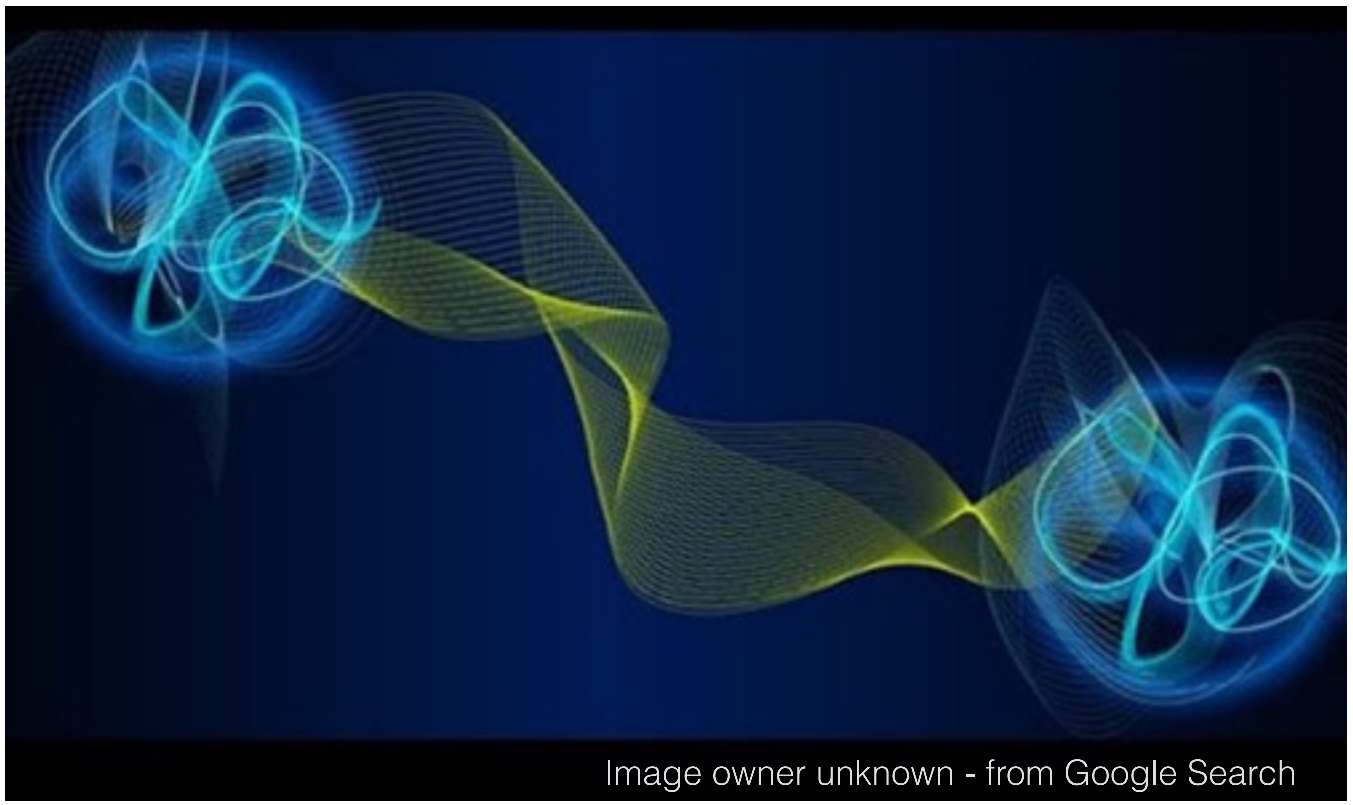
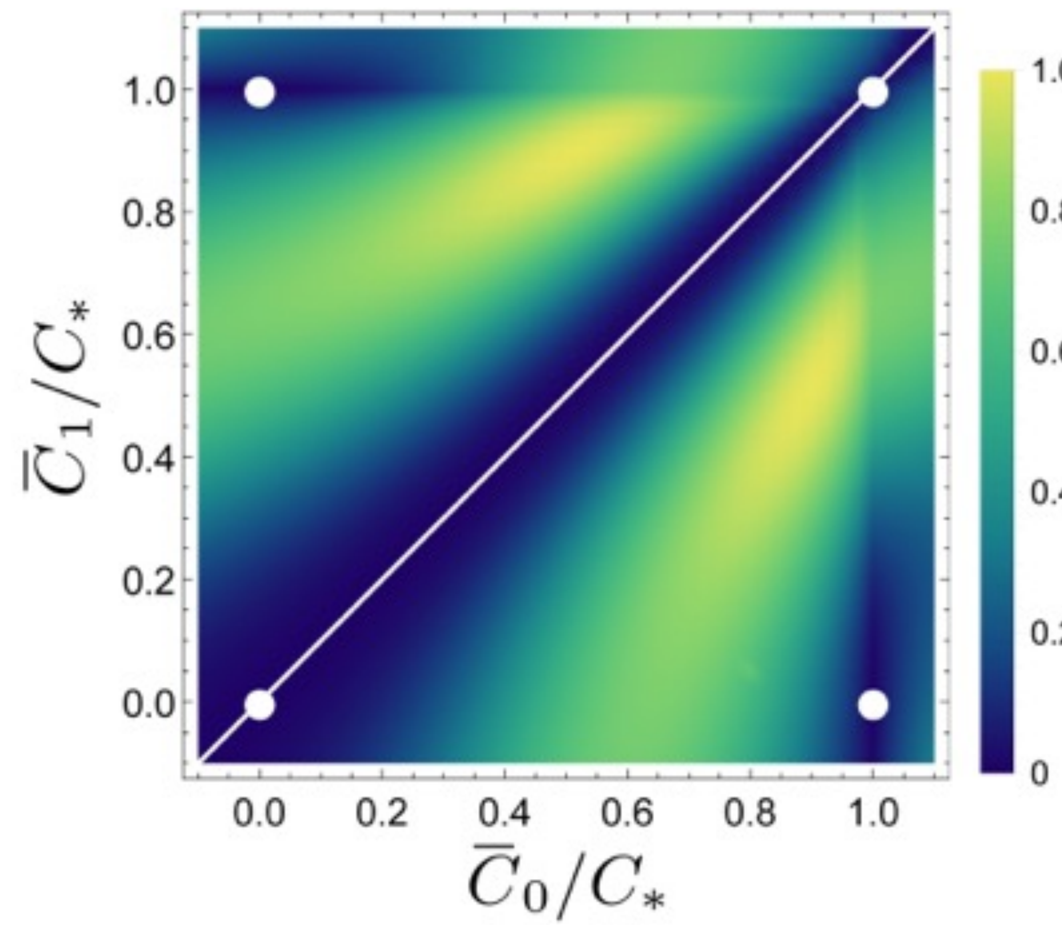
Silas Beane David Kaplan Natalie Klco + MJS
Phys. Rev. Lett. 122 (2019) no.10, 102001, arXiv: 1812.03138





Entanglement and Emergent Symmetries in QCD

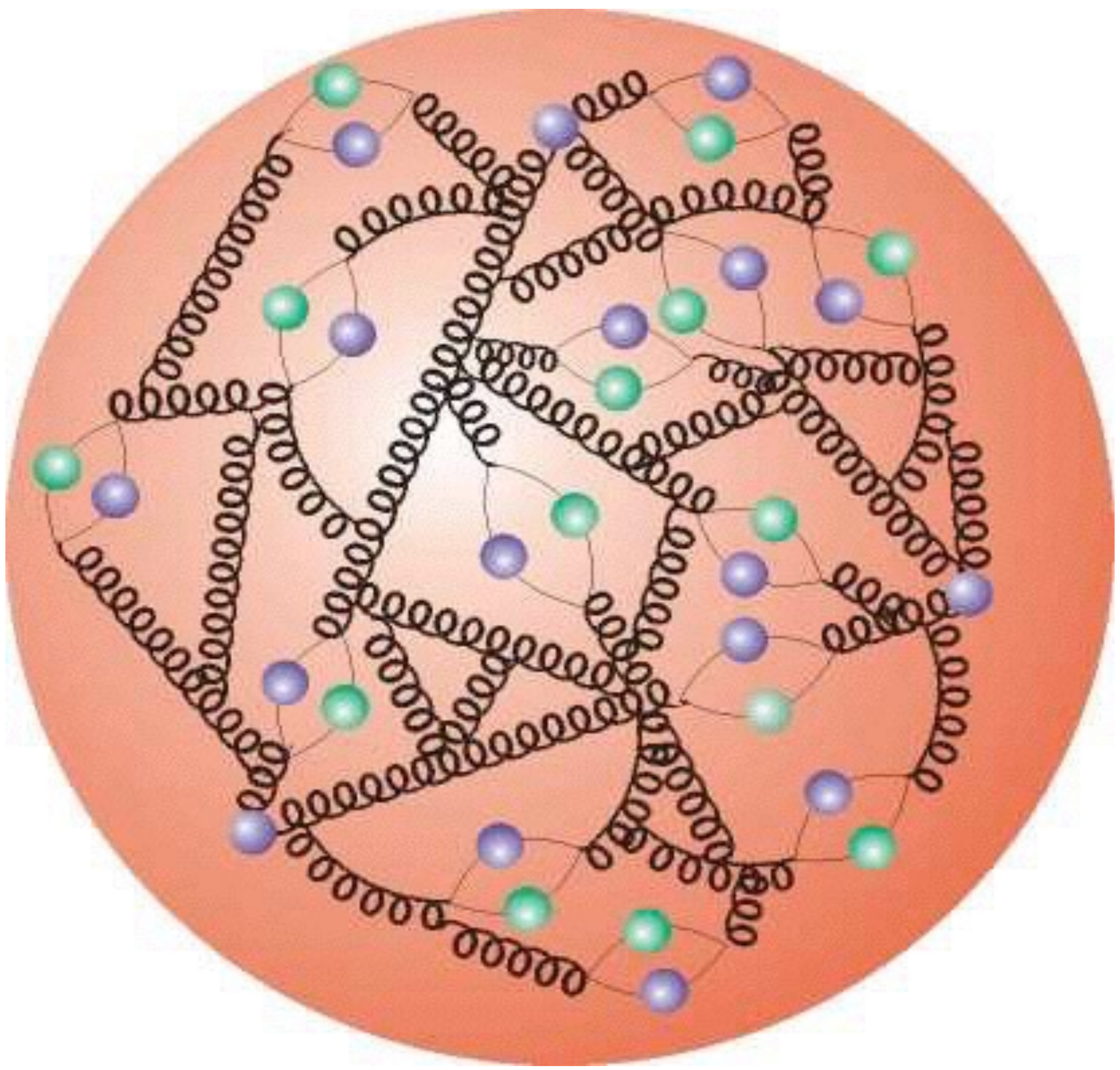
- special to QCD or generic of confining theories?

Martin J Savage

INSTITUTE for NUCLEAR THEORY

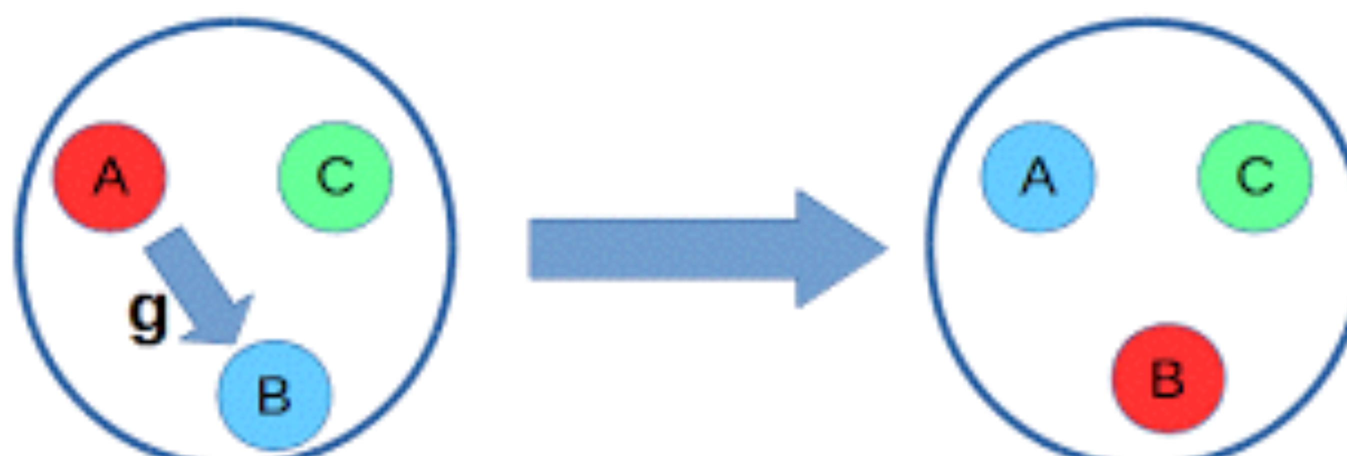
JLab, June 4 (2019)

