frac{A_2}{M+m}$$
$$a_- = -\frac{2m}{t}\left(\frac{M+m}{2m}A_1 - \frac{M-m}{2m}A_2 - A_0\right) \tag{4}$$

$$\mathcal{F}_1 = \frac{1}{m}\left[2k^2 t a_+ - \frac{1}{2}(t - M^2 + m^2) f\right]$$
$$\mathcal{F}_2 = \frac{1}{m}\left[f + (M^2 - m^2)a_+ + t a_-\right] \tag{5}$$

where $t = q^2$, $k^2 = \frac{(t_+ - t)(t_- - t)}{4t}$, $t_\pm = (M \pm m)^2$.
There is an additional constraint: $\mathcal{F}_1(t_-) = (M-m)f(t_-)$

What are the ingredients?

What are the ingredients?

LQCD

FV, m_q, a, NRQCD

What are the ingredients?

LQCD

FV, m_q, a, NRQCD

HQSS Relations

$\mathcal{O}(\frac{\Lambda_{QCD}}{m_c}, \frac{m_c}{m_b})$

What are the ingredients?

LQCD

FV, m_q, a, NRQCD

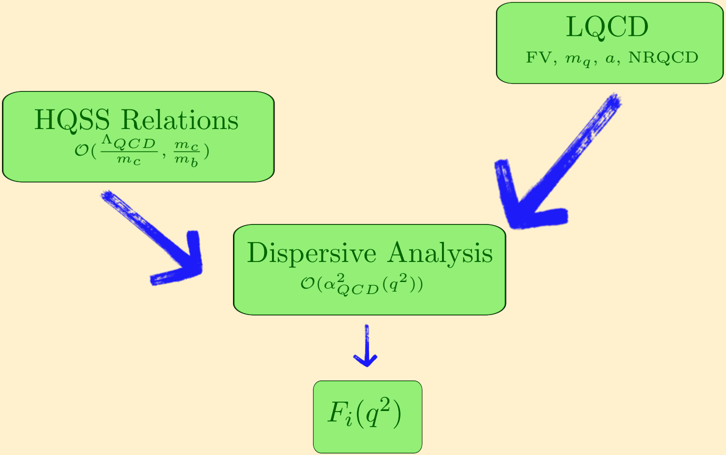
HQSS Relations

$\mathcal{O}(\frac{\Lambda_{QCD}}{m_c}, \frac{m_c}{m_b})$

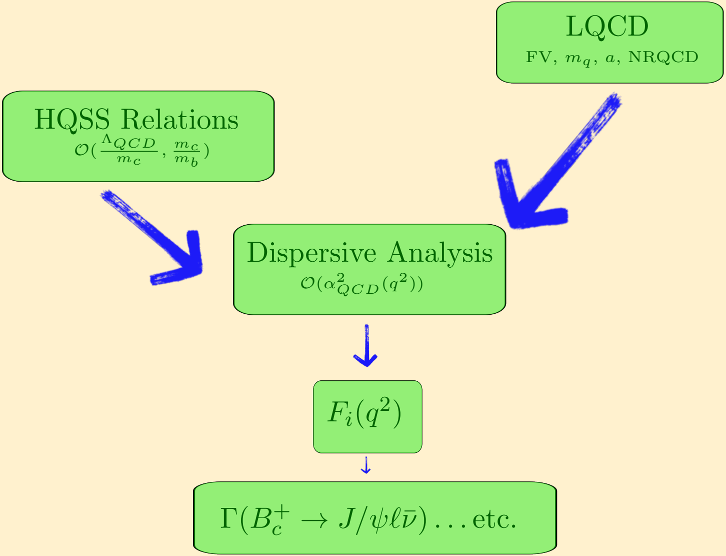
Dispersive Analysis

$\mathcal{O}(\alpha^2_{QCD}(q^2))$

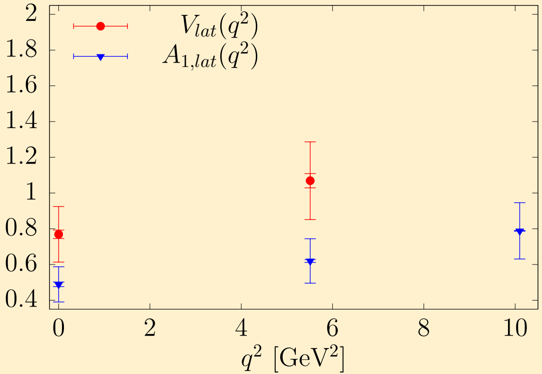
What are the ingredients?



What are the ingredients?



Lattice NRQCD results provide limited input[2]



2+1+1 HISQ, $a = 0.09$ fm, $m_s/m_l \approx 5$ from MILC with NRQCD for b

No data for $\mathcal{F}_1$ and $\mathcal{F}_2$

[2] B. Colquhoun et al. "B_c decays from highly improved staggered quarks and NRQCD". In: *PoS* LATTICE2016 (2016), p. 281. arXiv: 1611.01987 [hep-lat].

HQET reduced heavy-light decays to one form factor[4]

$$\langle B|\bar{b}\Gamma^{\mu}b|B)\rangle$$

[3] S. Rudaz and M. B. Voloshin. “On Exclusive weak decays of Lambda(b)”. In: *Phys. Lett.* B252 (1990), pp. 443–446.

[4] N. Isgur and M. B. Wise. “Weak Decays of Heavy Mesons in the Static Quark Approximation”. In: *Phys. Lett.* B232 (1989), pp. 113–117.

HQET reduced heavy-light decays to one form factor[4]

$$\langle B|\bar{b}\Gamma^\mu b|B)\rangle$$

[3] S. Rudaz and M. B. Voloshin. "On Exclusive weak decays of Lambda(b)". In: *Phys. Lett.* B252 (1990), pp. 443–446.

[4] N. Isgur and M. B. Wise. "Weak Decays of Heavy Mesons in the Static Quark Approximation". In: *Phys. Lett.* B232 (1989), pp. 113–117.

HQET reduced heavy-light decays to one form factor[4]

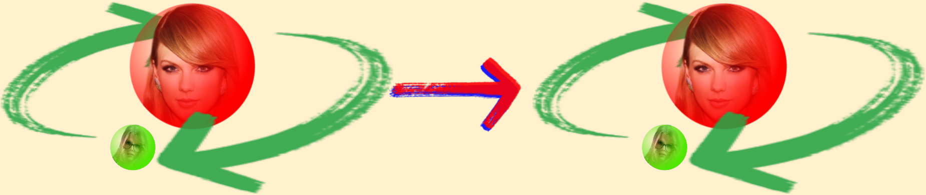
$$\langle B|\bar{b}\Gamma^{\mu}b|B)\rangle$$

[3] S. Rudaz and M. B. Voloshin. "On Exclusive weak decays of Lambda(b)". In: *Phys. Lett.* B252 (1990), pp. 443–446.

[4] N. Isgur and M. B. Wise. "Weak Decays of Heavy Mesons in the Static Quark Approximation". In: *Phys. Lett.* B232 (1989), pp. 113–117.

HQET reduced heavy-light decays to one form factor[4]

$$\langle B|\bar{b}\Gamma^\mu b|B)\rangle$$

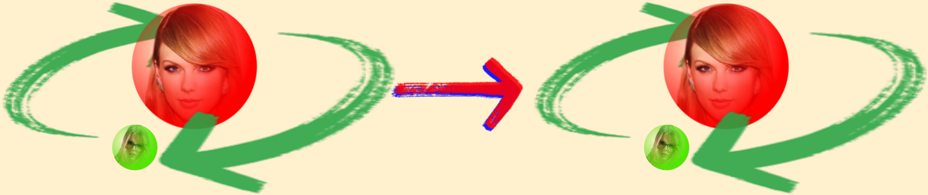


[3] S. Rudaz and M. B. Voloshin. "On Exclusive weak decays of Lambda(b)". In: *Phys. Lett.* B252 (1990), pp. 443–446.

[4] N. Isgur and M. B. Wise. "Weak Decays of Heavy Mesons in the Static Quark Approximation". In: *Phys. Lett.* B232 (1989), pp. 113–117.

