

LIGHT MESONS FROM TAU DECAYS

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BASED ON:

S. GONZÁLEZ-SOLÍS AND P. ROIG; EUR. PHYS. J. C79 (2019) 436,

R. ESCRIBANO, S. GONZÁLEZ-SOLÍS, M. JAMIN AND P. ROIG; JHEP 1409 (2014) 042,

A. FRIEDLAND, S. GONZÁLEZ-SOLÍS AND E. PASSEMAR; TO APPEAR SOON.

JLAB, USA, OCTOBER 14, 2019

1 Introduction

2 Tau decays into two mesons

- $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$: The Pion vector form factor
- $\tau^- \rightarrow K^- K_S \nu_\tau$: The Kaon vector form factor
- $\tau^- \rightarrow K_S \pi^- \nu_\tau$ and $\tau^- \rightarrow K^- \eta \nu_\tau$: $K\pi$ form factor

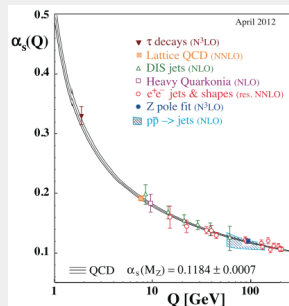
3 $\tau \rightarrow 3\pi \nu_\tau$ (preliminary results)

4 Outlook

INTRODUCTION

QUANTUM CHROMODYNAMICS

- **asymptotic freedom:**
"like QED", but only at high energies
- **confinement:**
at low energies the gluons bind the quarks together to form the hadrons



- Approaches to describe the Low-energy regime of QCD:
 1. Lattice QCD simulations: determination of SM fundamental parameters from first principles (quark masses, α_s)
 2. Chiral Perturbation theory
 3. S-matrix theory: based on analyticity and unitarity arguments (dispersion relations)

TEST OF QCD AND ELECTROWEAK INTERACTIONS

- Inclusive decays: $\tau^- \rightarrow (\bar{u}d, \bar{u}s)\nu_\tau$

Full hadron spectra (precision physics)



Fundamental SM parameters:

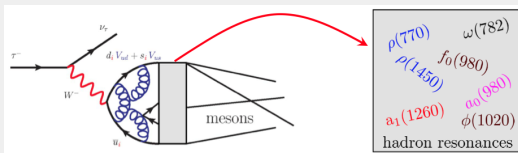
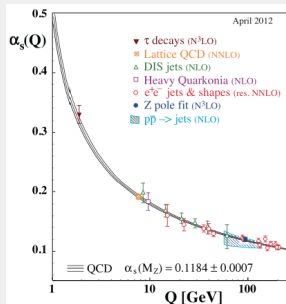
$$\alpha_s(m_\tau), m_s, |V_{us}|$$

- Exclusive decays: $\tau^- \rightarrow (PP, PPP, \dots)\nu_\tau$

specific hadron spectrum (approximate physics)

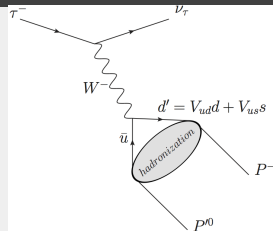


Hadronization of QCD currents, study of Form Factors, resonance parameters (M_R, Γ_R)



τ DECAYS INTO TWO MESONS

- $\tau^- \rightarrow P^- \nu_\tau : F_{\pi,K}$ (decay constants)
- $\tau^- \rightarrow (2P)^- \nu_\tau$: good control
- $\tau^- \rightarrow (3P)^- \nu_\tau$: reasonable good control
- $\tau^- \rightarrow (> 3P)^- \nu_\tau$: poor knowledge



$$\mathcal{M}(\tau^- \rightarrow P^- P'^0 \nu_\tau) = \frac{G_F}{\sqrt{2}} V_{\text{CKM}} \bar{u}(p_{\nu_\tau}) \gamma^\mu (1 - \gamma^5) u(p_\tau) \langle P^- P'^0 | d' \gamma^\mu u | 0 \rangle,$$

$$\langle P^- P'^0 | d' \gamma^\mu u | 0 \rangle = C_{P-P'^0} \left\{ \left(p_- - p_0 - \frac{\Delta_{P-P'^0}}{s} q \right)^\mu F_V^{P^- P'^0}(s) + \frac{\Delta_{P-P'^0}}{s} q^\mu F_S^{P^- P'^0}(s) \right\}$$

$$\begin{aligned} \frac{d\Gamma(\tau^- \rightarrow P^- P'^0 \nu_\tau)}{ds} &= \frac{G_F^2 |V_{ui}|^2 m_\tau^3}{768 \pi^3} S_{EW}^{\text{had}} C_{PP'}^2 \left(1 - \frac{s}{M_\tau^2} \right)^2 \\ &\times \left\{ \left(1 + \frac{2s}{m_\tau^2} \right) \lambda_{P-P'^0}^{3/2}(s) |F_V^{P^- P'^0}(s)|^2 + 3 \frac{\Delta_{P-P'^0}^2}{s^2} \lambda_{P-P'^0}^{1/2}(s) |F_S^{P^- P'^0}(s)|^2 \right\} \end{aligned}$$

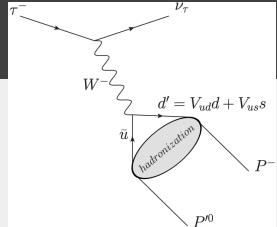
$$\Delta_{P-P'^0} = m_{P^-}^2 - m_{P'^0}^2, \quad C_{P'P^0} : \text{Clebsch - Gordon}$$