The Proton - A Complicated Pure State

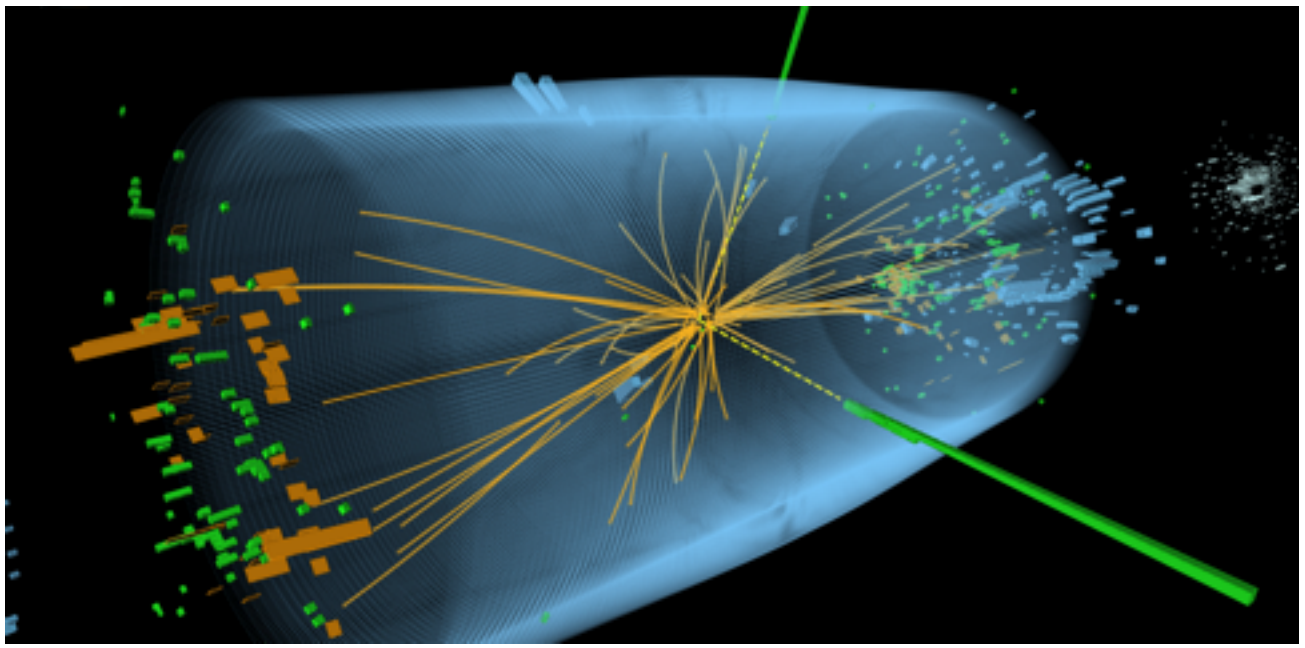


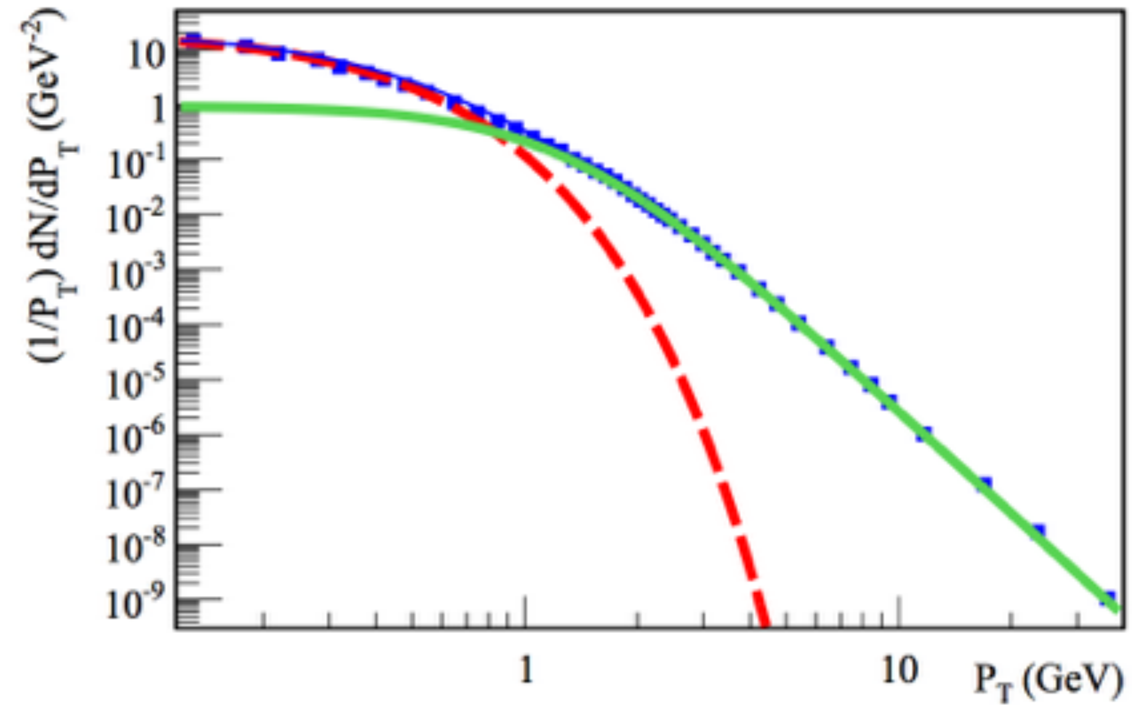
Comprised of quarks and gluons

- gluons are massless - gauge field
- light quarks are light - matter fields
- confined into hadrons by the strong force
- indefinite particle number
- chiral symmetry breaking



Entanglement in High-Energy Collisions

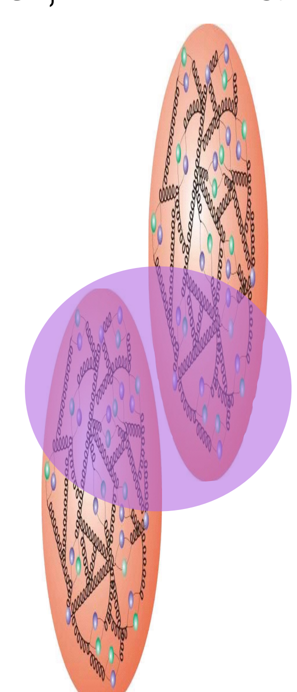




Thermal radiation and entanglement in proton-proton collisions at the LHC
O.K. Baker, D.E. Kharzeev

e.g.

- proton-proton collisions at LHC
  - thermal distributions - Kharzeev, Baker, …
- heavy-ion collisions - Hsu et al (2015), …
- coherence in fragmentation functions
- deep Inelastic Scattering and PDFs (Kharzeev)



A Trail of Bread Crumbs from Standard Model Perturbation Theory

(Alba Cervera-Lierta *et al* , 2017)

Maximal Entanglement in High Energy Physics

Alba Cervera-Lierta,[1] José I. Latorre,[1,2] Juan Rojo,[3] and Luca Rottoli[4]

[1] *Dept. Física Quàntica i Astrofísica, Universitat de Barcelona, Diagonal 645, 08028 Barcelona, Spain.*
[2] *Center for Quantum Technologies, National University of Singapore.*
[3] *Department of Physics and Astronomy, VU University, De Boelelaan 1081, 1081HV Amsterdam, and Nikhef, Science Park 105, NL-1098 XG Amsterdam, The Netherlands*
[4] *Rudolf Peierls Centre for Theoretical Physics, University of Oxford, 1 Keble Road, Oxford OX1 3NP, UK.*

(Dated: November 27, 2017)

Abstract

We analyze how maximal entanglement is generated at the fundamental level in QED by studying correlations between helicity states in tree-level scattering processes at high energy. We demonstrate that two mechanisms for the generation of maximal entanglement are at work: *i)* s-channel processes where the virtual photon carries equal overlaps of the helicities of the final state particles, and *ii)* the indistinguishable superposition between t- and u-channels. We then study whether *requiring* maximal entanglement constrains the coupling structure of QED and the weak interactions. In the case of photon-electron interactions unconstrained by gauge symmetry, we show how this requirement allows to reproduce QED. For Z-mediated weak scattering, the maximal entanglement principle leads to non-trivial predictions for the value of the weak mixing angle θ_W. Our results are a first step towards understanding the connections between maximal entanglement and the fundamental symmetries of high-energy physics.

Maximal entanglement predicts:

$\Theta_W = \pi/6$, $\sin^2\Theta_W = 0.25$

Experiment:

$\sin^2\Theta_W = 0.2223(21)$

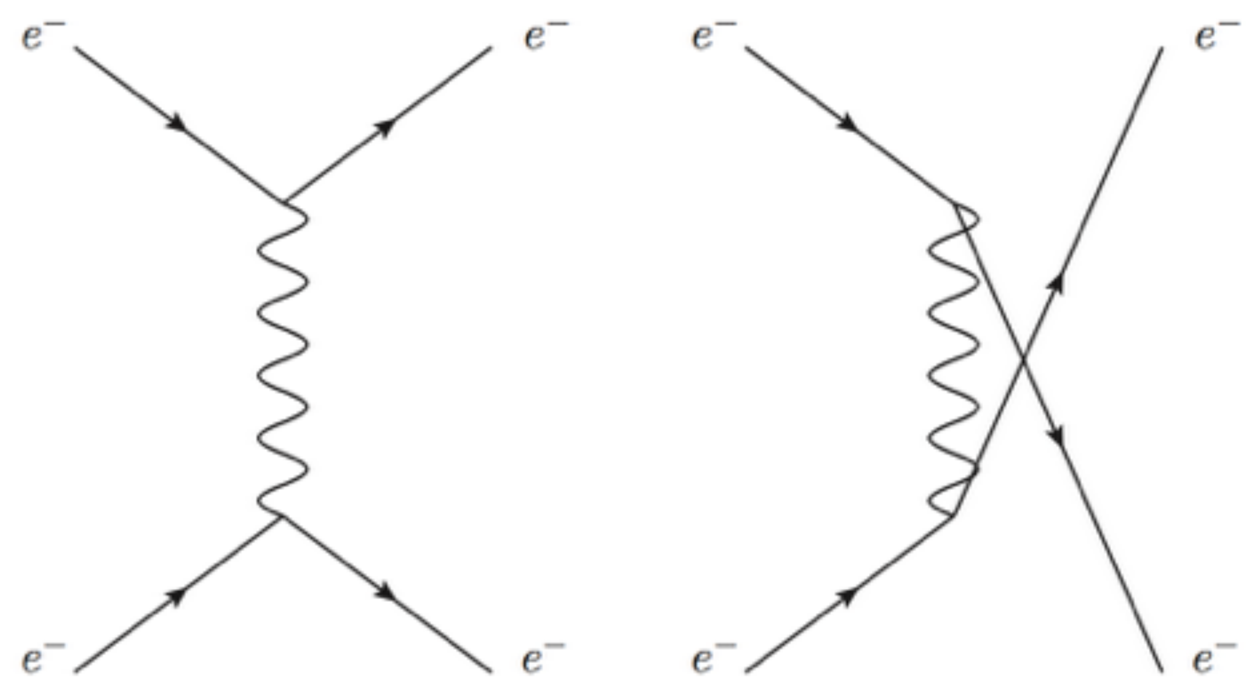
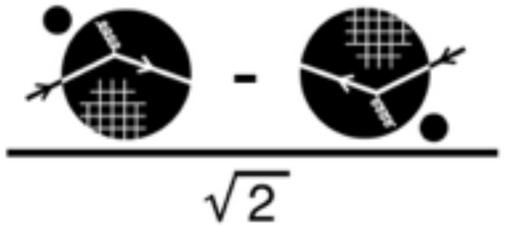
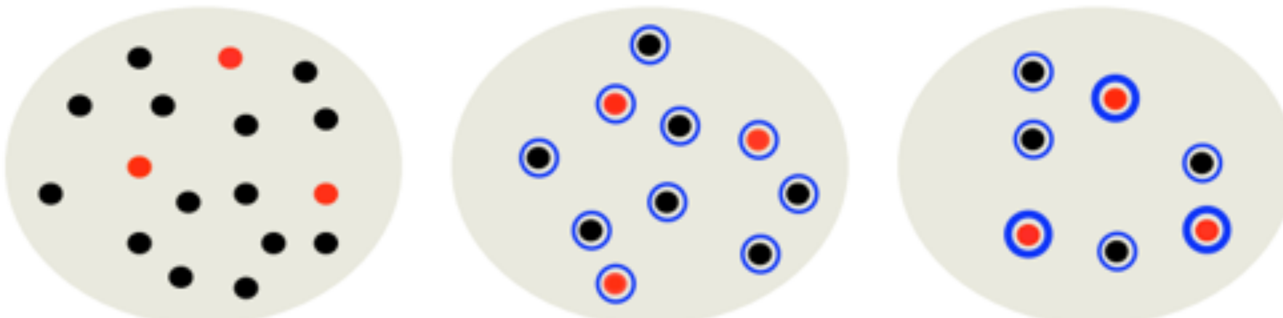


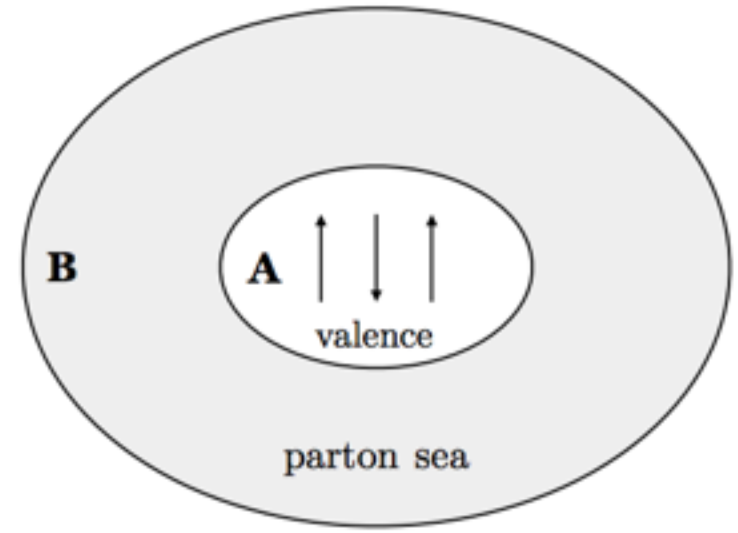
FIG. 2: Feynman diagrams for Møller scattering, $e^-e^- \to e^-e^-$, in the t (left) and u (right) channels.