HQET reduced heavy-light decays to one form factor[4]

$$\langle B|\bar{b}\Gamma^\mu b|B)\rangle = \mathcal{N}_\Gamma(M,M)\times\xi(w)$$



[3] S. Rudaz and M. B. Voloshin. "On Exclusive weak decays of Lambda(b)". In: *Phys. Lett.* B252 (1990), pp. 443–446.

[4] N. Isgur and M. B. Wise. "Weak Decays of Heavy Mesons in the Static Quark Approximation". In: *Phys. Lett.* B232 (1989), pp. 113–117.

HQET reduced heavy-light decays to one form factor[4]

$$\langle B|\bar{b}\Gamma^\mu b|B)\rangle = \mathcal{N}_\Gamma(M,M)\times\xi(w)$$

$$\langle D^*|\bar{c}\Gamma^\mu b|B)\rangle$$

[3] S. Rudaz and M. B. Voloshin. "On Exclusive weak decays of Lambda(b)". In: *Phys. Lett.* B252 (1990), pp. 443–446.

[4] N. Isgur and M. B. Wise. "Weak Decays of Heavy Mesons in the Static Quark Approximation". In: *Phys. Lett.* B232 (1989), pp. 113–117.

HQET reduced heavy-light decays to one form factor[4]

$$\langle B|\bar{b}\Gamma^{\mu}b|B)\rangle = \mathcal{N}_{\Gamma}(M,M)\times\xi(w)$$

$$\langle D^*|\bar{c}\Gamma^{\mu}b|B)\rangle$$

[3] S. Rudaz and M. B. Voloshin. "On Exclusive weak decays of Lambda(b)". In: *Phys. Lett.* B252 (1990), pp. 443–446.

[4] N. Isgur and M. B. Wise. "Weak Decays of Heavy Mesons in the Static Quark Approximation". In: *Phys. Lett.* B232 (1989), pp. 113–117.

HQET reduced heavy-light decays to one form factor[4]

$$\langle B|\bar{b}\Gamma^\mu b|B\rangle\rangle = \mathcal{N}_\Gamma(M,M)\times\xi(w)$$

$$\langle D^*|\bar{c}\Gamma^\mu b|B\rangle\rangle$$

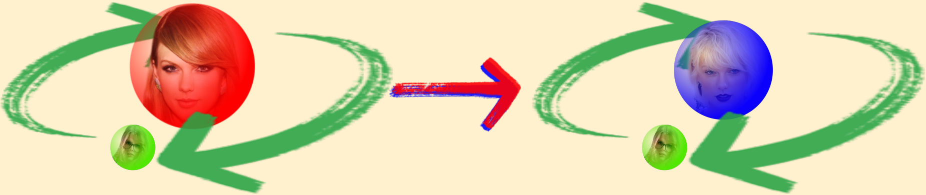
[3] S. Rudaz and M. B. Voloshin. "On Exclusive weak decays of Lambda(b)". In: *Phys. Lett.* B252 (1990), pp. 443–446.

[4] N. Isgur and M. B. Wise. "Weak Decays of Heavy Mesons in the Static Quark Approximation". In: *Phys. Lett.* B232 (1989), pp. 113–117.

HQET reduced heavy-light decays to one form factor[4]

$$\langle B|\bar{b}\Gamma^\mu b|B\rangle\rangle = \mathcal{N}_\Gamma(M,M) \times \xi(w)$$

$$\langle D^*|\bar{c}\Gamma^\mu b|B\rangle\rangle$$



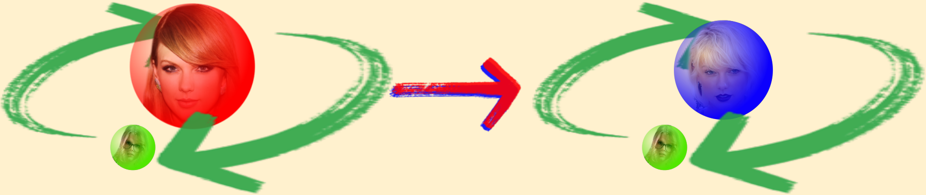
[3] S. Rudaz and M. B. Voloshin. "On Exclusive weak decays of Lambda(b)". In: *Phys. Lett.* B252 (1990), pp. 443–446.

[4] N. Isgur and M. B. Wise. "Weak Decays of Heavy Mesons in the Static Quark Approximation". In: *Phys. Lett.* B232 (1989), pp. 113–117.

HQET reduced heavy-light decays to one form factor[4]

$$\langle B|\bar{b}\Gamma^{\mu}b|B\rangle\rangle = \mathcal{N}_{\Gamma}(M,M)\times\xi(w)$$

$$\langle D^{*}|\bar{c}\Gamma^{\mu}b|B\rangle\rangle$$



static heavy quark in a baryon by another, in which the case the light quarks are not disturbed and there is no "shake-off" damping of the overlap of the light-quark wavefunctions (use of this language in no way

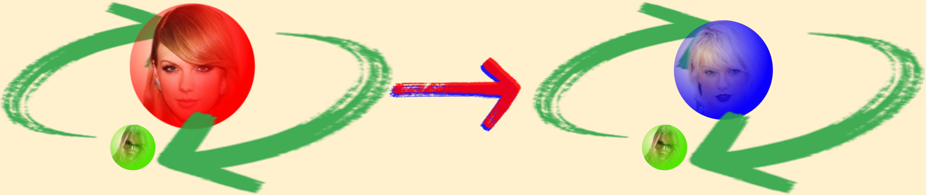
Reproduced from[3]

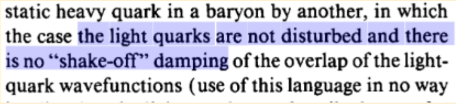
[3] S. Rudaz and M. B. Voloshin. "On Exclusive weak decays of Lambda(b)". In: *Phys. Lett.* B252 (1990), pp. 443–446.

[4] N. Isgur and M. B. Wise. "Weak Decays of Heavy Mesons in the Static Quark Approximation". In: *Phys. Lett.* B232 (1989), pp. 113–117.

HQET reduced heavy-light decays to one form factor[4]

$$\langle B|\bar{b}\Gamma^\mu b|B\rangle\rangle = \mathcal{N}_\Gamma(M,M) \times \xi(w)$$
$$\langle D^*|\bar{c}\Gamma^\mu b|B\rangle\rangle \approx \mathcal{N}_\Gamma(M,m) \times \xi(w)\left[1+\mathcal{O}(\frac{\Lambda}{m_c})\right]$$




static heavy quark in a baryon by another, in which the case the light quarks are not disturbed and there is no "shake-off" damping of the overlap of the light-quark wavefunctions (use of this language in no way

Reproduced from[3]

[3] S. Rudaz and M. B. Voloshin. "On Exclusive weak decays of Lambda(b)". In: *Phys. Lett.* B252 (1990), pp. 443–446.

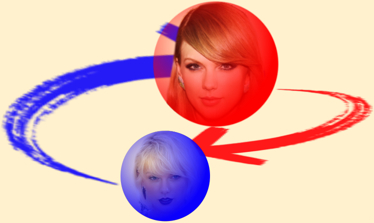
[4] N. Isgur and M. B. Wise. "Weak Decays of Heavy Mesons in the Static Quark Approximation". In: *Phys. Lett.* B232 (1989), pp. 113–117.