τ DECAYS INTO TWO MESONS

$$\frac{d\Gamma(\tau^- \rightarrow P^- P^0 \nu_\tau)}{ds} = \frac{G_F^2 |V_{ui}|^2 m_\tau^3}{768 \pi^3} S_{EW}^{\text{had}} C_{PP'}^2 \left(1 - \frac{s}{M_\tau^2}\right)^2$$

$$\left\{ \left(1 + \frac{2s}{m_\tau^2}\right) \lambda_{P^- P^0}^{3/2}(s) |F_V^{P^- P^0}(s)|^2 + 3 \frac{\Delta_{P^- P^0}^2}{s^2} \lambda_{P^- P^0}^{1/2}(s) |F_S^{P^- P^0}(s)|^2 \right\}$$

$$\Delta_{P^- P^0} = m_{P^-}^2 - m_{P^0}^2$$

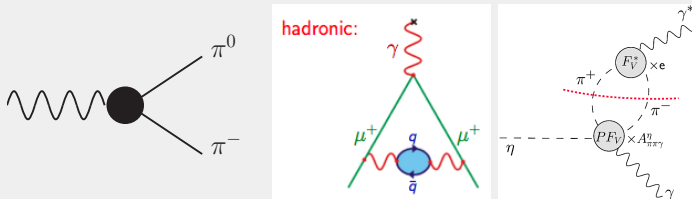


- $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$: Pion vector form factor, $\rho(770)$, $\rho(1450)$, $\rho(1700)$
- $\tau^- \rightarrow K^- K_S \nu_\tau$: Kaon vector form factor, $\rho(770)$, $\rho(1450)$, $\rho(1700)$
- $\tau^- \rightarrow K_S \pi^- \nu_\tau$: $K\pi$ form factor, $K^*(892)$, $K^*(1410)$, $K_{\ell 3}$, V_{us}
- $\tau^- \rightarrow K^- \eta^{(\prime)} \nu_\tau$: $K^*(1410)$, V_{us}
- $\tau^- \rightarrow \pi^- \eta^{(\prime)} \nu_\tau$: isospin violation

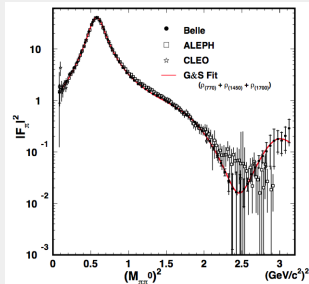
TAU DECAYS INTO TWO MESONS

THE PION VECTOR FORM FACTOR: MOTIVATION

- Enters the description of many physical processes



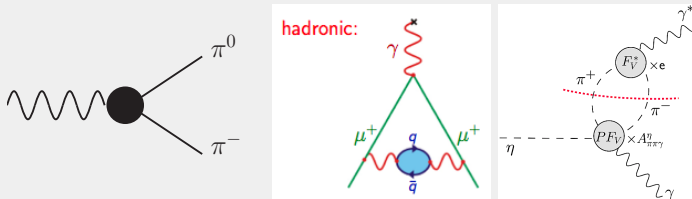
- Belle measurement of the pion form factor (PRD 78 (2008) 072006)



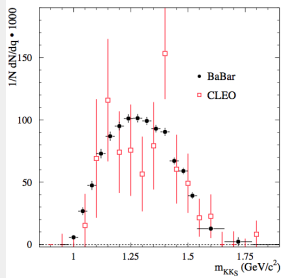
- high-statistics data until de τ mass
- sensitive to $\rho(1450)$ and $\rho(1700)$
- our aim: to improve the description of the $\rho(1450)$ and $\rho(1700)$ region

THE PION VECTOR FORM FACTOR: MOTIVATION

- Enters the description of many physical processes



- BaBar measurement of $\tau^- \rightarrow K^- K_S \nu_\tau$ (PRD 98 (2018) no.3, 032010)

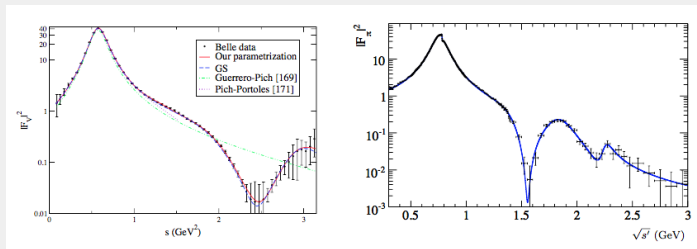


- good quality data
- sensitive to $\rho(1450)$ and $\rho(1700)$
- our aim: to improve the description of the $\rho(1450)$ and $\rho(1700)$ region

THE PION VECTOR FORM FACTOR $F_V^\pi(s)$

■ How to determine $F_V^\pi(s)$ experimentally?

- ▶ $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ (Belle PRD 78 (2008) 072006) and $e^+e^- \rightarrow \pi^+\pi^-$ (BaBar PRD 86 (2012) 032013)

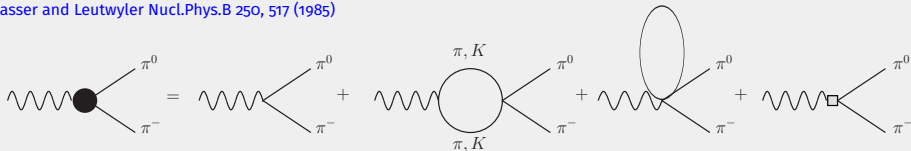


■ What do we know theoretically on the form factor?