Chiral Symmetry Breaking and Entanglement

(Silas Beane and Peter Ehlers, 2019)

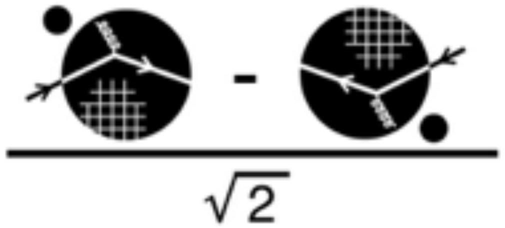




$$|N\,,\,\tfrac{1}{2}\,\rangle \;=\; |(\mathbf{2},\mathbf{3})_{\mathbf{2}}\,,\,\tfrac{1}{2}\,,\,\mathbf{0}\rangle \;=\; |(\mathbf{2},\mathbf{3})_{\mathbf{2}}\,,\,\tfrac{1}{2}\rangle \otimes |\,\mathbf{0}\,\rangle$$

Valence Sea

- **Large Nc**
  - no chiral symmetry breaking
  - tensor product wavefunction
  - vanishing entanglement entropy



Chiral Symmetry Breaking and Entanglement

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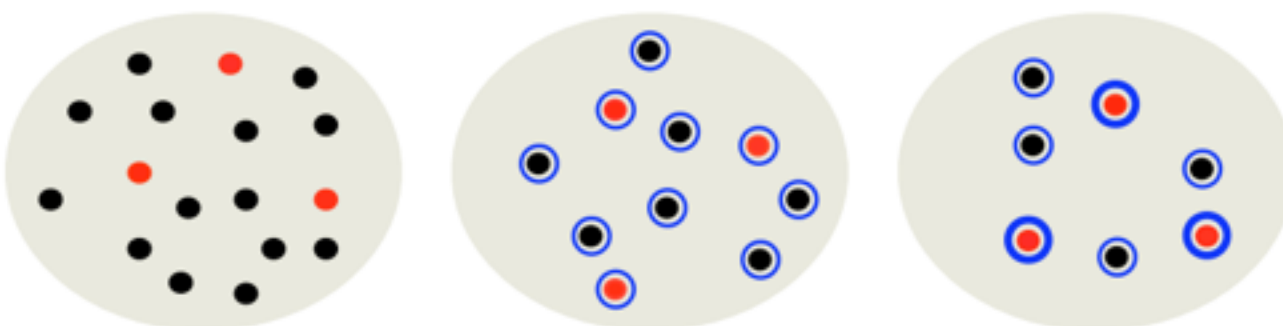
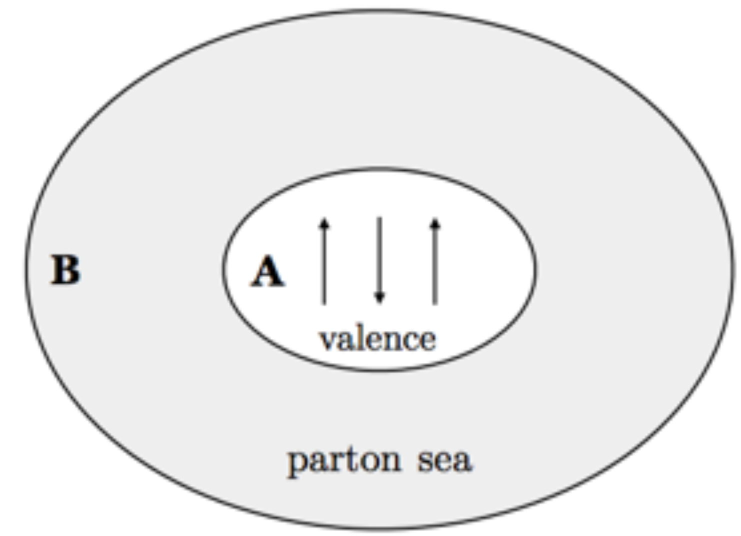




$$|\,N\,,\,\tfrac{1}{2}\,\rangle = \sin\psi\,|(\mathbf{1},\mathbf{2}),-\tfrac{1}{2},\mathbf{1}\rangle + \cos\psi\,|(\mathbf{2},\mathbf{3})_{\mathbf{2}},\tfrac{1}{2},\mathbf{0}\rangle$$

Valence Sea Valence Sea

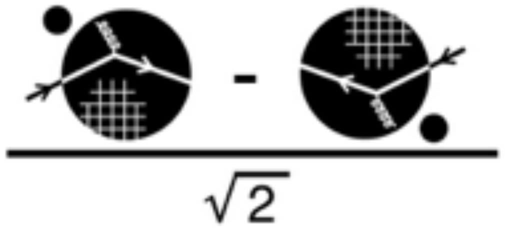
$C_{\Delta N}$: ψ=41(4)^{o}

g_A =1.38(5) [c/w/ 1.2724(23) expt]

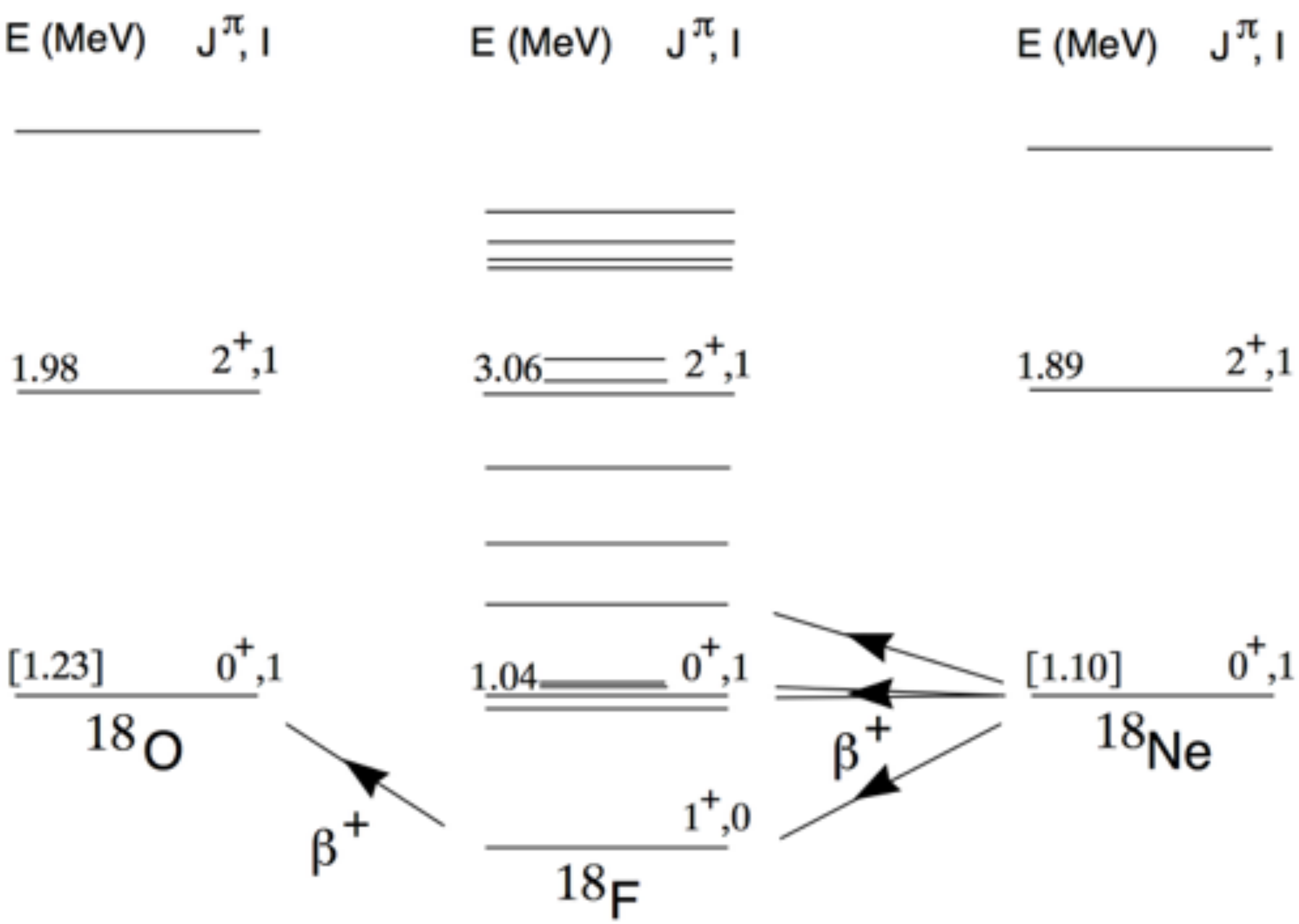
Σ_N = 0.14(13) [c/w 0.36(9) expt]

S_N = 0.98(02) **Maximal Entanglement**

Entanglement Entropy is an Order Parameter for Chiral Symmetry Breaking



Emergent Symmetries of QCD in Strong Forces

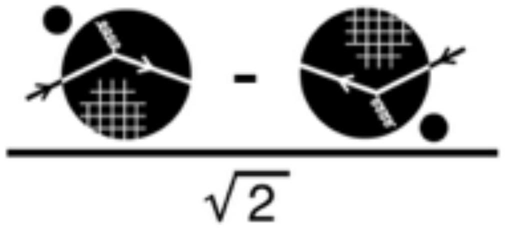


Wigner's SU(4) Symmetry in Light Nuclei ~1930's

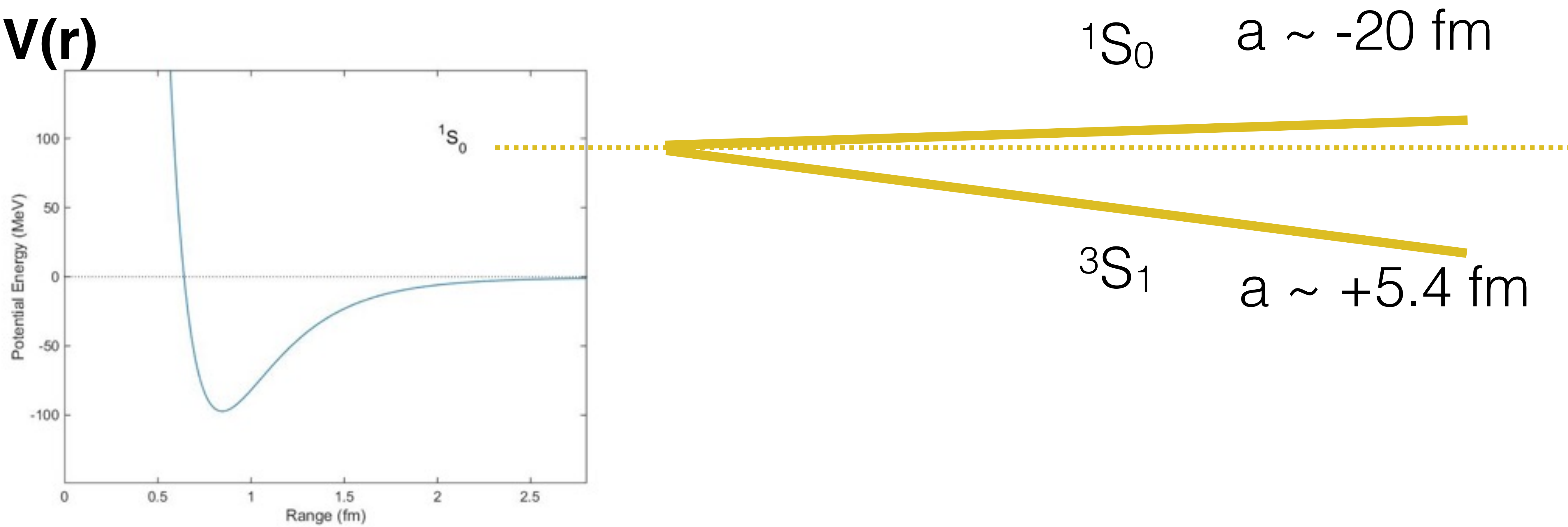
Not a symmetry of interactions between quarks and gluons !!