HQSS suggests relations on form factors for $B_c^+ \to J/\psi$

$$\langle J/\psi(\boldsymbol{p}=0)|\bar{c}\Gamma^\mu b|B_c^+(\boldsymbol{p}=0)\rangle$$

HQSS suggests relations on form factors for $B_c^+ \to J/\psi$

$$\langle J/\psi(\boldsymbol{p}=0)|\bar{c}\Gamma^\mu b|B_c^+(\boldsymbol{p}=0)\rangle$$

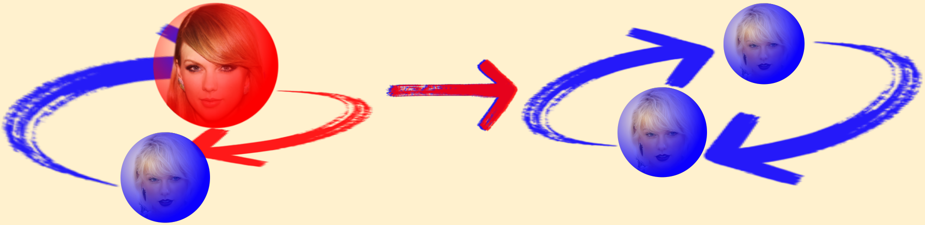


HQSS suggests relations on form factors for $B_c^+ \to J/\psi$

$\langle J/\psi(\boldsymbol{p}=0)|\bar{c}\Gamma^\mu b|B_c^+(\boldsymbol{p}=0)\rangle$

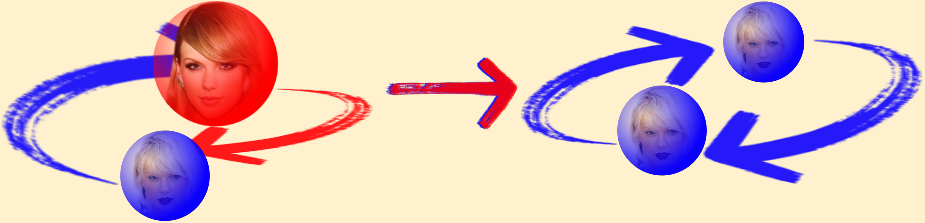
HQSS suggests relations on form factors for $B_c^+ \to J/\psi$

$\langle J/\psi(\boldsymbol{p}=0)|\bar{c}\Gamma^\mu b|B_c^+(\boldsymbol{p}=0)\rangle$



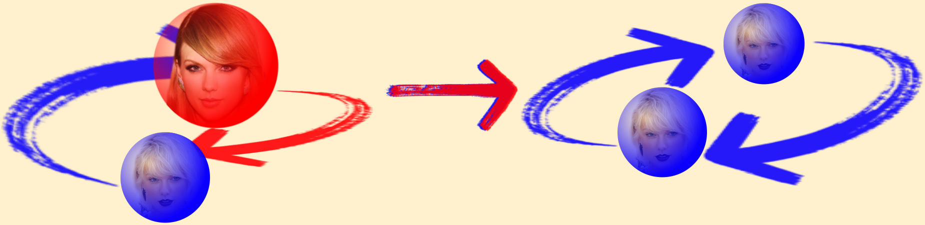
HQSS suggests relations on form factors for $B_c^+ \to J/\psi$

$$\langle J/\psi(\boldsymbol{p}=0)|\bar{c}\Gamma^{\mu}b|B_c^+(\boldsymbol{p}=0)\rangle \approx \mathcal{N}_{\Gamma}(M,m) \times h(1)\left[1+\mathcal{O}\left(\frac{\Lambda_{QCD}}{m_c},\frac{m_c}{m_b}\right)\right]$$



HQSS suggests relations on form factors for $B_c^+ \to J/\psi$

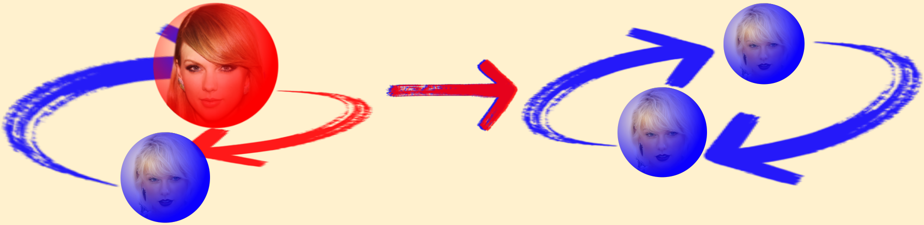
$$\langle J/\psi(\boldsymbol{p}=0)|\bar{c}\Gamma^\mu b|B_c^+(\boldsymbol{p}=0)\rangle \approx \mathcal{N}_\Gamma(M,m)\times h(1)\left[1+\mathcal{O}\left(\frac{\Lambda_{QCD}}{m_c},\frac{m_c}{m_b}\right)\right]$$



Form factors are zero-recoil are **related**!

HQSS suggests relations on form factors for $B_c^+ \to J/\psi$

$$\langle J/\psi(\boldsymbol{p}=0)|\bar{c}\Gamma^\mu b|B_c^+(\boldsymbol{p}=0)\rangle \approx \mathcal{N}_\Gamma(M,m) \times h(1)\left[1+\mathcal{O}\left(\frac{\Lambda_{QCD}}{m_c},\frac{m_c}{m_b}\right)\right]$$

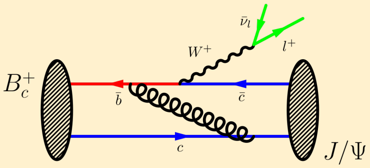


Form factors are zero-recoil are **related**!

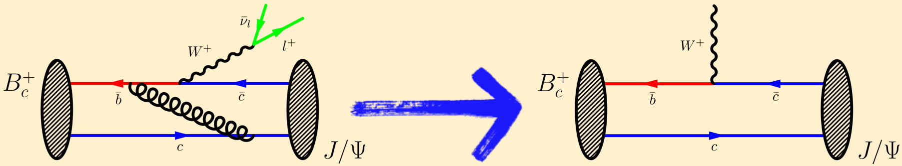
Do we know anything else?

Outline

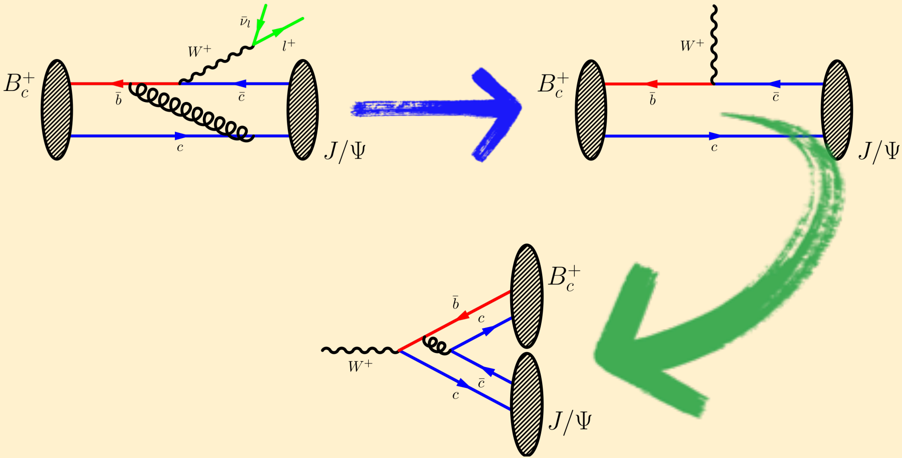
Crossing symmetry...it's a thing.



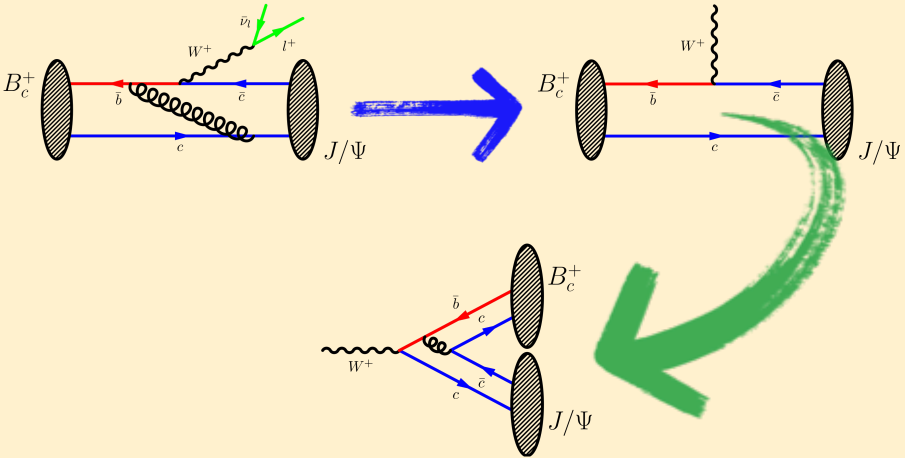
Crossing symmetry...it's a thing.



Crossing symmetry...it's a thing.



Crossing symmetry...it's a thing.