- ▶ Its low-energy behaviour: given by ChPT (Gasser&Leutwyler'85)
- ▶ Its high-energy behaviour ($\sim 1/s$): given by pQCD (Brodsky&Lepage'79)
- ▶ For the intermediate energy region: models

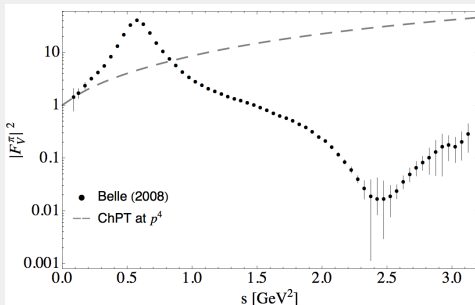
PION VECTOR FORM FACTOR: CHPT $\mathcal{O}(p^4)$

Gasser and Leutwyler Nucl.Phys.B 250, 517 (1985)



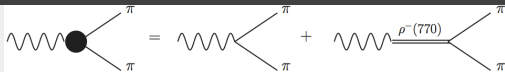
$$F_V^\pi(s)|_{\text{ChPT}} = 1 + \frac{2L_9^r(\mu)}{F_\pi^2} s - \frac{s}{96\pi^2 F_\pi^2} \left(A_\pi(s, \mu^2) + \frac{1}{2} A_K(s, \mu^2) \right),$$

$$A_P(s, \mu^2) = \log \frac{m_P^2}{\mu^2} + \frac{8m_P^2}{s} - \frac{5}{3} + \sigma_P^3(s) \log \left(\frac{\sigma_P(s) + 1}{\sigma_P(s) - 1} \right), \quad \sigma_P(s) = \sqrt{1 - 4 \frac{m_P^2}{s}}$$



PION VECTOR FORM FACTOR: CHPT WITH RESONANCES

■ Resonance Chiral Th. ($R_\chi T$)



$$F_V^\pi(s) = 1 + \frac{F_V G_V}{F_\pi^2} \frac{s}{M_\rho^2 - s} \xrightarrow{F_V G_V = F_\pi^2} \frac{M_\rho^2}{M_\rho^2 - s},$$

■ Expansion in s and comparing ChPT and $R_\chi T$

$$F_V^\pi(s) = 1 + \frac{2L_9^r(\mu)}{F_\pi^2} s - \frac{s}{96\pi^2 F_\pi^2} \left(A_\pi(s, \mu^2) + \frac{1}{2} A_K(s, \mu^2) \right)$$

$$F_V^\pi(s) = 1 + \left(\frac{s}{M_\rho^2} \right) + \left(\frac{s}{M_\rho^2} \right)^2 + \dots$$

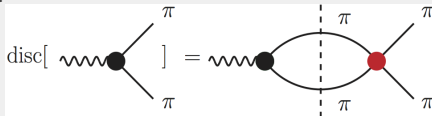
■ Chiral coupling estimate: $L_9^r(M_\rho) = \frac{F_V G_V}{2M_\rho^2} = \frac{F_\pi^2}{2M_\rho^2} \simeq 7.2 \times 10^{-3}$

■ Combining ChPT and $R_\chi T$

$$F_V^\pi(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 F_\pi^2} \left[A_\pi(s, \mu^2) + \frac{1}{2} A_K(s, \mu^2) \right],$$

DISPERSIVE REPRESENTATION

■ Dispersive approach

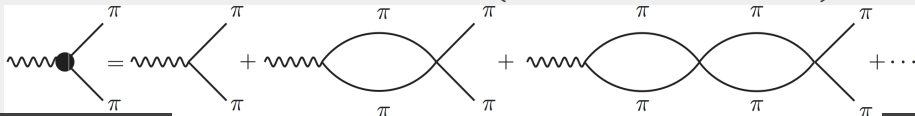


$$\text{disc}F_V(s) = 2i\sigma_\pi(s)F_V(s)T_1^{1*}(s) = 2iF_V(s)\sin\delta_1^1(s)e^{-i\delta_1^1(s)},$$

$$F_V(s) = \frac{1}{2i\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\text{disc}F_V(s')}{s' - s - i\varepsilon},$$

■ Analytic solution (Omnès equation)

$$F_V(s) = P(s)\Omega(s), \quad \Omega(s) = \exp \left\{ \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\delta_1^1(s')}{s'(s' - s - i\varepsilon)} \right\},$$



- Resummation of final-state interactions to all orders (Omnès)

$$F_V^\pi(s) = P_n(s) \exp \left\{ \frac{s^n}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{(s')^n} \frac{\delta_1^1(s')}{s' - s - i\varepsilon} \right\},$$

- Get a model for the phase from $\pi\pi \rightarrow \pi\pi$ scattering at $\mathcal{O}(p^2)$

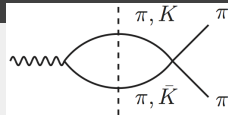
$$T(s) = \frac{s - m_\pi^2}{F_\pi^2} \rightarrow T_1^1(s) = \frac{s\sigma_\pi^2(s)}{96\pi F_\pi^2} \rightarrow \delta_1^1(s) = \sigma_\pi(s) T_1^1(s) = \frac{s\sigma_\pi^3(s)}{96\pi F_\pi^2},$$

- Omnès exponentiation of the full loop function

$$F_V^\pi(s) = \frac{M_\rho^2}{M_\rho^2 - s} \exp \left\{ - \frac{s}{96\pi^2 F_\pi^2} A_\pi(s, \mu^2) \right\}.$$