S=0 : pp, np, nn
S=1 : np

Global symmetries beyond those explicit in Standard Model Lagrange density



Relevant Features of Strong Forces



Large s-wave scattering lengths

- r/a << 1
- cancellations between all length scales of the nuclear forces
- Wigner SU(4) spin-flavor symmetry emerges

QCD viewpoint

- quark masses trade-off with strong scale
- Short, intermediate and long-range components of force all correlated by QCD
- Interesting EFT

Side Comment(s)

- **This fine-tuning is accomodated by an EFT near a nontrivial fixed point - expand around unitarity**
  - NNEFT not similar to NRQED
- **Implications for Quantum Computing**
  - chemistry algorithms and scalings may not readily translate to nonperturbative QCD
    - much more exploration is required

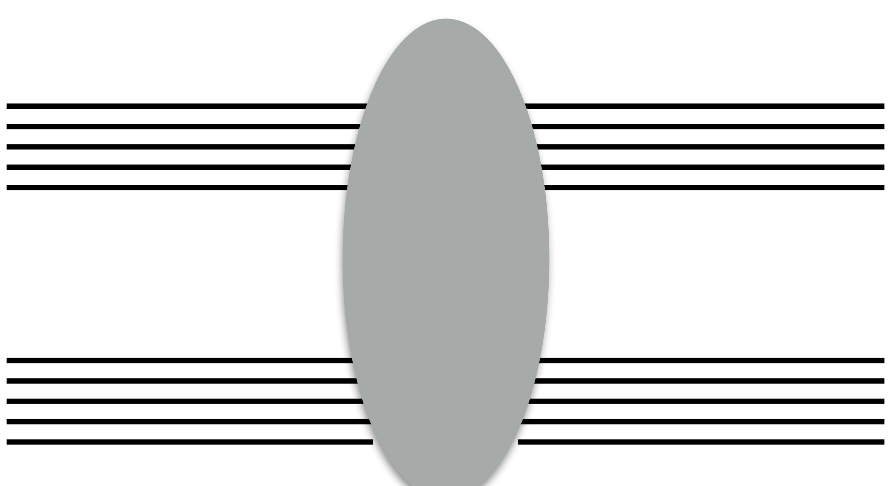
Analysis of Nuclear Forces

QCD and Electroweak parameters : Large Scattering Lengths

Wigner symmetry in the limit of large scattering lengths,
by Thomas Mehen, Iain W. Stewart, Mark B. Wise.
Phys.Rev.Lett. 83 (1999) 931-934

$$a_1\ ,\ a_3 \ \rightarrow\ \infty \qquad\qquad \text{SU(4) symmetry} \qquad\qquad \begin{pmatrix} p\uparrow \\ p\downarrow \\ n\uparrow \\ n\downarrow \end{pmatrix}$$

Large Nc Limit of QCD



$$(N^\dagger N)(N^\dagger N) \quad \sim 1$$

$$(N^\dagger \sigma N)(N^\dagger \sigma N) \sim 1/Nc^2$$

$$\begin{pmatrix} p\uparrow \\ p\downarrow \\ n\uparrow \\ n\downarrow \end{pmatrix}$$

SU(4) symmetry

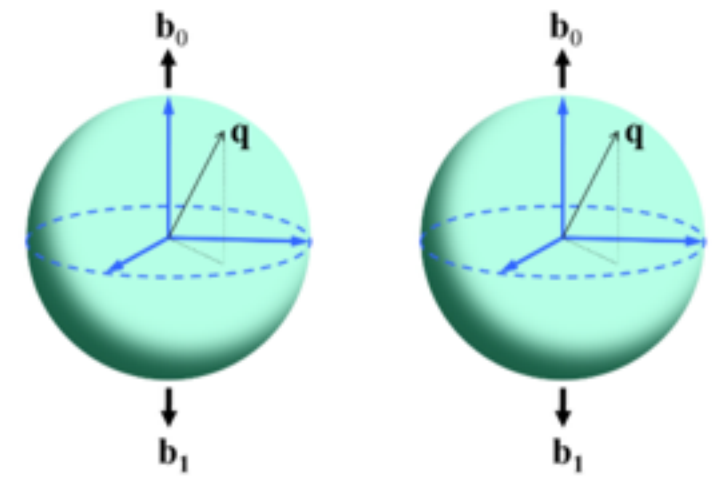
David Kaplan and MJS, Phys.Lett. B365 (1996) 244-251

Entanglement Power of an Operator

Ballard and Wu (2011)

$$|\Psi(\theta_i,\phi_i)\rangle \;=\; \hat{U}(\theta_1,\phi_1)|0\rangle_1 \otimes \hat{U}(\theta_2,\phi_2)|0\rangle_2$$

$$\hat{U}(\theta_1,\phi_1)|0\rangle_1 \;=\; \cos\left(\frac{\theta_1}{2}\right)|0\rangle_1 \;+\; e^{i\phi_1}\sin\left(\frac{\theta_1}{2}\right)|1\rangle_1$$

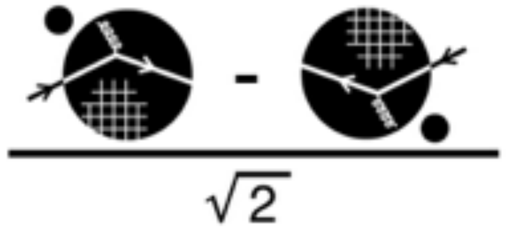


$$|\overline{\Psi}(\theta_i,\phi_i)\rangle \;=\; \hat{A}|\Psi(\theta_i,\phi_i)\rangle \qquad \hat{\rho}_{\hat{A};12}(\theta_i,\phi_i) \;=\; |\overline{\Psi}\rangle\langle\overline{\Psi}|$$

Unitary Operator

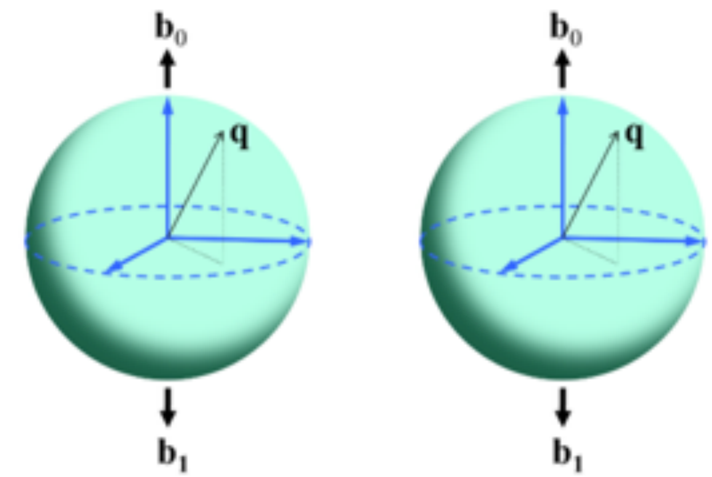
$$\hat{\rho}_{\hat{A};1}(\theta_i,\phi_i) \;=\; \mathrm{Tr}_2[\hat{\rho}_{\hat{A};12}(\theta_i,\phi_i)] \qquad E_{\hat{A}}(\theta_i,\phi_i) \;=\; 1-\mathrm{Tr}_1[\,(\hat{\rho}_{\hat{A};1}(\theta_i,\phi_i))^2\,]$$

$$e_p(\hat{A}) \;=\; \overline{E_{\hat{A};p}} \;=\; \frac{1}{(4\pi)^2}\int_{-1}^{+1} d\cos\theta_1 \int_0^{2\pi} d\phi_1 \int_{-1}^{+1} d\cos\theta_2 \int_0^{2\pi} d\phi_2\; E_{\hat{A}}(\theta_i,\phi_i)\; p(\theta_i,\phi_i)$$



e.g., Entanglement Power of CNOT.H : Setting the ``Scale"

Act with Hadamard gate, followed by a CNOT gate:

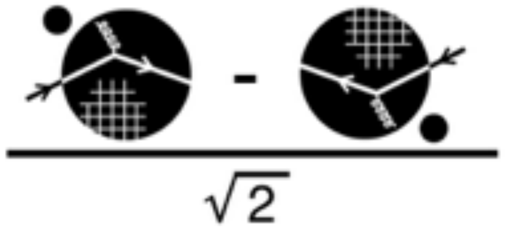


$$
\begin{aligned}
|\overline{\Psi}(\theta_i,\phi_i)\rangle \;=\; & \frac{1}{\sqrt{2}}\cos(\frac{\theta_2}{2})\left(\cos(\frac{\theta_1}{2}) + e^{i\phi_1}\sin(\frac{\theta_1}{2})\right)|00\rangle \\
+ & \frac{1}{\sqrt{2}}e^{i\phi_2}\sin(\frac{\theta_2}{2})\left(\cos(\frac{\theta_1}{2}) - e^{i\phi_1}\sin(\frac{\theta_1}{2})\right)|10\rangle \\
+ & \frac{1}{\sqrt{2}}e^{i\phi_2}\sin(\frac{\theta_2}{2})\left(\cos(\frac{\theta_1}{2}) + e^{i\phi_1}\sin(\frac{\theta_1}{2})\right)|01\rangle \\
+ & \frac{1}{\sqrt{2}}\cos(\frac{\theta_2}{2})\left(\cos(\frac{\theta_1}{2}) - e^{i\phi_1}\sin(\frac{\theta_1}{2})\right)|11\rangle \quad .
\end{aligned}
$$

$$
\mathrm{Tr}_1\left[\rho_1^2\right] \;=\; \frac{1}{2}\left[\, 1 + \sin^2\theta_1\cos^2\phi_1 + \sin^2\theta_2\cos^2\phi_2\left(1 - \sin^2\theta_1\cos^2\phi_1\right)\right]
$$

$$
e_p(\hat{A}) \;=\; \overline{E_{\hat{A};p}} \;=\; \frac{1}{(4\pi)^2}\int_{-1}^{+1} d\cos\theta_1 \int_0^{2\pi} d\phi_1 \int_{-1}^{+1} d\cos\theta_2 \int_0^{2\pi} d\phi_2 \; E_{\hat{A}}(\theta_i,\phi_i)\; p(\theta_i,\phi_i)
$$

CNOT.H $\longrightarrow$ 2/9 1



Entanglement by Low-Energy Strong S-Matrix

Two-hadron s-wave scattering

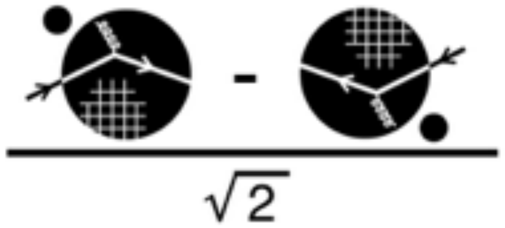
- Two-body S-matrix is a unitary operator below inelastic threshold(s)
- neglect non-central forces

$$\hat{\mathbf{S}}_\sigma = \frac{1}{4}\left(3e^{i2\delta_3} + e^{i2\delta_1}\right)\hat{\mathbf{1}} + \frac{1}{4}\left(e^{i2\delta_3} - e^{i2\delta_1}\right)\hat{\boldsymbol{\sigma}}\cdot\hat{\boldsymbol{\sigma}}$$

Define the ***Entanglement Power*** of the S-Matrix to be

$$e_p(\hat{A}) \longrightarrow \mathcal{E}(\hat{\mathbf{S}}_\sigma) = \frac{1}{6}\ \sin^2\left(2(\delta_3 - \delta_1)\right)$$

- The average value of fluctuations in entanglement induced by strong forces
- vanishes for identical phase shifts, or those that differ by n $\boldsymbol{\pi}/2$
- vanishes in the large-Nc limit and unitary limit