Transition form factors are related to production

Let's talk about analytic structure

Let's talk about analytic structure

Consider a flavor-changing vector-like current between two quarks

$$J^\mu \equiv \bar{c}\Gamma^\mu b\,, \tag{6}$$

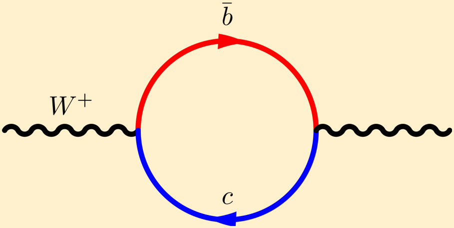
Let's talk about analytic structure

Consider a flavor-changing vector-like current between two quarks

$$J^\mu \equiv \bar{c}\Gamma^\mu b\,, \tag{6}$$

the Green's function, $\Pi_J^{\mu\nu}$, is split into spin-1 (Π_J^T) and spin-0 (Π_J^L)

$$\begin{aligned}\Pi_J^{\mu\nu}(q) &\equiv i\int d^4x\, e^{iqx}\left\langle 0\left|T J^\mu(x) J^{\dagger\nu}(0)\right|0\right\rangle \\ &= \frac{1}{q^2}\left(q^\mu q^\nu - q^2 g^{\mu\nu}\right)\Pi_J^T(q^2) + \frac{q^\mu q^\nu}{q^2}\Pi_J^L(q^2)\,.\end{aligned} \tag{7}$$



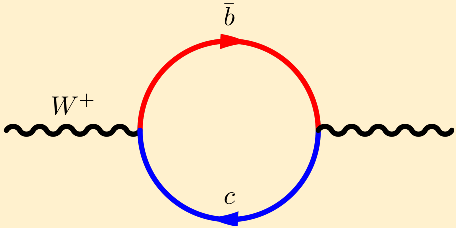
Let's talk about analytic structure

Consider a flavor-changing vector-like current between two quarks

$$J^\mu \equiv \bar{c}\Gamma^\mu b\,, \tag{6}$$

the Green's function, $\Pi_J^{\mu\nu}$, is split into spin-1 (Π_J^T) and spin-0 (Π_J^L)

$$\begin{aligned}\Pi_J^{\mu\nu}(q) &\equiv i\int d^4x\, e^{iqx}\left\langle 0\left|T J^\mu(x) J^{\dagger\nu}(0)\right|0\right\rangle \\ &= \frac{1}{q^2}\left(q^\mu q^\nu - q^2 g^{\mu\nu}\right)\Pi_J^T(q^2) + \frac{q^\mu q^\nu}{q^2}\Pi_J^L(q^2)\,.\end{aligned} \tag{7}$$



Hidden inside $\Pi_J^{T,L}(q^2)$ are the form factors of all states coupling to J^μ

Finite relations require subtractions

Finite relations require subtractions

Asymptotics of QCD tell us $\Pi_J^{L,T}$ contain **divergences** and must undergo **subtractions.**

Finite relations require subtractions

Asymptotics of QCD tell us $\Pi_J^{L,T}$ contain **divergences** and must undergo **subtractions.**

$$\chi_J^L(q^2) \equiv \frac{\partial \Pi_J^L}{\partial q^2} = \frac{1}{\pi}\int_0^\infty dt\, \frac{\mathrm{Im}\,\Pi_J^L(t)}{(t-q^2)^2}\,,$$
$$\chi_J^T(q^2) \equiv \frac{1}{2}\frac{\partial^2 \Pi_J^T}{\partial (q^2)^2} = \frac{1}{\pi}\int_0^\infty dt\, \frac{\mathrm{Im}\,\Pi_J^T(t)}{(t-q^2)^3}\,. \quad (8)$$

where $\mathrm{Im}\,\Pi_J^{L,T}(t) \geq 0$ are spectral functions which contain FFs.

Finite relations require subtractions

Asymptotics of QCD tell us $\Pi_J^{L,T}$ contain **divergences** and must undergo **subtractions.**

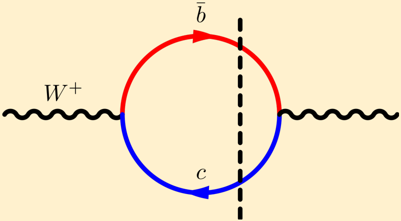
$$\chi_J^L(q^2) \equiv \frac{\partial \Pi_J^L}{\partial q^2} = \frac{1}{\pi}\int_0^\infty dt\,\frac{\mathrm{Im}\,\Pi_J^L(t)}{(t-q^2)^2}\,,$$
$$\chi_J^T(q^2) \equiv \frac{1}{2}\frac{\partial^2 \Pi_J^T}{\partial (q^2)^2} = \frac{1}{\pi}\int_0^\infty dt\,\frac{\mathrm{Im}\,\Pi_J^T(t)}{(t-q^2)^3}\,. \qquad (8)$$

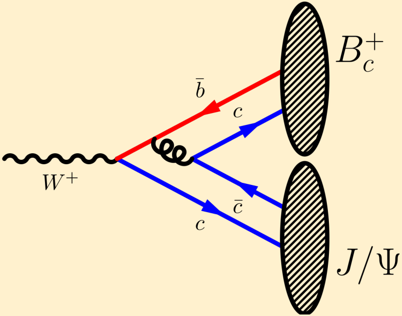
where $\mathrm{Im}\,\Pi_J^{L,T}(t) \geq 0$ are spectral functions which contain FFs.
We **need** $\chi_J^{L,T}(q^2)$, but note that **arbitrary** q^2 isn't the same as t!

What can be done about $\Pi_J^{L,T}$?

$\mathrm{Im}\,\Pi_J$ can be evaluated by the insertion into the dispersion relation of a complete set of states X that can couple the current J to the vacuum

$$\mathrm{Im}\,\Pi_J^{T,L}(q^2) = \frac{1}{2}\sum_X (2\pi)^4 \delta^4(q - p_X)\,|\langle 0\,|J|X\rangle|^2\ . \qquad (9)$$





...and we have bounds from dispersion relations[5]

Since $\mathrm{Im}\,\Pi_J^{L,T}(t) \geq 0$, any restriction to subset of hadronic states is a **strict inequality**, i.e.

$$\frac{1}{\pi\chi^T(q^2)}\int_{t_+}^{\infty} \mathrm{d}t \frac{W(t)|F(t)|^2}{(t-q^2)^3} < 1 \tag{10}$$

where known $W(t)$ depends on $F(t)$ considered.

[5] C. G. Boyd, B. Grinstein, and R. F. Lebed. "Precision corrections to dispersive bounds on form-factors". In: *Phys. Rev.* D56 (1997), pp. 6895–6911. arXiv: hep-ph/9705252 [hep-ph].

...and we have bounds from dispersion relations[5]

Since $\mathrm{Im}\,\Pi_J^{L,T}(t) \geq 0$, any restriction to subset of hadronic states is a **strict inequality**, i.e.

$$\frac{1}{\pi\chi^T(q^2)}\int_{t_+}^{\infty} \mathrm{d}t \frac{W(t)|F(t)|^2}{(t-q^2)^3} < 1 \tag{10}$$

where known $W(t)$ depends on $F(t)$ considered.

Intuition: Fraction of the W vacuum polarization given by subset, implying 1 is a **very** conservative bound for any single state

[5] C. G. Boyd, B. Grinstein, and R. F. Lebed. "Precision corrections to dispersive bounds on form-factors". In: *Phys. Rev.* D56 (1997), pp. 6895–6911. arXiv: hep-ph/9705252 [hep-ph].

...and we have bounds from dispersion relations[5]

Since $\mathrm{Im}\,\Pi_J^{L,T}(t) \geq 0$, any restriction to subset of hadronic states is a **strict inequality**, i.e.

$$\frac{1}{\pi\chi^T(q^2)}\int_{t_+}^{\infty} \mathrm{d}t \frac{W(t)|F(t)|^2}{(t-q^2)^3} < 1 \tag{10}$$

where known $W(t)$ depends on $F(t)$ considered.

Intuition: Fraction of the W vacuum polarization given by subset, implying 1 is a **very** conservative bound for any single state

But how do we get $\chi(q^2)$?

[5] C. G. Boyd, B. Grinstein, and R. F. Lebed. "Precision corrections to dispersive bounds on form-factors". In: *Phys. Rev.* D56 (1997), pp. 6895–6911. arXiv: hep-ph/9705252 [hep-ph].