R χ T + OMNÈS: EXPONENTIAL REPRESENTATION

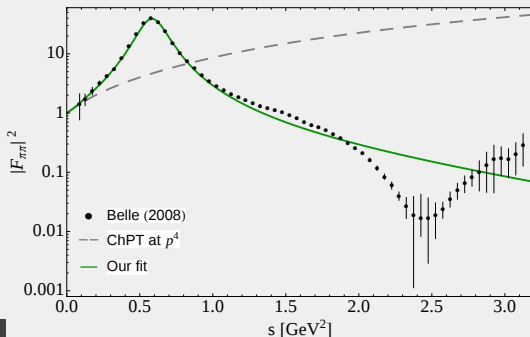
■ Incorporation of the (off-shell) ρ width



$$\Gamma_\rho(s) = -\frac{M_\rho s}{96\pi^2 F_\pi^2} \text{Im} \left[A_\pi(s) + \frac{1}{2} A_K(s) \right] = \frac{M_\rho s}{96\pi F_\pi^2} \left[\sigma_\pi(s)^3 \theta(s - 4m_\pi^2) + \sigma_K(s)^3 \theta(s - 4m_K^2) \right].$$

Guerrero, Pich PLB 412, 382 (1997)

$$F_V^\pi(s)|_{\text{expo}}^{\text{1 res}} = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left\{ -\frac{s}{96\pi^2 F_\pi^2} \text{Re} \left[A_\pi(s, \mu^2) + \frac{1}{2} A_K(s, \mu^2) \right] \right\}.$$



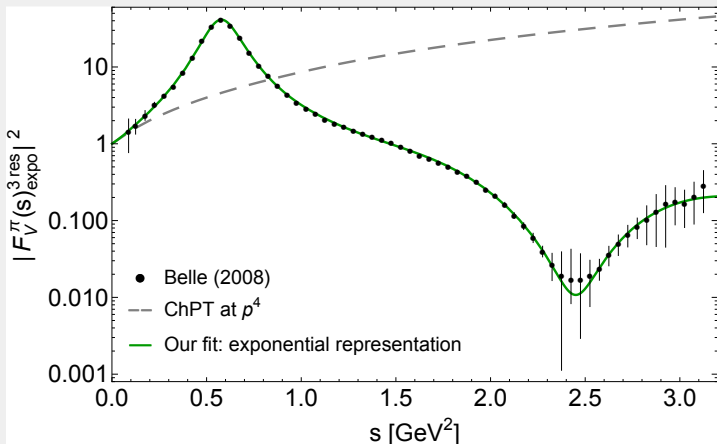
■ Incorporation of the $\rho'(1450)$, $\rho''(1700)$

$$\begin{aligned}
 F_V^\pi(s)|_{\text{expo}}^{\text{3 res}} = & \frac{M_\rho^2 + s (\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left\{ \text{Re} \left[-\frac{s}{96\pi^2 F_\pi^2} \left(A_\pi(s) + \frac{1}{2} A_K(s) \right) \right] \right\} \\
 & - \gamma \frac{s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'} \Gamma_{\rho'}(s)} \exp \left\{ -\frac{s \Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)} \text{Re} A_\pi(s) \right\} \\
 & - \delta \frac{s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''} \Gamma_{\rho''}(s)} \exp \left\{ -\frac{s \Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)} \text{Re} A_\pi(s) \right\},
 \end{aligned}$$

where

$$\Gamma_{\rho', \rho''}(s) = \Gamma_{\rho', \rho''} \frac{M_{\rho', \rho''}}{\sqrt{s}} \frac{\sigma_\pi^3(s)}{\sigma_\pi^3(M_{\rho', \rho''}^2)}.$$

R χ T + OMNÈS: EXPONENTIAL REPRESENTATION

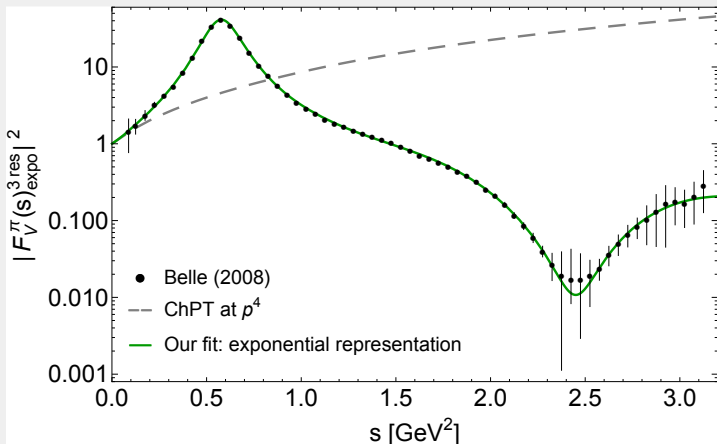


$$M_\rho = 775.2(4) \text{ MeV}, \quad \gamma = 0.15(4), \quad \phi_1 = -0.36(24),$$

$$M_{\rho'} = 1438(39) \text{ MeV}, \quad \Gamma_{\rho'} = 535(63) \text{ MeV}, \quad \delta = -0.12(4), \quad \phi_2 = -0.02(45),$$

$$M_{\rho''} = 1754(91) \text{ MeV}, \quad \Gamma_{\rho''} = 412(102) \text{ MeV}, \quad \chi^2_{\text{dof}} = 0.92$$

$R_{\chi T} + \text{OMNÈS}$: EXPONENTIAL REPRESENTATION



$$M_{\rho}^{\text{pole}} = 762.0(3) \text{ MeV}, \quad \Gamma_{\rho}^{\text{pole}} = 143.0(2) \text{ MeV},$$

$$M_{\rho'}^{\text{pole}} = 1366(38) \text{ MeV}, \quad \Gamma_{\rho'}^{\text{pole}} = 488(48) \text{ MeV},$$

$$M_{\rho''}^{\text{pole}} = 1718(82) \text{ MeV}, \quad \Gamma_{\rho''}^{\text{pole}} = 397(88) \text{ MeV},$$