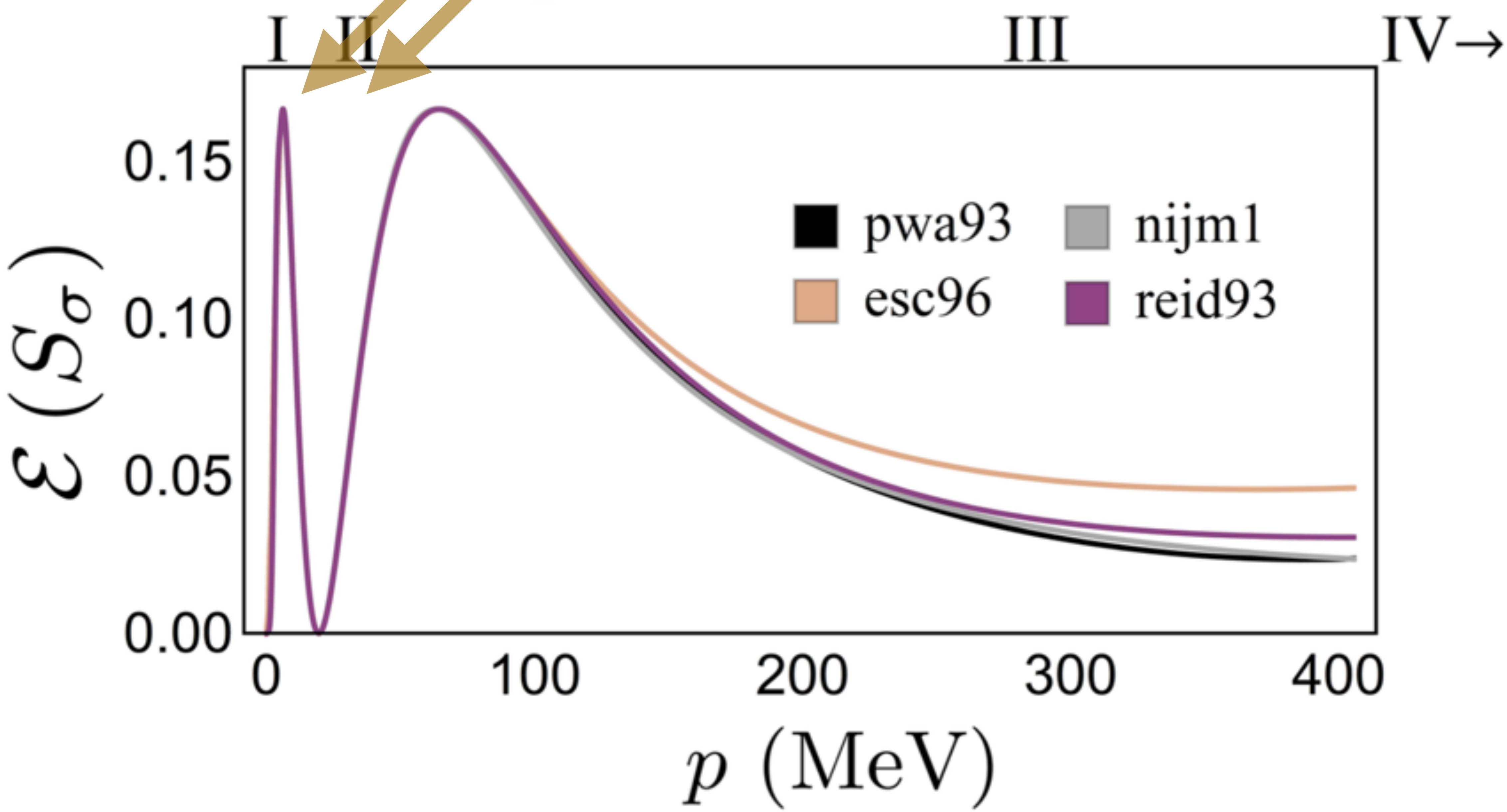


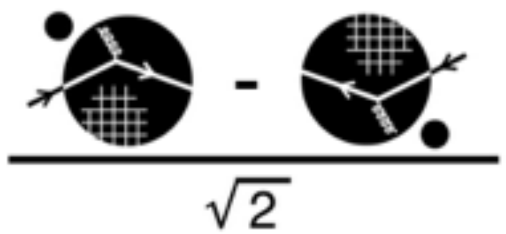
Entanglement by Low-Energy Strong S-Matrix

max @ ~10 MeV

min @ ~20 MeV

Experiments ?

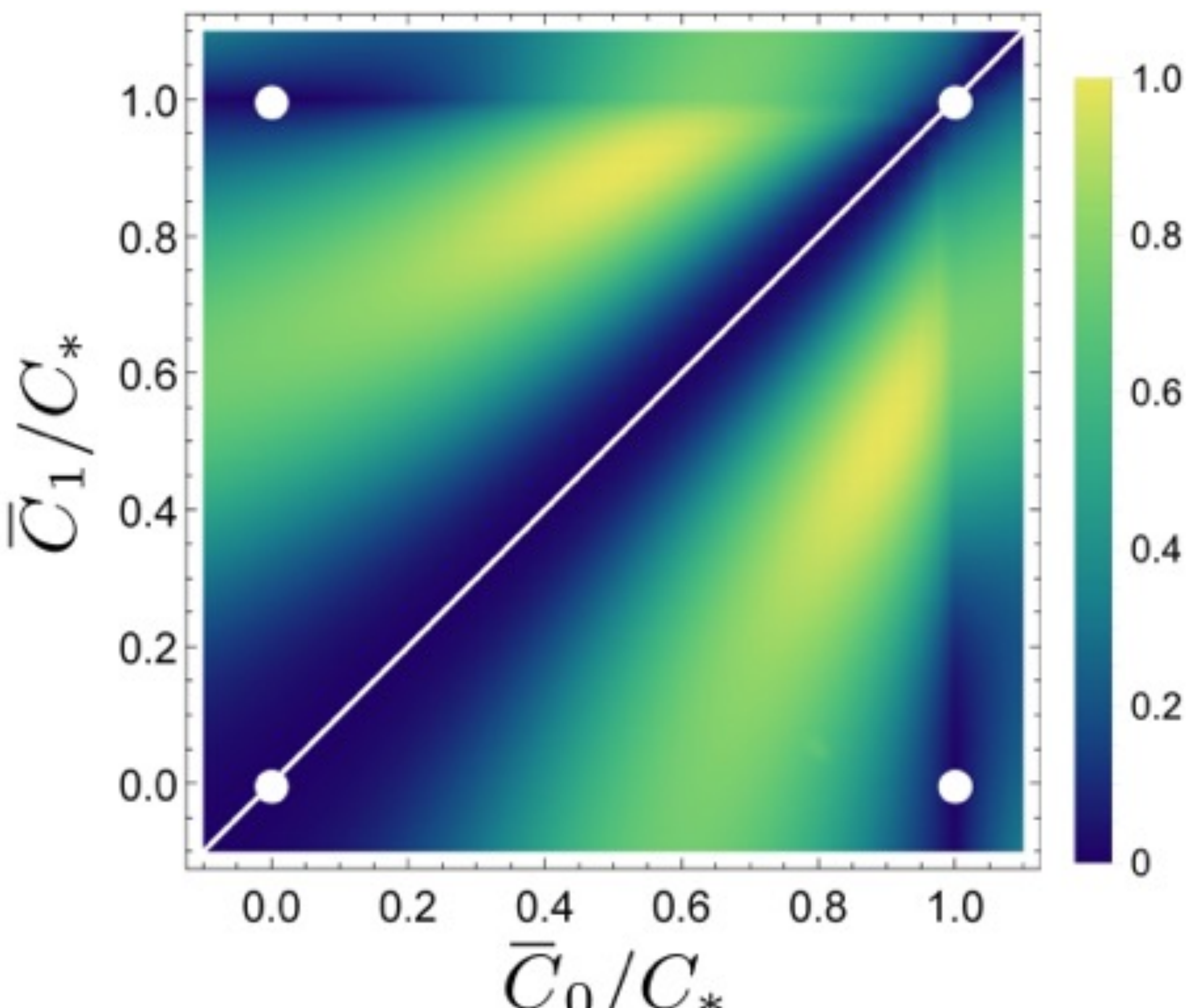




Entanglement by Low-Energy Strong S-Matrix n_f=2

$$\mathcal{L}_{\rm LO}^{n_f=2} = -\frac{1}{2}C_S(N^\dagger N)^2 - \frac{1}{2}C_T\left(N^\dagger \boldsymbol{\sigma} N\right)\cdot\left(N^\dagger \boldsymbol{\sigma} N\right)$$

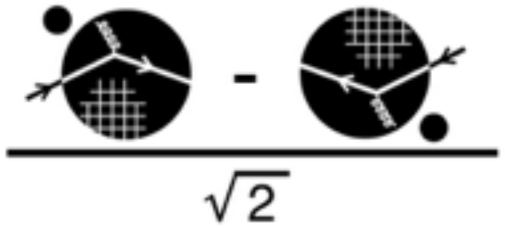
$$\overline{C}_0 = (C_S - 3C_T) \text{ and } \overline{C}_1 = (C_S + C_T)$$



$C_\star$ is coupling for system at unitarity

conformal points

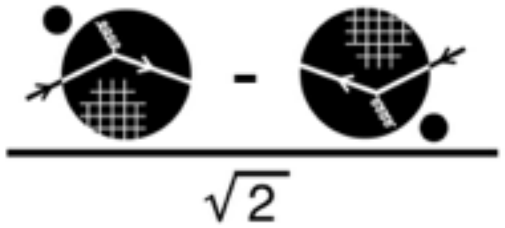
Wigner symmetry



Entanglement by Low-Energy Strong S-Matrix
n_f=2

Entanglement Suppression in low-energy NN forces is consistent with SU(4) Wigner symmetry, large-Nc limit

Is the* Suppression of Fluctuations in Entanglement *a general defining principle of strongly interacting theories ?



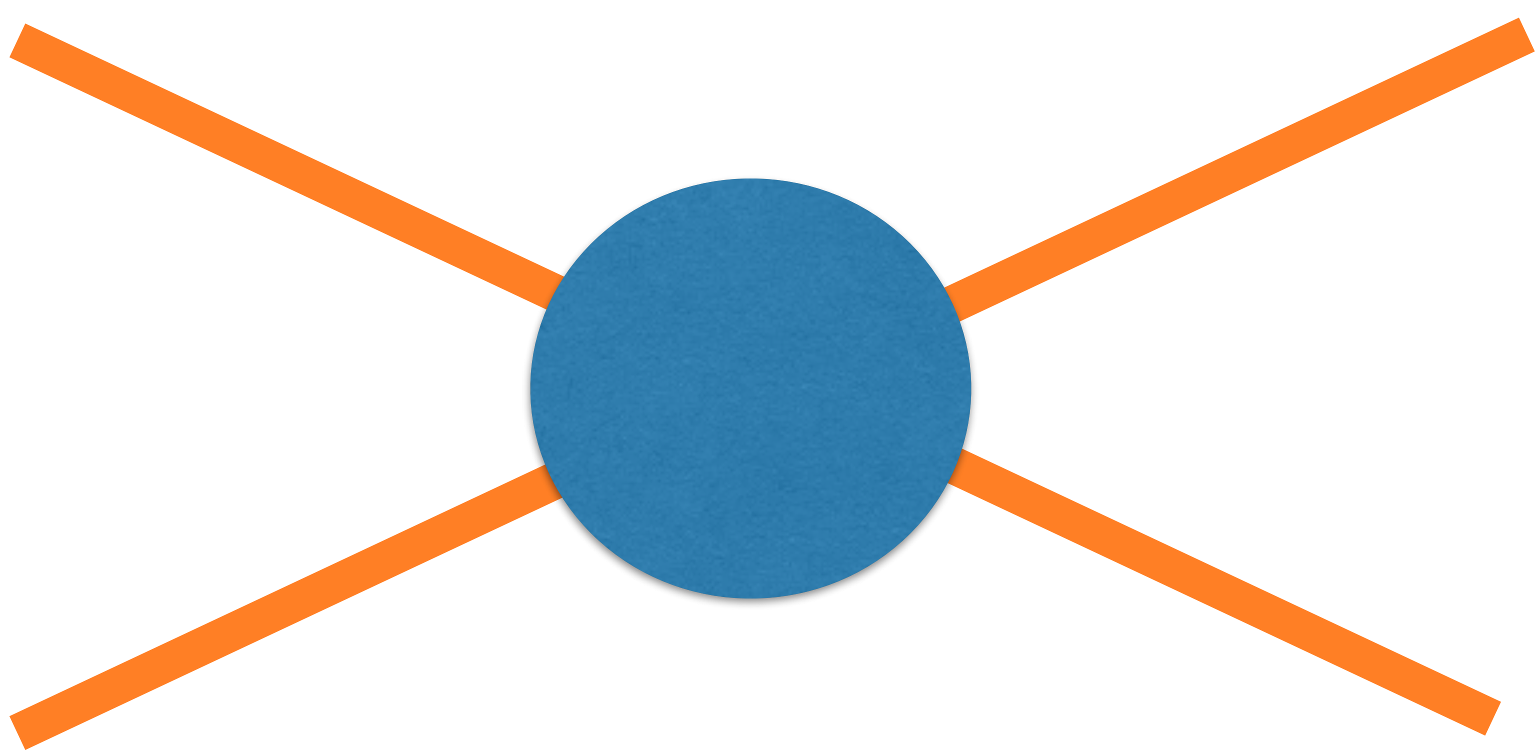
Entanglement by Low-Energy Strong S-Matrix n_f=3