An application of pQCD?!? Really?

[6] S. C. Generalis. “QCD sum rules. 1: Perturbative results for current correlators”. In: *J. Phys.* G16 (1990), pp. 785–793.

[7] L. J. Reinders, S. Yazaki, and H. R. Rubinstein. “Two Point Functions for Flavor Changing Currents in QCD”. In: *Phys. Lett.* 103B (1981), pp. 63–67.

An application of pQCD?!? Really?

In pQCD, $\chi(q^2)$ computable reliably at q^2 far from NP regime!

[6] S. C. Generalis. "QCD sum rules. 1: Perturbative results for current correlators". In: *J. Phys.* G16 (1990), pp. 785–793.

[7] L. J. Reinders, S. Yazaki, and H. R. Rubinstein. "Two Point Functions for Flavor Changing Currents in QCD". In: *Phys. Lett.* 103B (1981), pp. 63–67.

An application of pQCD?!? Really?

In pQCD, $\chi(q^2)$ computable reliably at q^2 far from NP regime!
The **formal** condition is

$$(m_Q + m'_Q)\Lambda_{\text{QCD}} \ll (m_Q + m'_Q)^2 - q^2 \tag{11}$$

[6] S. C. Generalis. "QCD sum rules. 1: Perturbative results for current correlators". In: *J. Phys.* G16 (1990), pp. 785–793.

[7] L. J. Reinders, S. Yazaki, and H. R. Rubinstein. "Two Point Functions for Flavor Changing Currents in QCD". In: *Phys. Lett.* 103B (1981), pp. 63–67.

An application of pQCD?!? Really?

In pQCD, $\chi(q^2)$ computable reliably at q^2 far from NP regime!
The **formal** condition is

$$(m_Q + m'_Q)\Lambda_{\text{QCD}} \ll (m_Q + m'_Q)^2 - q^2 \tag{11}$$

Taking $q^2 = 0$ is sufficient for $Q, Q' = c, b$, where two-loop calculations exist[6][7].

$$\begin{aligned} \chi^T(+u) &= 0.0125388 \qquad \chi^T(-u) = 0.0071680 \\ \chi^L(+u) &= 0.0044512 \qquad \chi^L(-u) = 0.0250418 \end{aligned} \tag{12}$$

where $u = m_c/m_b$

[6] S. C. Generalis. "QCD sum rules. 1: Perturbative results for current correlators". In: *J. Phys.* G16 (1990), pp. 785–793.

[7] L. J. Reinders, S. Yazaki, and H. R. Rubinstein. "Two Point Functions for Flavor Changing Currents in QCD". In: *Phys. Lett.* 103B (1981), pp. 63–67.

An application of pQCD?!? Really?

In pQCD, $\chi(q^2)$ computable reliably at q^2 far from NP regime! The **formal** condition is

$$(m_Q + m_Q')\Lambda_{\mathrm{QCD}} \ll (m_Q + m_Q')^2 - q^2 \qquad (11)$$

Taking $q^2 = 0$ is sufficient for $Q, Q' = c, b$, where two-loop calculations exist[6][7].

$$\begin{aligned} \chi^T(+u) &= 0.0125388 \qquad \chi^T(-u) = 0.0071680 \\ \chi^L(+u) &= 0.0044512 \qquad \chi^L(-u) = 0.0250418 \end{aligned} \qquad (12)$$

where $u = m_c/m_b$

But how can we parameterize $F(t)$?

[6] S. C. Generalis. "QCD sum rules. 1: Perturbative results for current correlators". In: *J. Phys.* G16 (1990), pp. 785–793.

[7] L. J. Reinders, S. Yazaki, and H. R. Rubinstein. "Two Point Functions for Flavor Changing Currents in QCD". In: *Phys. Lett.* 103B (1981), pp. 63–67.

Mapping $t \to z$

Use a **conformal** variable transformation

Mapping $t \to z$

Use a **conformal** variable transformation

$$z(t;t_0) \equiv \frac{\sqrt{t_{\rm bc}-t}-\sqrt{t_{\rm bc}-t_0}}{\sqrt{t_{\rm bc}-t}+\sqrt{t_{\rm bc}-t_0}}, \tag{13}$$

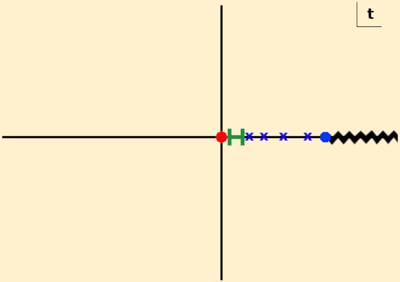
$t_{\rm bc}$ is production threshold of lightest states in channel, $BD^{(*)}$, t_0 defined to improve convergence. z is real for $t \leq t_{\rm bc}$ and a pure phase for $t \geq t_{\rm bc}$.

Mapping $t \to z$

Use a **conformal** variable transformation

$$z(t;t_0) \equiv \frac{\sqrt{t_{\rm bc}-t}-\sqrt{t_{\rm bc}-t_0}}{\sqrt{t_{\rm bc}-t}+\sqrt{t_{\rm bc}-t_0}}, \tag{13}$$

$t_{\rm bc}$ is production threshold of lightest states in channel, $BD^{(*)}$, t_0 defined to improve convergence. z is real for $t \le t_{\rm bc}$ and a pure phase for $t \ge t_{\rm bc}$.

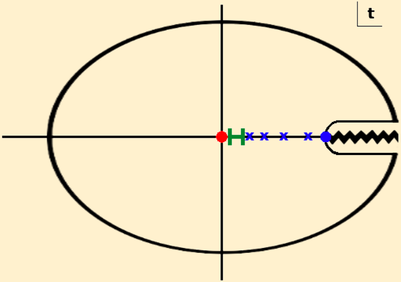


Mapping $t \to z$

Use a **conformal** variable transformation

$$z(t; t_0) \equiv \frac{\sqrt{t_{\rm bc} - t} - \sqrt{t_{\rm bc} - t_0}}{\sqrt{t_{\rm bc} - t} + \sqrt{t_{\rm bc} - t_0}}, \qquad (13)$$

$t_{\rm bc}$ is production threshold of lightest states in channel, $BD^{(*)}$, t_0 defined to improve convergence. z is real for $t \leq t_{\rm bc}$ and a pure phase for $t \geq t_{\rm bc}$.

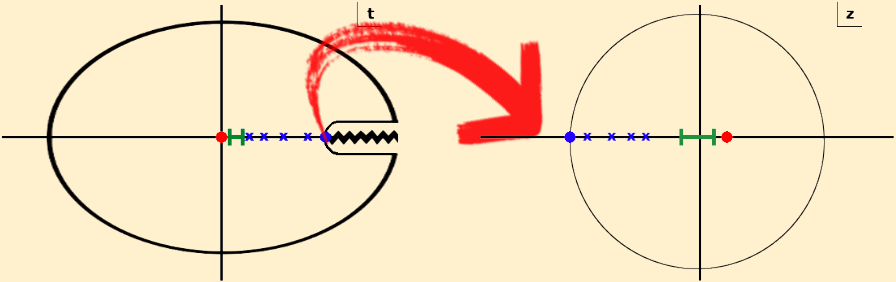


Mapping $t \to z$

Use a **conformal** variable transformation

$$z(t;t_0) \equiv \frac{\sqrt{t_{\rm bc}-t}-\sqrt{t_{\rm bc}-t_0}}{\sqrt{t_{\rm bc}-t}+\sqrt{t_{\rm bc}-t_0}}, \tag{13}$$

$t_{\rm bc}$ is production threshold of lightest states in channel, $BD^{(*)}$, t_0 defined to improve convergence. z is real for $t \leq t_{\rm bc}$ and a pure phase for $t \geq t_{\rm bc}$.