DISPERSIVE REPRESENTATION

- Drawbacks: Constraints from analyticity, unitarity and chiral symmetry not fully respected
- Dispersion relation with subtractions:

$$F_V^\pi(s) = \exp \left[\alpha_1 s + \frac{\alpha_2}{2} s^2 + \frac{s^3}{\pi} \int_{4m_\pi^2}^{S_{\text{cut}}} ds' \frac{\phi(s')}{(s')^3 (s' - s - i\varepsilon)} \right],$$

- Low-energy observables

$$F_V^\pi(s) = 1 + \frac{1}{6} \langle r^2 \rangle_V^\pi s + c_V^\pi s^2 + d_V^\pi s^3 + \dots,$$

$$\langle r^2 \rangle_V^\pi |_{\text{ChPT}}^{\mathcal{O}(p^4)} = \frac{12L_9^r(\mu)}{F_\pi^2} - \frac{1}{32\pi^2 F_\pi^2} \left[2 \log \left(\frac{M_\pi^2}{\mu^2} \right) + \log \left(\frac{M_K^2}{\mu^2} \right) + 3 \right],$$

$$\langle r^2 \rangle_V^\pi = 6\alpha_1, \quad c_V^\pi = \frac{1}{2} (\alpha_2 + \alpha_1^2), \quad \alpha_k = \frac{k!}{\pi} \int_{4m_\pi^2}^{S_{\text{cut}}} ds' \frac{\phi(s')}{s'^{k+1}}.$$

- S_{cut} : cut-off to check stability

DISPERSIVE REPRESENTATION

■ Dispersion relation with subtractions:

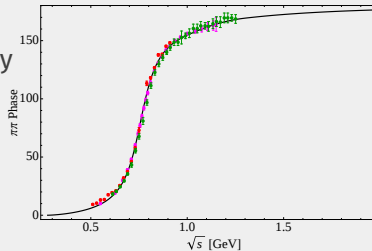
$$F_V^\pi(s) = \exp \left[\alpha_1 s + \frac{\alpha_2}{2} s^2 + \frac{s^3}{\pi} \int_{4m_\pi^2}^{s_{\text{cut}}} ds' \frac{\phi(s')}{(s')^3 (s' - s - i\epsilon)} \right],$$

■ Form Factor phase $\phi(s)$

- ▶ $4m_\pi^2 < s < 1 \text{ GeV}$: $\pi\pi$ phase from Roy
(García-Martín et.al PRD 83, 074004 (2011))
- ▶ $1 < s < m_\tau^2$: "Pheno" phase shift

$$\tan \phi(s) = \frac{\text{Im} F_V^\pi(s)|_{\text{exp}}^{\text{res}}}{\text{Re} F_V^\pi(s)|_{\text{exp}}^{\text{res}}},$$