$$\mathcal{L}_{\mathrm{LO}}^{n_f=3} = -c_1\langle B_i^\dagger B_i B_j^\dagger B_j\rangle - c_2\langle B_i^\dagger B_j B_j^\dagger B_i\rangle$$
$$-c_3\langle B_i^\dagger B_j^\dagger B_i B_j\rangle - c_4\langle B_i^\dagger B_j^\dagger B_j B_i\rangle$$
$$-c_5\langle B_i^\dagger B_i\rangle\langle B_j^\dagger B_j\rangle - c_6\langle B_i^\dagger B_j\rangle\langle B_j^\dagger B_i\rangle$$

$$B = \begin{bmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{bmatrix}$$

$SU(3)_{\mathrm{flavor}} \otimes SU(2)_{\mathrm{spin}}$

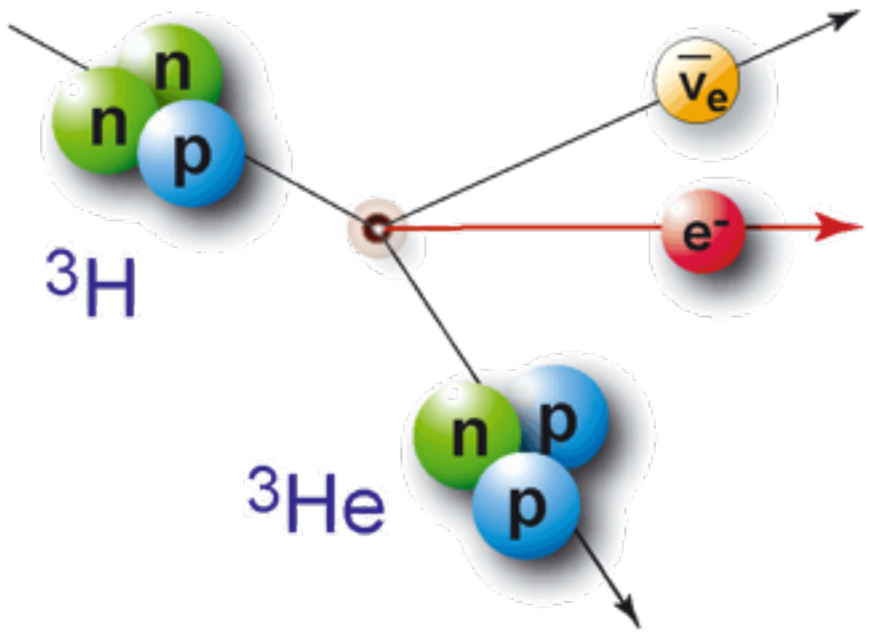
Baryon octet

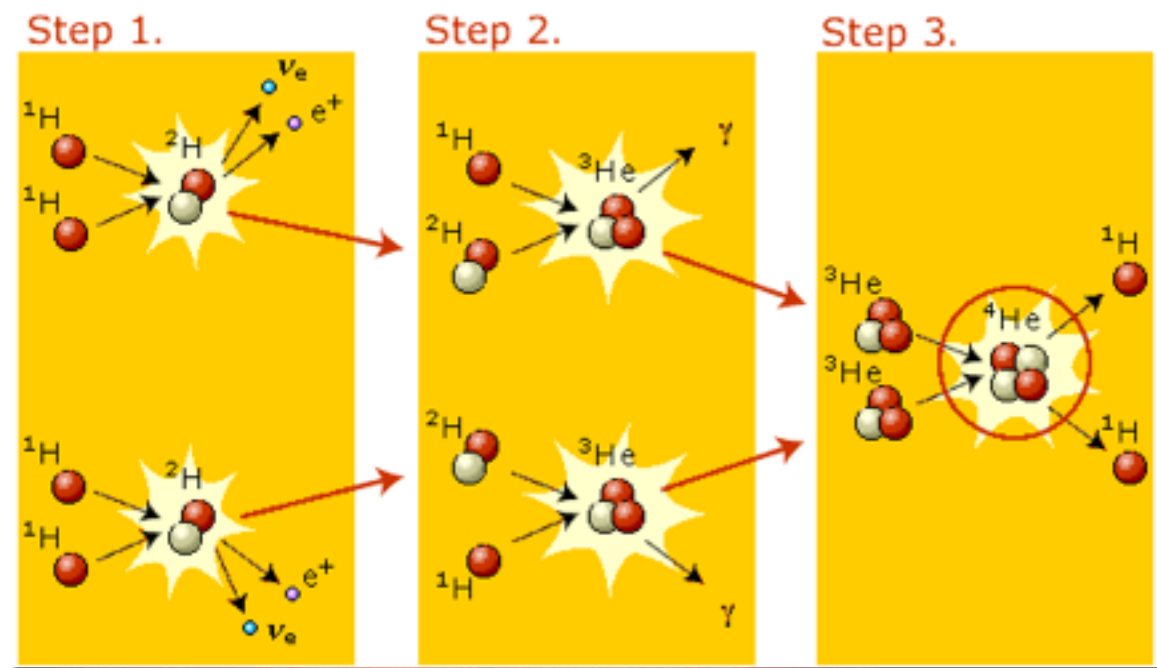


$$C(p^2,\mu)_\sigma = \overline{C}_S\, \hat{\mathbf{1}} + \overline{C}_T\, \hat{\boldsymbol{\sigma}} \cdot \hat{\boldsymbol{\sigma}}$$


NPLQCD
founded 2004

Lattice QCD
Proton-Proton Fusion and
Nuclear Axial Matrix Elements



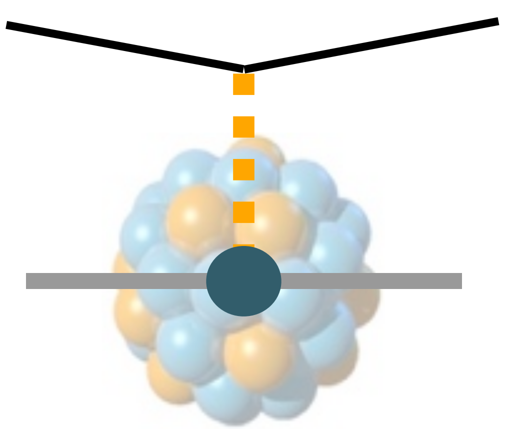


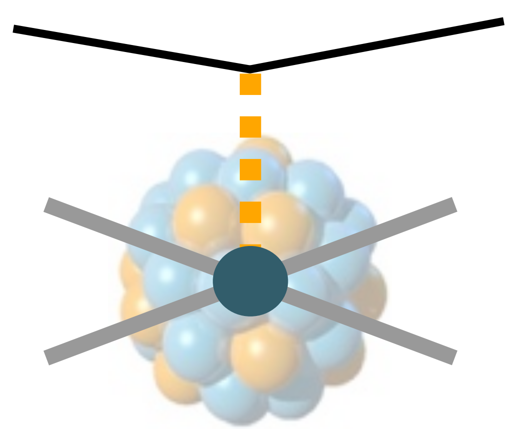
pionless theory

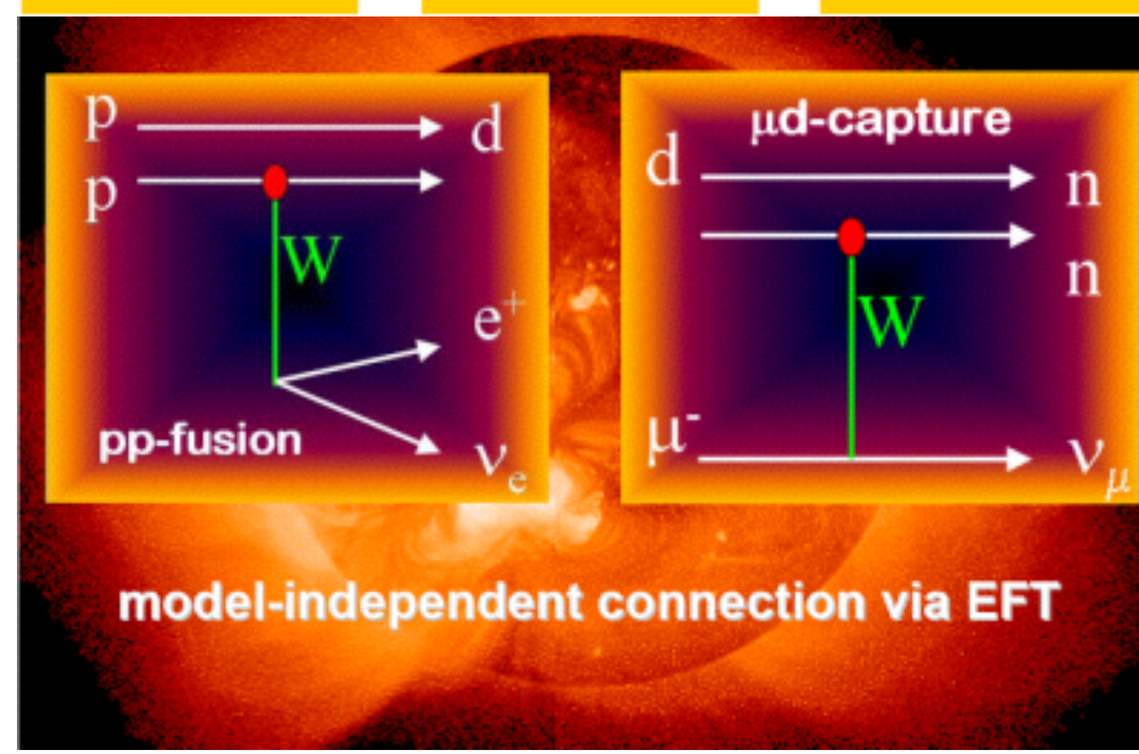
$$\delta\mathcal{L} = -gW\,\frac{g_A}{2}\,N^\dagger\sigma^z\tau^3N - \frac{gW\,l_{1,A}}{2M\sqrt{r_1r_3}}\left[t_3^\dagger s_3 + \mathrm{h.c.}\right] + \ldots$$