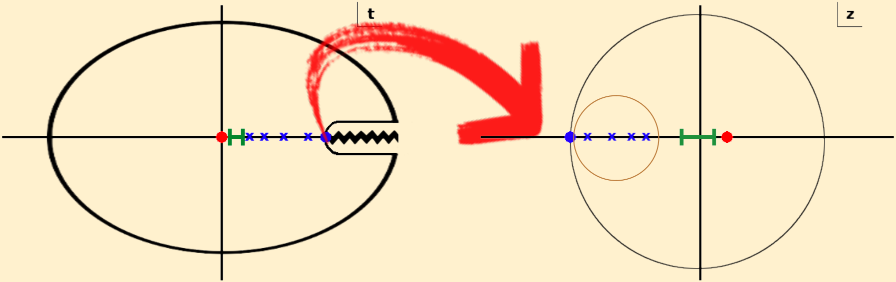


Mapping $t \to z$

Use a **conformal** variable transformation

$$z(t;t_0) \equiv \frac{\sqrt{t_{\rm bc} - t} - \sqrt{t_{\rm bc} - t_0}}{\sqrt{t_{\rm bc} - t} + \sqrt{t_{\rm bc} - t_0}}, \tag{13}$$

$t_{\rm bc}$ is production threshold of lightest states in channel, $BD^{(*)}$, t_0 defined to improve convergence. z is real for $t \le t_{\rm bc}$ and a pure phase for $t \ge t_{\rm bc}$.



Removing the poles

Removing the poles

Rewritting the bound on $F_i(t)$ as

$$\frac{1}{\pi}\sum_i \int_{t_{\rm bc}}^{\infty} dt \left|\frac{dz(t;t_0)}{dt}\right| \left|\phi_i(t;t_0)P_i(t)F_i(t)\right|^2 \leq 1\,, \tag{14}$$

where $\phi_i(t;t_0)$ is an **outer function**,

Removing the poles

Rewritting the bound on $F_i(t)$ as

$$\frac{1}{\pi}\sum_i \int_{t_{\rm bc}}^{\infty} dt \left|\frac{dz(t;t_0)}{dt}\right| |\phi_i(t;t_0)P_i(t)F_i(t)|^2 \leq 1\,, \tag{14}$$

where $\phi_i(t;t_0)$ is an **outer function**, given by

$$\phi_i(t;t_0) = \tilde{P}_i(t)\left[\frac{W_i(t)}{|dz(t;t_0)/dt|\,\chi^j(q^2)(t-q^2)^n}\right]^{1/2}\,, \tag{15}$$

$\tilde{P}_i(t)$ is product of $z(t;t_s)$ and $\sqrt{z(t;t_s)}$ removing **kinematical** singularities

Removing the poles

Rewritting the bound on $F_i(t)$ as

$$\frac{1}{\pi}\sum_i \int_{t_{\rm bc}}^{\infty} dt \left|\frac{dz(t;t_0)}{dt}\right| |\phi_i(t;t_0)P_i(t)F_i(t)|^2 \leq 1\,, \tag{14}$$

where $\phi_i(t;t_0)$ is an **outer function**, given by

$$\phi_i(t;t_0) = \tilde{P}_i(t)\left[\frac{W_i(t)}{|dz(t;t_0)/dt|\,\chi^j(q^2)(t-q^2)^n}\right]^{1/2}\,, \tag{15}$$

$\tilde{P}_i(t)$ is product of $z(t;t_s)$ and $\sqrt{z(t;t_s)}$ removing **kinematical** singularities
$P_i(t)$ is product of Blaschke factors $z(t;t_p)$ that **remove poles**.

What is the outer function?

Written **less** opaquely

$$\phi_i(t;t_0) = \sqrt{\frac{2}{K\pi\chi}} \left(\frac{t_{\rm bc}-t}{t_{\rm bc}-t_0}\right)^{\frac{1}{4}} \left(\sqrt{t_{\rm bc}-t}+\sqrt{t_{\rm bc}-t_0}\right)(t_{\rm bc}-t)^{\frac{a}{4}}$$
$$\times \left(\sqrt{t_{\rm bc}-t}+\sqrt{t_{\rm bc}-t_-}\right)^{\frac{b}{2}} \left(\sqrt{t_{\rm bc}-t}+\sqrt{t_{\rm bc}}\right)^{-(c+3)}. \quad (16)$$

What is the outer function?

Written less opaquely

$$\phi_i(t;t_0) = \sqrt{\frac{2}{K\pi\chi}} \left(\frac{t_{\rm bc}-t}{t_{\rm bc}-t_0}\right)^{\frac{1}{4}} \left(\sqrt{t_{\rm bc}-t}+\sqrt{t_{\rm bc}-t_0}\right)(t_{\rm bc}-t)^{\frac{a}{4}}$$

$$\times\left(\sqrt{t_{\rm bc}-t}+\sqrt{t_{\rm bc}-t_-}\right)^{\frac{b}{2}}\left(\sqrt{t_{\rm bc}-t}+\sqrt{t_{\rm bc}}\right)^{-(c+3)}. \quad (16)$$

Factors entering $\phi_i(t,t_0)$ for the meson form factors F_i

F_i	K	χ	a	b	c
f	24	$\chi^T(-u)$	1	1	1
$\mathcal{F}_1$	48	$\chi^T(-u)$	1	1	2
g	96	$\chi^T(+u)$	3	3	1
$\mathcal{F}_2$	64	$\chi^L(-u)$	3	3	1

Now that its analytic, so what?

Now that its analytic, so what?

$$\frac{1}{2\pi i}\sum_i \oint_C \frac{dz}{z}|\phi_i(z)P_i(z)F_i(z)|^2 \leq 1\,, \tag{17}$$

Now that its analytic, so what?

$$\frac{1}{2\pi i}\sum_i \oint_C \frac{dz}{z}|\phi_i(z)P_i(z)F_i(z)|^2 \leq 1\,, \tag{17}$$

we can now take an **expansion** around $z \approx 0$ ($z_{\max} = 0.027$)

$$F_i(t) = \frac{1}{|P_i(t)|\phi_i(t;t_0)}\sum_{n=0}^{\infty} a_{in} z(t;t_0)^n\,, \tag{18}$$

Now that its analytic, so what?

$$\frac{1}{2\pi i}\sum_i \oint_C \frac{dz}{z}|\phi_i(z)P_i(z)F_i(z)|^2 \leq 1\,, \tag{17}$$

we can now take an **expansion** around $z \approx 0$ ($z_{\text{max}} = 0.027$)

$$F_i(t) = \frac{1}{|P_i(t)|\phi_i(t;t_0)}\sum_{n=0}^{\infty} a_{in} z(t;t_0)^n\,, \tag{18}$$

with the bound now expressed as

$$\sum_{i;n=0}^{\infty} a_{in}^2 \leq 1\,. \tag{19}$$

Now that its analytic, so what?

$$\frac{1}{2\pi i}\sum_i \oint_C \frac{dz}{z}|\phi_i(z)P_i(z)F_i(z)|^2 \leq 1\,, \tag{17}$$

we can now take an **expansion** around $z \approx 0$ ($z_{\mathrm{max}} = 0.027$)

$$F_i(t) = \frac{1}{|P_i(t)|\phi_i(t;t_0)}\sum_{n=0}^{\infty} a_{in} z(t;t_0)^n\,, \tag{18}$$

with the bound now expressed as

$$\sum_{i;n=0}^{\infty} a_{in}^2 \leq 1\,. \tag{19}$$

Form factors **cannot** change arbitrarily fast!

How many poles are we dealing with?

Lowest B_c^+ states for the channels from model[8], except for the two measured by LHCb. Bold values indicate ones needed for Blaschke factors with $t < t_{\rm bc}$

Type	J^P	M [GeV]
Vector	1^-	**6.337, 6.899, 7.012,** 7.280
		7.350, 7.594, 7.646, 7.872, 7.913
Axial	1^+	**6.730, 6.736, 7.135, 7.142**
		7.470, 7.470, 7.757, 7.757
Scalar	0^+	**6.700, 7.108,** 7.470, 7.757
Pseudoscalar	0^-	**6.2749(8), 6.842(9),** 7.244, 7.562
		7.844

[8] E. J. Eichten and C. Quigg. "Mesons with beauty and charm: Spectroscopy". In: *Phys. Rev.* D49 (1994), pp. 5845–5856. arXiv: hep-ph/9402210 [hep-ph].

Any problems?

Any problems?

- Lattice data for $V(q^2), A_1(q^2)$ aren't wildly off

Any problems?