- ▶ $m_\tau^2 < s$: phase guided smoothly to π

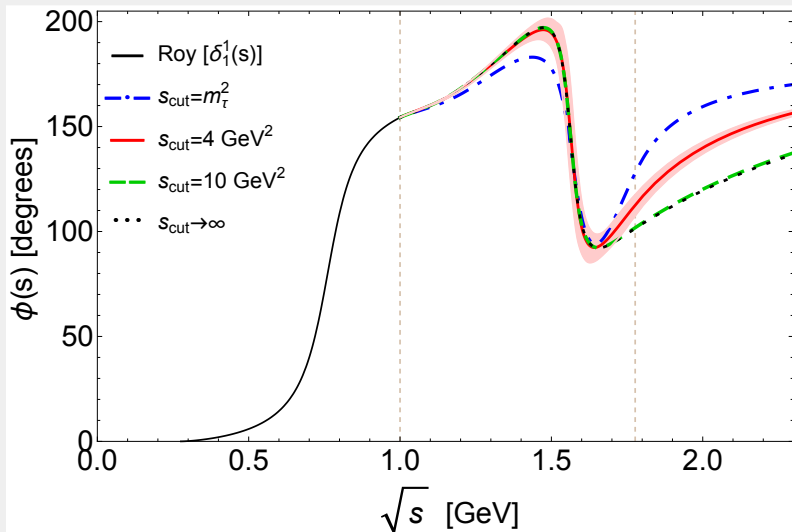


DISPERSIVE FITS TO THE PION VECTOR FORM FACTOR

■ Fits for different values of s_{cut} and matching at 1 GeV

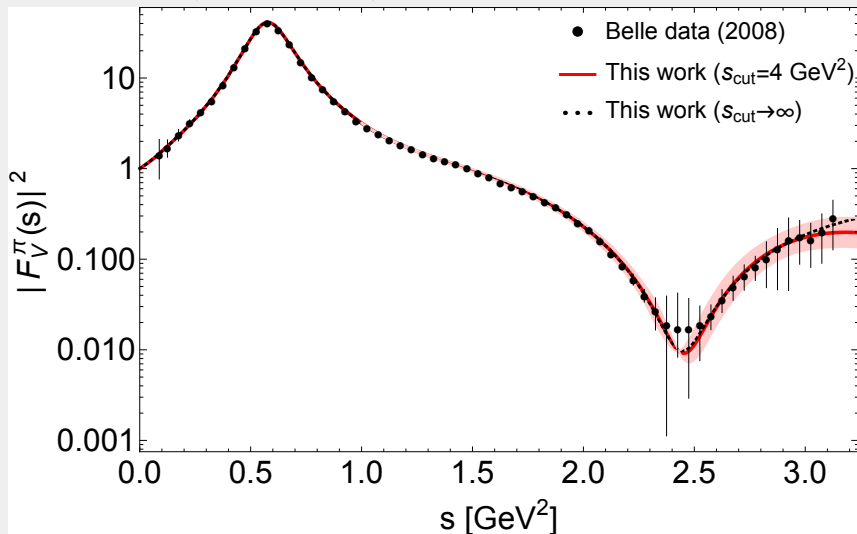
Fits	Parameter	s_{cut} [GeV ²]			
		m_τ^2	4 (reference fit)	10	∞
Fit 1	α_1 [GeV ⁻²]	1.87(1)	1.88(1)	1.89(1)	1.89(1)
	α_2 [GeV ⁻⁴]	4.40(1)	4.34(1)	4.32(1)	4.32(1)
	m_ρ [MeV]	= 773.6(9)	= 773.6(9)	= 773.6(9)	= 773.6(9)
	M_ρ [MeV]	= m_ρ	= m_ρ	= m_ρ	= m_ρ
	$M_{\rho'}$ [MeV]	1365(15)	1376(6)	1313(15)	1311(5)
	$\Gamma_{\rho'}$ [MeV]	562(55)	603(22)	700(6)	701(28)
	$M_{\rho''}$ [MeV]	1727(12)	1718(4)	1660(9)	1658(1)
	$\Gamma_{\rho''}$ [MeV]	278(1)	465(9)	601(39)	602(3)
	γ	0.12(2)	0.15(1)	0.16(1)	0.16(1)
	ϕ_1	-0.69(1)	-0.66(1)	-1.36(10)	-1.39(1)
	δ	-0.09(1)	-0.13(1)	-0.16(1)	-0.17(1)
	ϕ_2	-0.17(5)	-0.44(3)	-1.01(5)	-1.03(2)
	$\chi^2/\text{d.o.f}$	1.47	0.70	0.64	0.64

■ Form Factor phase shift for different values of s_{cut}



■ The results can be found in tables provided as ancillary material in [1902.02273 \[hep-ph\]](#)

■ Modulus squared of the pion vector form factor



■ The results can be found in tables provided as ancillary material in [1902.02273 \[hep-ph\]](#)

VARIANT (I)

■ Fits for different matching point and with $s_{\text{cut}} = 4 \text{ GeV}$

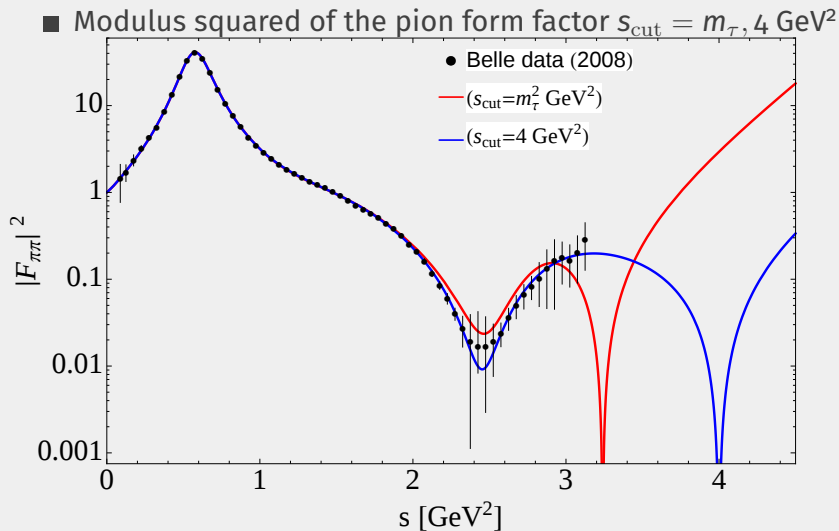
Fits	Parameter	Matching point [GeV]			
		0.85	0.9	0.95	1 (reference fit)
Fit I	$\alpha_1 [\text{GeV}^{-2}]$	1.88(1)	1.88(1)	1.88(1)	1.88(1)
	$\alpha_2 [\text{GeV}^{-4}]$	4.35(1)	4.35(1)	4.34(1)	4.34(1)
	$m_\rho [\text{MeV}]$	$= 773.6(9)$	$= 773.6(9)$	$= 773.6(9)$	$= 773.6(9)$
	$M_\rho [\text{MeV}]$	$= m_\rho$	$= m_\rho$	$= m_\rho$	$= m_\rho$
	$M_{\rho'} [\text{MeV}]$	1394(6)	1374(8)	1351(5)	1376(6)
	$\Gamma_{\rho'} [\text{MeV}]$	592(19)	583(27)	592(2)	603(22)
	$M_{\rho''} [\text{MeV}]$	1733(9)	1715(1)	1697(3)	1718(4)
	$\Gamma_{\rho''} [\text{MeV}]$	562(3)	541(45)	486(7)	465(9)
	γ	0.12(1)	0.12(1)	0.13(1)	0.15(1)
	ϕ_1	-0.44(3)	-0.60(1)	-0.80(1)	-0.66(1)
	δ	-0.13(1)	-0.13(1)	-0.13(1)	-0.13(1)
	ϕ_2	-0.38(3)	-0.51(2)	-0.62(1)	-0.44(3)
	$\chi^2/\text{d.o.f}$	0.75	0.74	0.68	0.70