single-nucleon interaction

leading two-nucleon interaction

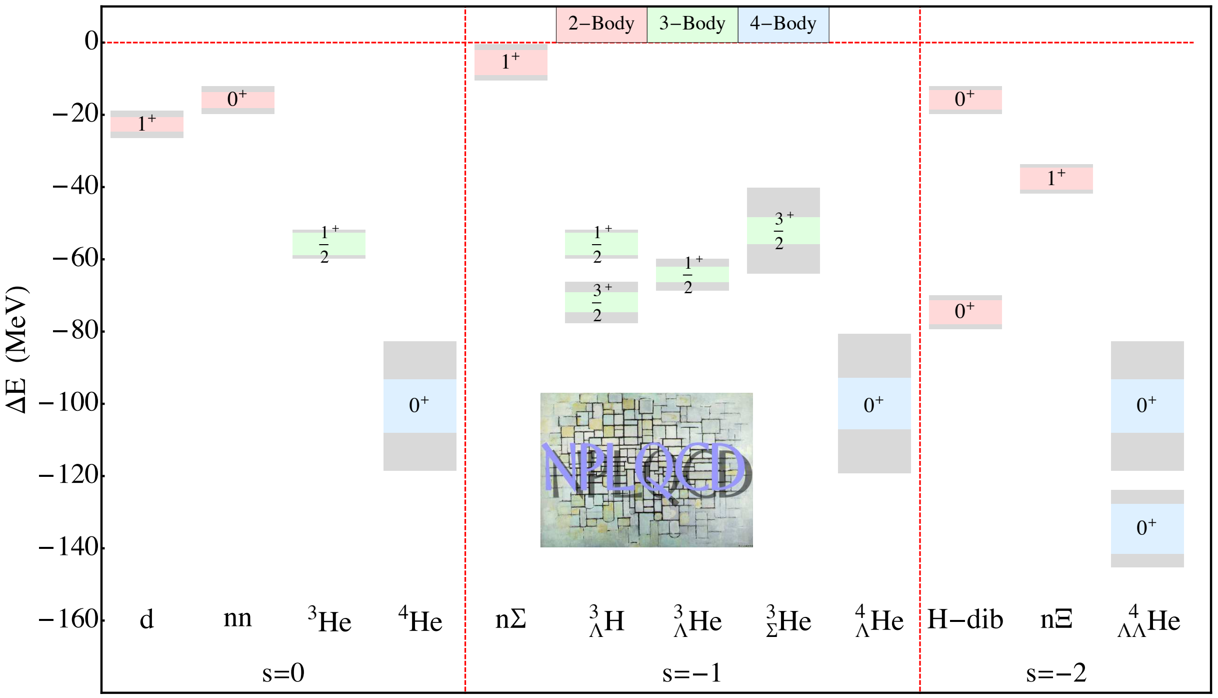


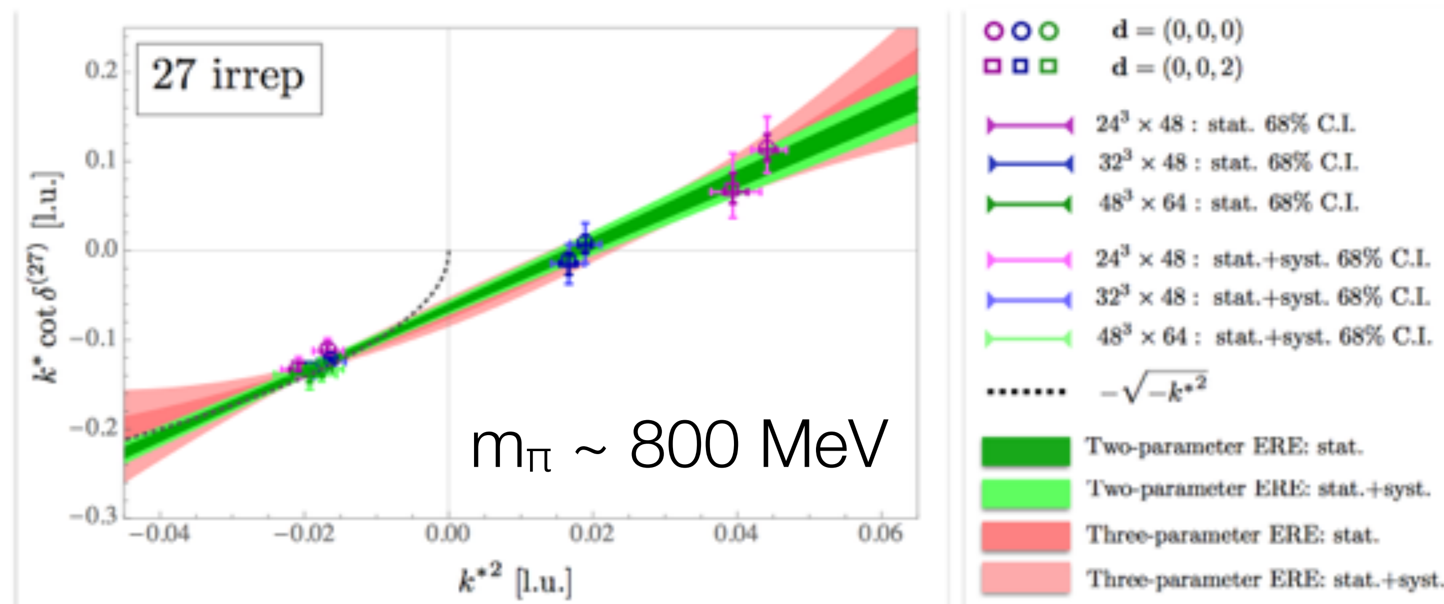


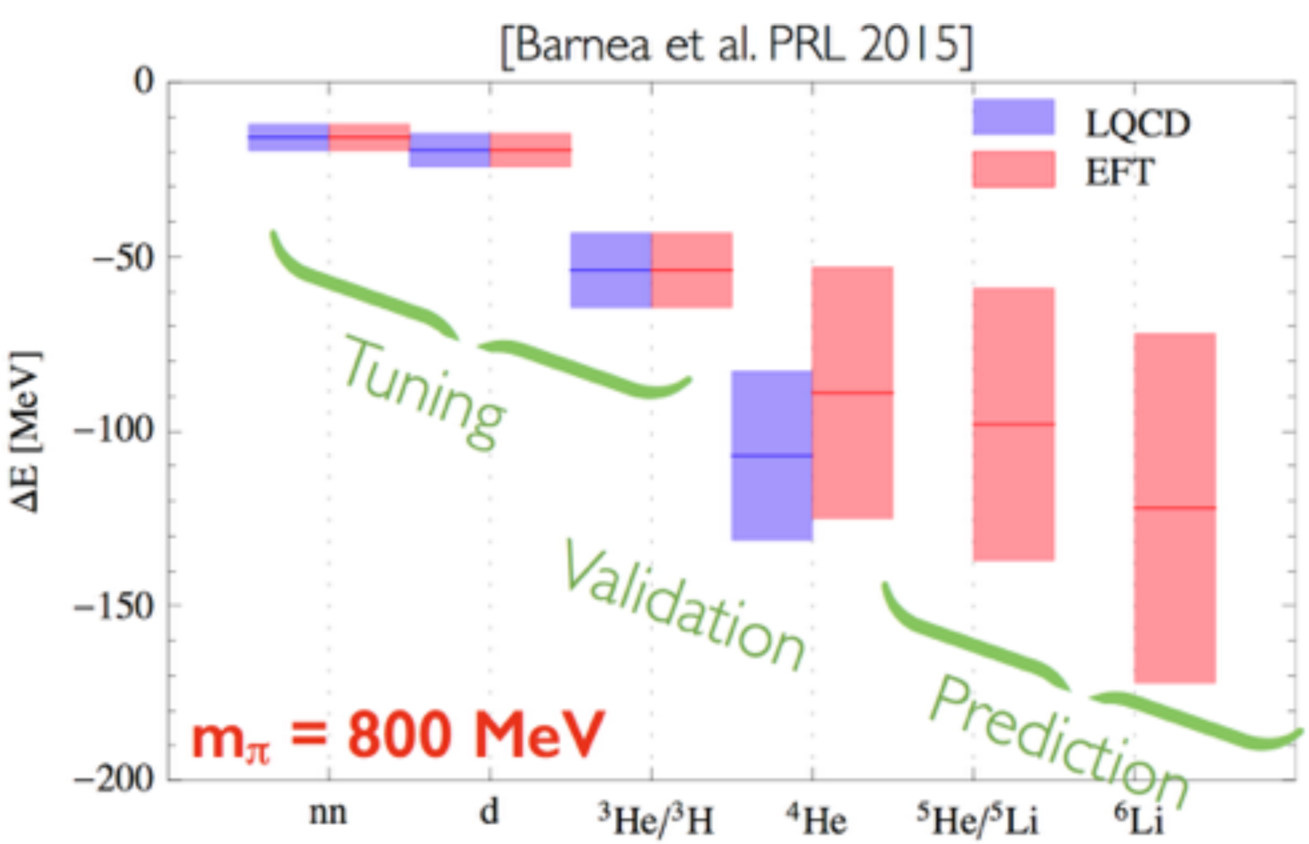


$\Lambda(0) = 2.6585(06)(72)(25)$ and $L_{1,A} = 3.9(0.1)(1.0)(0.3)(0.9)$ fm^3

Lattice QCD
QCD to Forces: First Steps





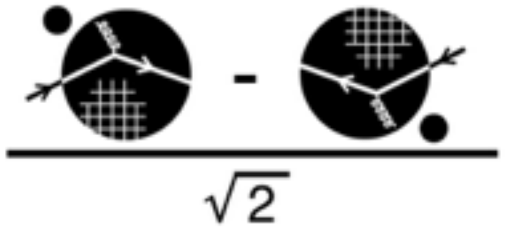


Finite-Volume Energy Eigenvalues

Unnatural systems with approximate SU(6) and SU(16) Spin-Flavor Symmetries

Must calculate over a range of quark masses to refine nuclear forces

Ground-State Properties of 4He and 16O Extrapolated from Lattice QCD with Pionless EFT
L. Contessi, A. Lovato, F. Pederiva, A. Roggero, J. Kirscher, U. van Kolck.,

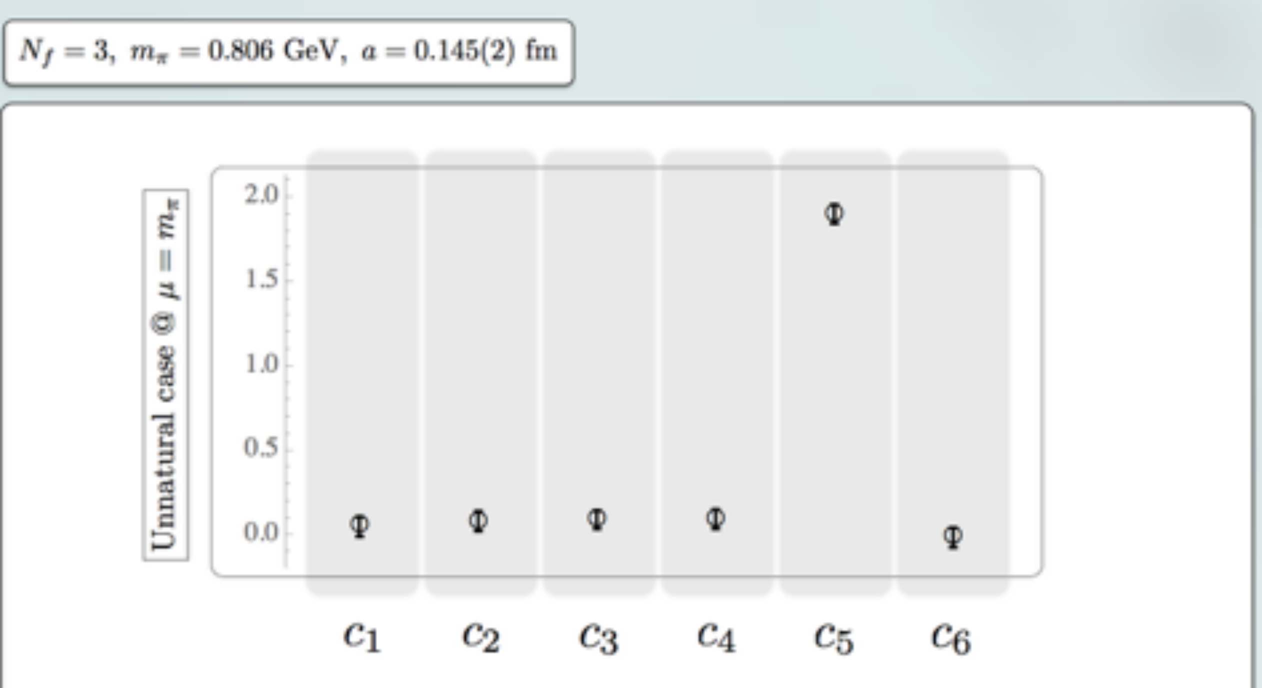


Three flavors @ Leading Order 2-Body - Limits of the Large-Nc Limits

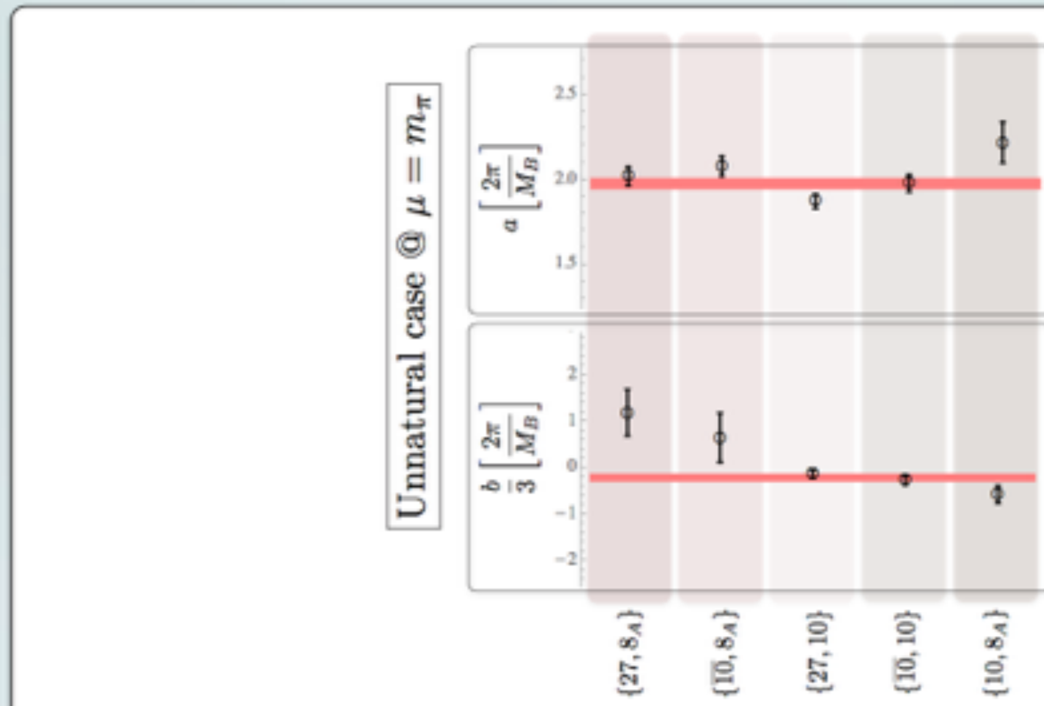
NPLQCD analysis led by Zohreh Davoudi (UMD)