- Lattice data for $V(q^2), A_1(q^2)$ aren't wildly off
- Coefficient bounds from dispersive relations: $\sum_{i,n=0} a_{in}^2 \leq 1$

Any problems?

- Lattice data for $V(q^2), A_1(q^2)$ aren't wildly off
- Coefficient bounds from dispersive relations: $\sum_{i,n=0} a_{in}^2 \leq 1$
- HQSS relations between $F_i(q^2_{max})$ are satisfied with 50%

Any problems?

- Lattice data for $V(q^2), A_1(q^2)$ aren't wildly off
- Coefficient bounds from dispersive relations: $\sum_{i,n=0} a_{in}^2 \leq 1$
- HQSS relations between $F_i(q_{max}^2)$ are satisfied with 50%
- Form factors are maximal at q_{max}^2

Any problems?

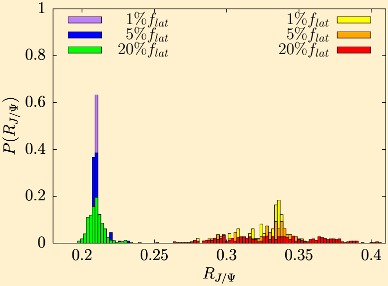
- Lattice data for $V(q^2), A_1(q^2)$ aren't wildly off
- Coefficient bounds from dispersive relations: $\sum_{i,n=0} a_{in}^2 \leq 1$
- HQSS relations between $F_i(q^2_{max})$ are satisfied with 50%
- Form factors are maximal at q^2_{max}
- Kinematic relations are exact

Any problems?

- Lattice data for $V(q^2), A_1(q^2)$ aren't wildly off
- Coefficient bounds from dispersive relations: $\sum_{i,n=0} a_{in}^2 \leq 1$
- HQSS relations between $F_i(q^2_{max})$ are satisfied with 50%
- Form factors are maximal at q^2_{max}
- Kinematic relations are exact

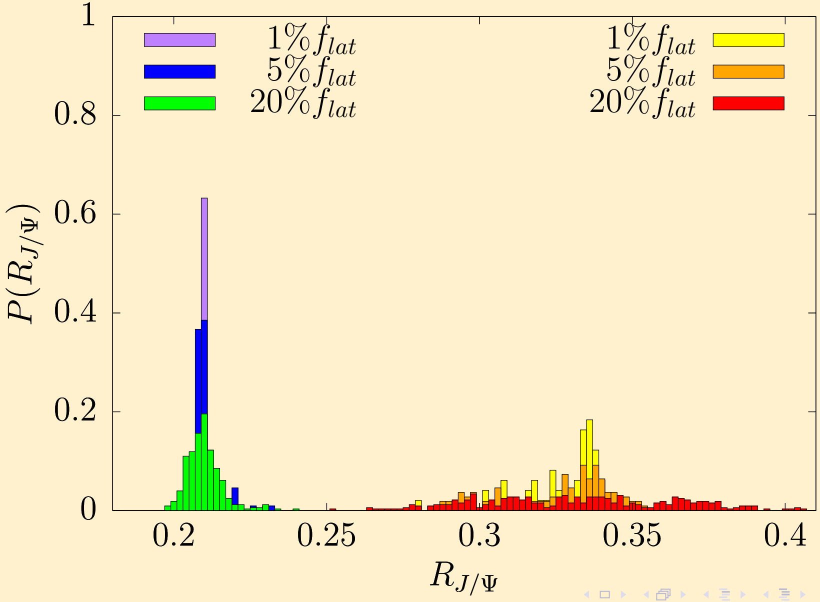
Strict prediction would require **additional assumptions** about priors, but min/max values are **independent** of this

Outline



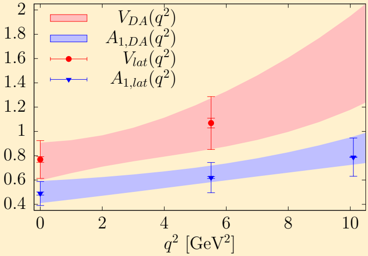
1%f_{lat}
5%f_{lat}
20%f_{lat}
1%f_{lat}
5%f_{lat}
20%f_{lat}
$P(R_{J/\Psi})$
$R_{J/\Psi}$
0
0.2
0.4
0.6
0.8
1
0.2
0.25
0.3
0.35
0.4

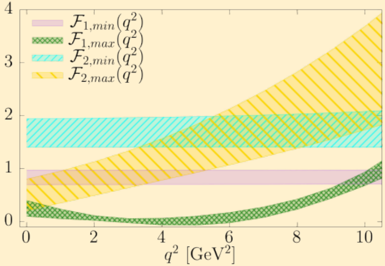
n=2 bounds



1
0.8
0.6
0.4
0.2
0
$P(R_{J/\Psi})$
0.2
0.25
0.3
0.35
0.4
$R_{J/\Psi}$
$1\% f_{lat}$
$5\% f_{lat}$
$20\% f_{lat}$
$1\% f_{lat}$
$5\% f_{lat}$
$20\% f_{lat}$

So what does the Standard Model allow?

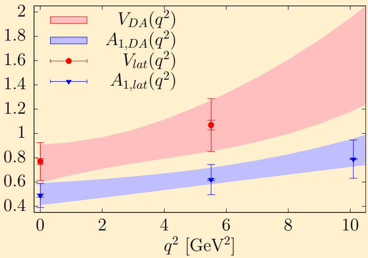


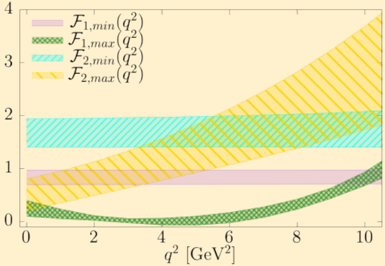


95% CL bounds on $R_{J/\psi}$ as a function of the truncation power n and the systematic lattice uncertainty f_{lat}.

f_{lat}	$n = 1$	$n = 2$
1	[0.21, 0.33]	[0.20, 0.35]
5	[0.20, 0.33]	[0.20, 0.35]
20	[0.20, 0.36]	[0.20, 0.39]

So what does the Standard Model allow?



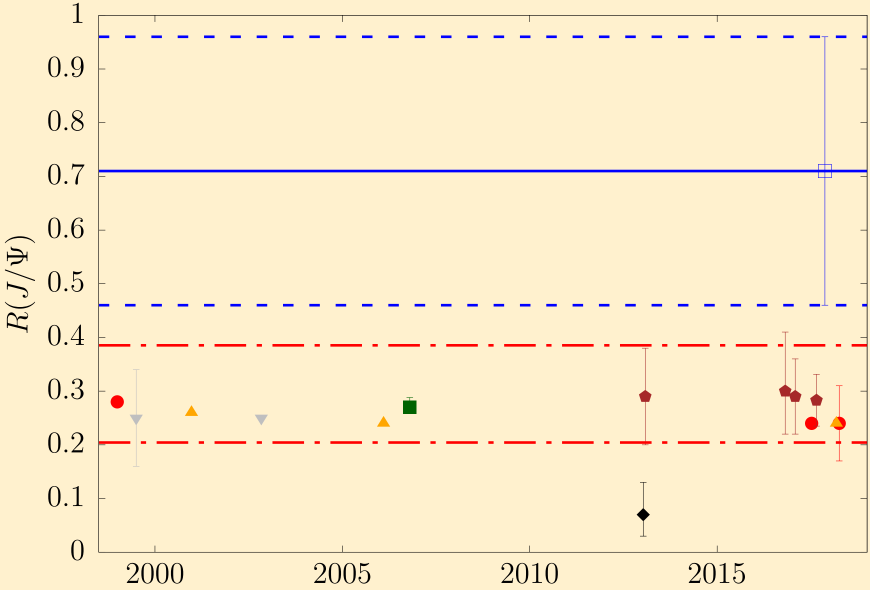


95% CL bounds on $R_{J/\psi}$ as a function of the truncation power n and the systematic lattice uncertainty f_{lat}.