VARIANT (II): INTERMEDIATE STATES OTHER THAN $\pi\pi$

■ Fit A: $\rho' \rightarrow K\bar{K}$ and $\rho'' \rightarrow K\bar{K}$

■ Fit B: $\rho' \rightarrow K\bar{K} + \rho' \rightarrow \omega\pi$

Parameter	$s_{\text{cut}} = 4 \text{ GeV}^2$		
	Fit A	Fit B	reference fit
$\alpha_1 [\text{GeV}^{-2}]$	1.87(1)	1.88(1)	1.88(1)
$\alpha_2 [\text{GeV}^{-4}]$	4.37(1)	4.35(1)	4.34(1)
$m_\rho [\text{MeV}]$	$= 773.6(9)$	$= 773.6(9)$	$= 773.6(9)$
$M_\rho [\text{MeV}]$	$= m_\rho$	$= m_\rho$	$= m_\rho$
$M_{\rho'} [\text{MeV}]$	1373(5)	1441(3)	1376(6)
$\Gamma_{\rho'} [\text{MeV}]$	462(14)	576(33)	603(22)
$M_{\rho''} [\text{MeV}]$	1775(1)	1733(9)	1718(4)
$\Gamma_{\rho''} [\text{MeV}]$	412(27)	349(52)	465(9)
γ	0.13(1)	0.15(3)	0.15(1)
ϕ_1	-0.80(1)	-0.53(5)	-0.66(1)
δ	-0.14(1)	-0.14(1)	-0.13(1)
ϕ_2	-0.44(2)	-0.46(3)	-0.44(3)
$\chi^2/\text{d.o.f}$	0.93	0.70	0.70



VARIANT (III)

■ Dispersive representation of the pion vector form factor

$$F_V^\pi(s) = \exp \left[\frac{s}{\pi} \int_{4m_\pi^2}^{s_{\text{cut}}} ds' \frac{\delta_1^1(s')}{(s')(s' - s - i\varepsilon)} + \frac{s}{\pi} \int_{s_{\text{cut}}}^{\infty} ds' \frac{\delta_{\text{eff}}(s')}{(s')(s' - s - i\varepsilon)} \right] \Sigma(s)$$

■ Properties for $\delta_{\text{eff}}(s)$

- $\delta_{\text{eff}}(s_{\text{cut}}) = \delta_1^1(s_{\text{cut}})$ and $\delta_{\text{eff}}(s) \rightarrow \pi$ for large s to recover $1/s$

$$\delta_{\text{eff}}(s) = \pi + (\delta_1^1(s_{\text{cut}}) - \pi) \frac{s_{\text{cut}}}{s}$$

- Integrating the piece with $\delta_{\text{eff}}(s)$

$$\begin{aligned} F_V^\pi(s) &= e^{1 - \frac{\delta_1^1(s_{\text{cut}})}{\pi}} \left(1 - \frac{s}{s_{\text{cut}}}\right)^{\left(1 - \frac{\delta_1^1(s_{\text{cut}})}{\pi}\right) \frac{s_{\text{cut}}}{s}} \left(1 - \frac{s}{s_{\text{cut}}}\right)^{-1} \\ &\quad \times \exp \left[\frac{s}{\pi} \int_{4m_\pi^2}^{s_{\text{cut}}} ds' \frac{\delta_1^1(s')}{(s')(s' - s - i\varepsilon)} \right] \Sigma(s) \\ \Sigma(s) &= \sum_{i=0}^{\infty} a_i \omega^i(s), \quad \omega(s) = \frac{\sqrt{s_{\text{cut}}} - \sqrt{s_{\text{cut}} - s}}{\sqrt{s_{\text{cut}}} + \sqrt{s_{\text{cut}} - s}} \end{aligned}$$

VARIANT (III)

The resulting fit parameters are found to be

$$a_1 = 2.99(12) ,$$

$$M_{\rho'} = 1261(7) \text{ MeV} , \quad \Gamma_{\rho'} = 855(15) \text{ MeV} ,$$

$$M_{\rho''} = 1600(1) \text{ MeV} , \quad \Gamma_{\rho''} = 486(26) \text{ MeV} ,$$

$$\gamma = 0.25(2) , \quad \phi_1 = -1.90(6) ,$$

$$\delta = -0.15(1) , \quad \phi_2 = -1.60(4) ,$$

with a $\chi^2/\text{d.o.f} = 32.3/53 \sim 0.61$ for the one-parameter fit, and

$$a_1 = 3.03(20) , \quad a_2 = 1.04(2.10) ,$$

$$M_{\rho'} = 1303(19) \text{ MeV} , \quad \Gamma_{\rho'} = 839(102) \text{ MeV} ,$$

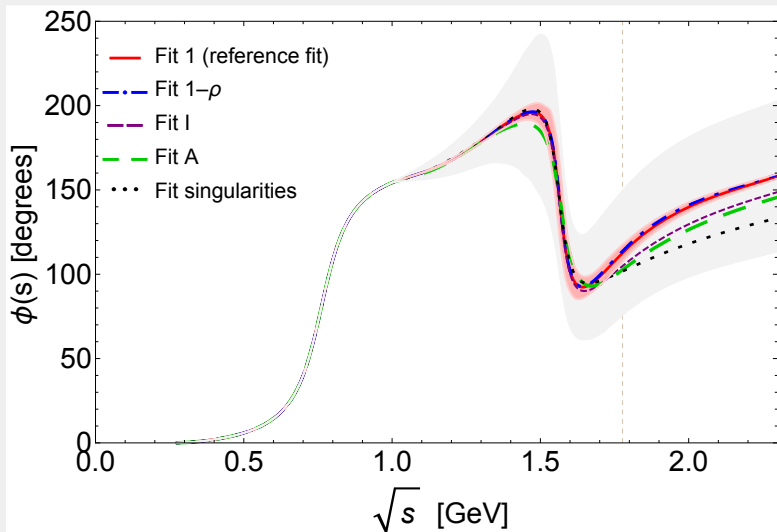
$$M_{\rho''} = 1624(1) \text{ MeV} , \quad \Gamma_{\rho''} = 570(99) \text{ MeV}$$