Large-Nc
$$c_1 = -\frac{7}{27}b\ ,\ c_2 = \frac{1}{9}b\ ,\ c_3 = \frac{10}{81}b\ ,$$
$$c_4 = -\frac{14}{81}b\ ,\ c_5 = a + \frac{2}{9}b\ ,\ c_6 = -\frac{1}{9}b\ .$$

Lattice QCD calculations at SU(3) symmetric point [NPLQCD]



$$\mathcal{L}^{(0)}_{BB} = -c_1 \mathrm{Tr}(B_i^\dagger B_i B_j^\dagger B_j) - c_2 \mathrm{Tr}(B_i^\dagger B_j B_j^\dagger B_i) - c_3 \mathrm{Tr}(B_i^\dagger B_j^\dagger B_i B_j) - c_4 \mathrm{Tr}(B_i^\dagger B_j^\dagger B_j B_i) - c_5 \mathrm{Tr}(B_i^\dagger B_i)\mathrm{Tr}(B_j^\dagger B_j) - c_6 \mathrm{Tr}(B_i^\dagger B_j)\mathrm{Tr}(B_j^\dagger B_i).$$

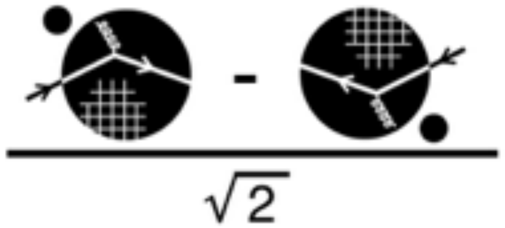


$$\mathcal{L}^{(0)}_{BB} = -a(\Psi^\dagger_{\mu\nu\rho}\Psi^{\mu\nu\rho})^2 - b(\Psi^\dagger_{\mu\nu\sigma}\Psi^{\mu\nu\tau})(\Psi^\dagger_{\rho\delta\tau}\Psi^{\rho\delta\sigma})$$

NPLQCD

Baryon-Baryon Interactions and Spin-Flavor Symmetry from Lattice Quantum Chromodynamics
Michael L. Wagman, Frank Winter, Emmanuel Chang, Zohreh Davoudi, William Detmold, Kostas Orginos, Martin J. Savage, Phiala E. Shanahan.
Phys.Rev. D96 (2017) no.11, 114510

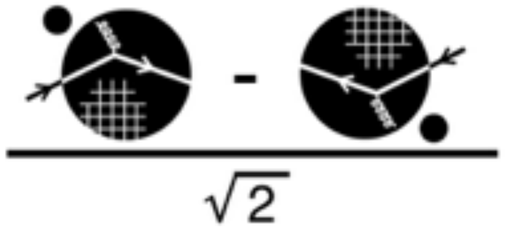
Consistent with predicted emergent SU(6) and a larger SU(16)



SU(16)

$$B = \begin{bmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{bmatrix}$$

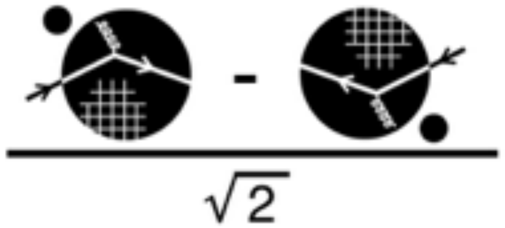
$$\begin{pmatrix} p\uparrow \\ p\downarrow \\ n\uparrow \\ n\downarrow \\ \Lambda\uparrow \\ \Lambda\downarrow \\ \Sigma^+\uparrow \\ \Sigma^+\downarrow \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$$



Three flavors @ Leading Order 2-Body
- Limits of the Large-Nc Limits

Emergent SU(6) spin-flavor symmetry, coefficients indicate very nearly SU(16) symmetry

An emergent SU(16) spin-flavor symmetry is consistent with, but larger than, predicted by large-Nc



Three flavors @ Leading Order 2-Body — Entanglement Power

$$\mathcal{L}_{\mathrm{LO}}^{n_f=3} = -c_1\langle B_i^\dagger B_i B_j^\dagger B_j\rangle - c_2\langle B_i^\dagger B_j B_j^\dagger B_i\rangle$$
$$-c_3\langle B_i^\dagger B_j^\dagger B_i B_j\rangle - c_4\langle B_i^\dagger B_j^\dagger B_j B_i\rangle$$
$$\boxed{-c_5\langle B_i^\dagger B_i\rangle\langle B_j^\dagger B_j\rangle} - c_6\langle B_i^\dagger B_j\rangle\langle B_j^\dagger B_i\rangle$$

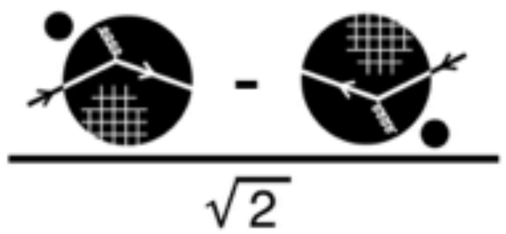
Requiring

$$\mathcal{E}(\hat{\mathbf{S}}_\sigma) = \frac{1}{6}\ \sin^2\left(2(\delta_3 - \delta_1)\right) = 0$$

$$c1 = c2 = c3 = c4 = c6 = 0$$

Emergent SU(16) spin-flavor symmetry

Consistent with Lattice QCD results, and stricter than large-Nc



Three flavors @ General 2-Body — Entanglement Power

Vanishing Entanglement Power implemented by 16 conserved charges:

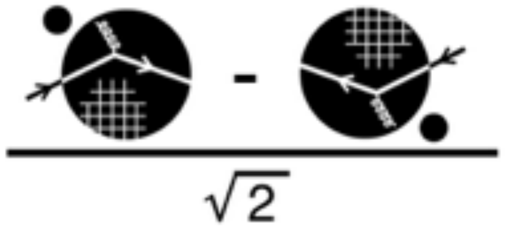
$$SU(3)_{flavor} \otimes SU(2)_{spin} \otimes U(1)^{16} \longrightarrow SU(16)$$

violated by spin-violating forces

Arbitrary operators that conserve spin only

Hamiltonian exhibits SU(16) spin-flavor symmetry including derivative terms

- **not restricted to only local operators**
- **vanishing entanglement is stronger condition than U(1) charge conservation**



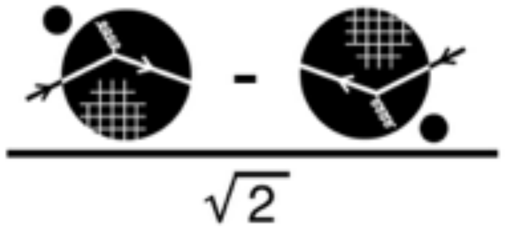
Three flavors @ General 2-Body — Entanglement Power

$|\Delta q|=1$

$$\hat{\Theta} = \int d^3\mathbf{v} d^3\mathbf{u} \ \left[f(\mathbf{v}-\mathbf{u}) \alpha^\dagger_{\mathbf{v}} \beta_{\mathbf{u}} + \text{h.c.} \right]$$

$$|\alpha_{\mathbf{x}}, \beta_{\mathbf{y}}, \gamma_{\mathbf{z}}\rangle \to \int d^3\mathbf{w} \ \left[f(\mathbf{w}-\mathbf{y}) |\alpha_{\mathbf{x}}, \alpha_{\mathbf{w}}, \gamma_{\mathbf{z}}\rangle + f^*(\mathbf{x}-\mathbf{w}) |\beta_{\mathbf{w}}, \beta_{\mathbf{y}}, \gamma_{\mathbf{z}}\rangle \right]$$

- **A one-body operator that changes U(1) charge and entangles**
- **There are higher body operators that conserve U(1) charges and entangle**
- **Vanishing entanglement requires U(1) charges conserved**



Three flavors @ General n-Body — Entanglement Power

Vanishing Entanglement Power *predicts* the dominant structure of all n-body operators to be simple

$$\mathcal{L}^{(n_f=2)} = -\sum_{n=2}^{4} \frac{1}{n!} C_S^{(n)} \left(N^\dagger N\right)^n$$

$$\mathcal{L}^{(n_f=3)} = -\sum_{n=2}^{16} \frac{1}{n!} c_S^{(n)} \left(\mathcal{B}^\dagger \mathcal{B}\right)^n$$

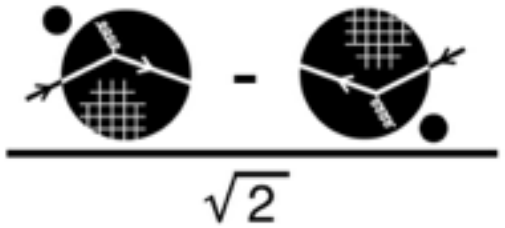
In general, many different operator structures

Constrains form of interactions with other matter fields

Sign Problems and Entanglement

- SU(4) limit used in initial numerical time evolution of EFT calculations on a lattice
  - no sign problem
  - zero fluctations in entanglement

- SU(4) breaking terms introduce a sign problem, which limits evolution time
  - fluctuations in entanglement



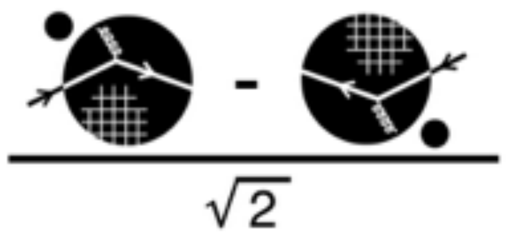
Summary

- Entanglement Power in QCD is suppressed at low energies
  - Why? Generic in confining theories?

- Induced emergent SU(4) and SU(16) spin-flavor symmetries consistent with QCD phenomenology

- Predicts form of many-body operators
  - expect stringent constraints on interactions with other matter fields

- Broader and more detailed analyses are desirable, and ongoing work



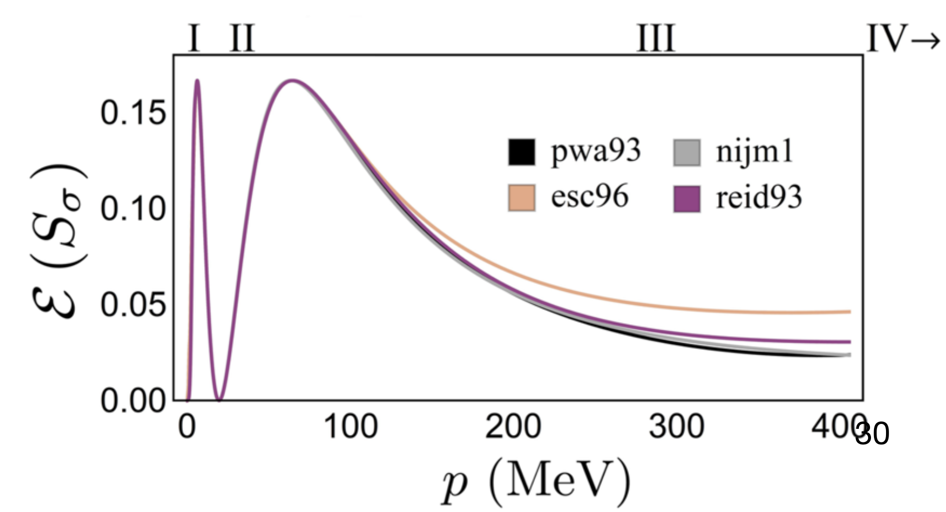
Questions

- Does nature choose quark masses to put systems near unitarity in order to reduce fluctuations in entanglement?

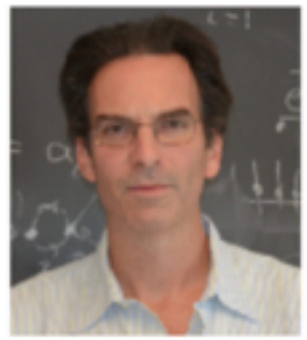
- Are the SU(4) and SU(16) symmetries implemented most ``efficiently'' with large scattering lengths?

- Is the SU(4) breaking, and rapid variation of physical NN Entanglement Power, the driver?

- Are these related features?



Acknowledgements

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