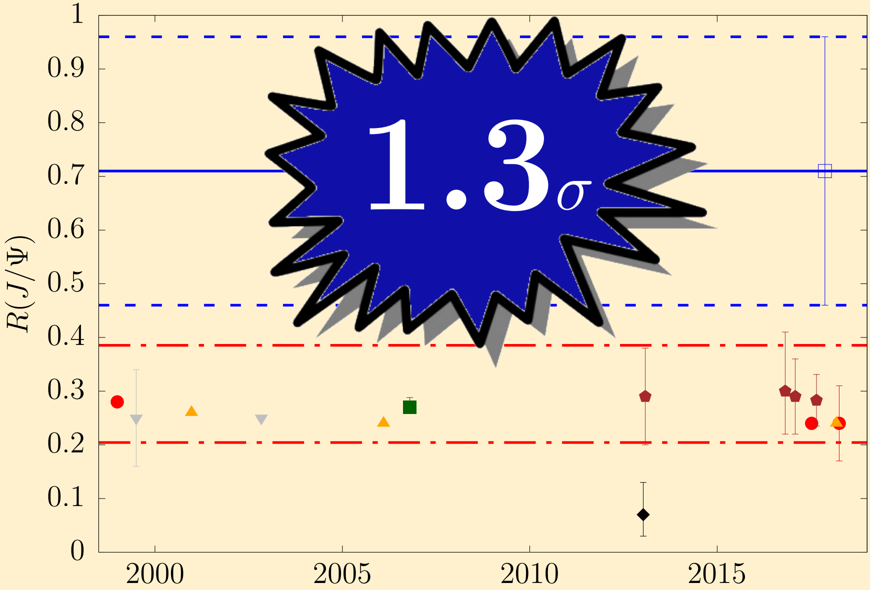
f_{lat}	$n = 1$	$n = 2$
1	[0.21, 0.33]	[0.20, 0.35]
5	[0.20, 0.33]	[0.20, 0.35]
20	[0.20, 0.36]	[0.20, 0.39]

$n > 2$ **unlikely** to affect bound, since $\frac{a_{n+1}}{a_n} \geq z_{\text{max}}^{-1} \approx 37$

Updated $R_{J/\psi}$ Plot



Updated $R_{J/\psi}$ Plot



What about $R(\eta_c)$?

What about $R(\eta_c)$?

$$\langle \eta_c(p)|(V-A)^\mu|B_c^+(P)\rangle = f_+(P+p)^\mu + f_-(P-p)^\mu$$

$$f_0(t) = (M^2-m^2)f_+ + tf_-(t)$$

What about $R(\eta_c)$?

$$\langle \eta_c(p) | (V-A)^\mu | B_c^+(P) \rangle = f_+(P+p)^\mu + f_-(P-p)^\mu$$

$$f_0(t) = (M^2 - m^2) f_+ + t f_-(t)$$

- Only two form factors

What about $R(\eta_c)$?

$$\langle \eta_c(p) | (V-A)^\mu | B_c^+(P) \rangle = f_+(P+p)^\mu + f_-(P-p)^\mu$$

$$f_0(t) = (M^2 - m^2) f_+ + t f_-(t)$$

- Only two form factors
- Both have LQCD results

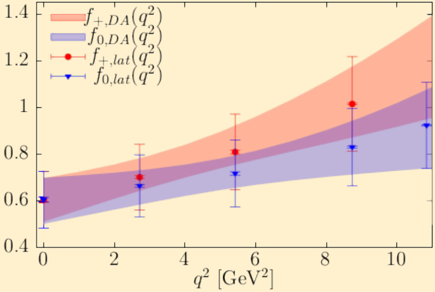
What about $R(\eta_c)$?

$$\langle \eta_c(p)|(V-A)^\mu|B_c^+(P)\rangle = f_+(P+p)^\mu + f_-(P-p)^\mu$$

$$f_0(t) = (M^2 - m^2)f_+ + tf_-(t)$$

- Only two form factors
- Both have LQCD results
- W. Avg. of Models $0.33^{+0.17}_{-0.17}$

What about $R(\eta_c)$?

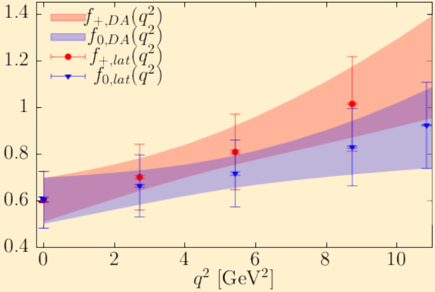


$$\langle \eta_c(p)|(V-A)^\mu|B_c^+(P)\rangle = f_+(P+p)^\mu + f_-(P-p)^\mu$$

$$f_0(t) = (M^2-m^2)f_+ + tf_-(t)$$

- Only two form factors
- Both have LQCD results
- W. Avg. of Models $0.33^{+0.17}_{-0.17}$

What about $R(\eta_c)$?



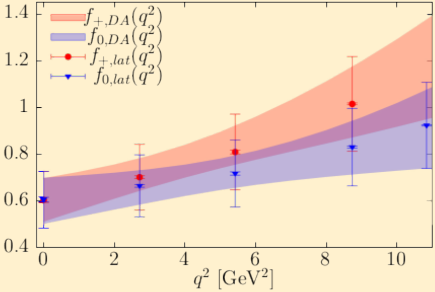
$$\langle \eta_c(p)|(V-A)^\mu|B_c^+(P)\rangle = f_+(P+p)^\mu + f_-(P-p)^\mu$$

$$f_0(t) = (M^2 - m^2)f_+ + tf_-(t)$$

f_{lat}	$n=2$	$n=3$
1	0.291(4)	0.290(4)
5	0.291(12)	0.29(2)
20	0.30(5)	0.29(5)

- Only two form factors
- Both have LQCD results
- W. Avg. of Models $0.33^{+0.17}_{-0.17}$

What about $R(\eta_c)$?



f_{lat}	$n = 2$	$n = 3$
1	0.291(4)	0.290(4)
5	0.291(12)	0.29(2)
20	0.30(5)	0.29(5)

$$\langle \eta_c(p)|(V-A)^\mu|B_c^+(P)\rangle = f_+(P+p)^\mu + f_-(P-p)^\mu$$

$$f_0(t) = (M^2 - m^2)f_+ + tf_-(t)$$

- Only two form factors
- Both have LQCD results
- W. Avg. of Models $0.33^{+0.17}_{-0.17}$
- No $J/\psi \to \mu\mu$, harder to detect

What is there to be done

What is there to be done

- Extend the NRQCD relations beyond q^2_{max}

What is there to be done

- Extend the NRQCD relations beyond q^2_{max}
- Sum rules could provide constraints on derivatives

What is there to be done

- Extend the NRQCD relations beyond q^2_{max}
- Sum rules could provide constraints on derivatives
- Global analysis by adding $B \to D^{(*)}, \Lambda_b \to \Lambda_c$

What is there to be done

- Extend the NRQCD relations beyond q^2_{max}
- Sum rules could provide constraints on derivatives
- Global analysis by adding $B \to D^{(*)}, \Lambda_b \to \Lambda_c$
- BSM bounds

A rigorous Standard Model bound now exists

A rigorous Standard Model bound now exists

- SM bounds of $0.20 \leq R(J/\Psi) \leq 0.39$ and $R(\eta_c) = 0.29(5)$ can be made **without** any recourse to models

A rigorous Standard Model bound now exists

- SM bounds of $0.20 \leq R(J/\Psi) \leq 0.39$ and $R(\eta_c) = 0.29(5)$ can be made **without** any recourse to models
- **Improvement** in existing lattice form factors, or **any** information about the remaining two can **substantially** shrink bounds

A rigorous Standard Model bound now exists

- SM bounds of $0.20 \leq R(J/\Psi) \leq 0.39$ and $R(\eta_c) = 0.29(5)$ can be made **without** any recourse to models
- **Improvement** in existing lattice form factors, or **any** information about the remaining two can **substantially** shrink bounds
- Including other channels could reduce bounds, since typical $\sum a_n^2 \approx 1$

A rigorous Standard Model bound now exists

- SM bounds of $0.20 \leq R(J/\Psi) \leq 0.39$ and $R(\eta_c) = 0.29(5)$ can be made **without** any recourse to models
- **Improvement** in existing lattice form factors, or **any** information about the remaining two can **substantially** shrink bounds
- Including other channels could reduce bounds, since typical $\sum a_n^2 \approx 1$

Questions?