$$\gamma = 0.22(10) , \quad \phi_1 = -1.65(4) ,$$

$$\delta = -0.18(1) , \quad \phi_2 = -1.34(14) ,$$

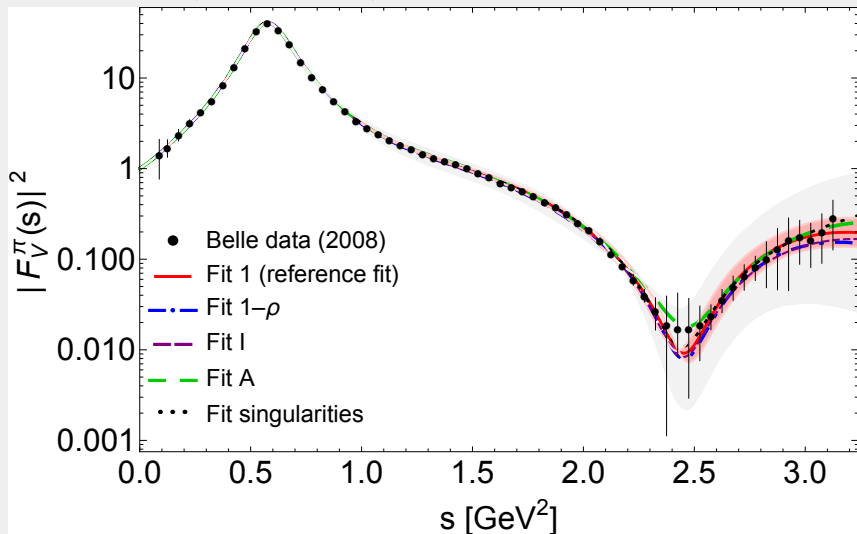
with a $\chi^2/\text{d.o.f} = 35.6/52 \sim 0.63$ for the two-parameter fit.

■ Form Factor phase shift for different parametrizations



■ The results can be found in tables provided as ancillary material in [1902.02273 \[hep-ph\]](#)

■ Modulus squared of the pion vector form factor



■ The results can be found in tables provided as ancillary material in [1902.02273 \[hep-ph\]](#)

CENTRAL RESULTS

■ Fit results (central value $\pm$ stat fit error $\pm$ syst th. error)

$$\alpha_1 = 1.88(1)(1) \text{ GeV}^{-2}, \alpha_2 = 4.34(1)(3) \text{ GeV}^{-4},$$

$$M_\rho \doteq 773.6 \pm 0.9 \pm 0.3 \text{ MeV},$$

$$M_{\rho'} = 1376 \pm 6_{-73}^{+18} \text{ MeV}, \quad \Gamma_{\rho'} = 603 \pm 22_{-141}^{+236} \text{ MeV},$$

$$M_{\rho''} = 1718 \pm 4_{-94}^{+57} \text{ MeV}, \quad \Gamma_{\rho''} = 465 \pm 9_{-53}^{+137} \text{ MeV},$$

$$\gamma = 0.15 \pm 0.01_{-0.03}^{+0.07}, \quad \phi_1 = -0.66 \pm 0.01_{-0.99}^{+0.22},$$

$$\delta = -0.13 \pm 0.01_{-0.05}^{+0.00}, \quad \phi_2 = -0.44 \pm 0.03_{-0.90}^{+0.06},$$

■ Physical pole mass and width

$$M_\rho^{\text{pole}} = 760.6 \pm 0.8 \text{ MeV}, \quad \Gamma_\rho^{\text{pole}} = 142.0 \pm 0.4 \text{ MeV},$$

$$M_{\rho'}^{\text{pole}} = 1289 \pm 8_{-143}^{+52} \text{ MeV}, \quad \Gamma_{\rho'}^{\text{pole}} = 540 \pm 16_{-111}^{+151} \text{ MeV},$$

$$M_{\rho''}^{\text{pole}} = 1673 \pm 4_{-125}^{+68} \text{ MeV}, \quad \Gamma_{\rho''}^{\text{pole}} = 445 \pm 8_{-49}^{+117} \text{ MeV},$$

$\rho(1450)$ AND $\rho(1700)$ RESONANCE PARAMETERS

Reference	Model parameters $M_{\rho'}, \Gamma_{\rho'} [\text{MeV}]$	Pole parameters $M_{\rho'}^{\text{pole}}, \Gamma_{\rho'}^{\text{pole}} [\text{MeV}]$	Data
ALEPH	$1328 \pm 15, 468 \pm 41$	$1268 \pm 19, 429 \pm 31$	τ
ALEPH	$1409 \pm 12, 501 \pm 37$	$1345 \pm 15, 459 \pm 28$	τ & e^-
Belle (fixed $ F_V^\pi(o) ^2$)	$1446 \pm 7 \pm 28, 434 \pm 16 \pm 60$	$1398 \pm 8 \pm 31, 408 \pm 13 \pm 50$	τ
Belle (all free)	$1428 \pm 15 \pm 26, 413 \pm 12 \pm 57$	$1384 \pm 16 \pm 29, 390 \pm 10 \pm 48$	τ
Dumm et. al.	—	$1440 \pm 80, 320 \pm 80$	τ
Celis et. al.	$1497 \pm 7, 785 \pm 51$	$1278 \pm 18, 525 \pm 16$	τ
Bartos et. al.	—	$1342 \pm 47, 492 \pm 138$	e^+e^-
Bartos et. al.	—	$1374 \pm 11, 341 \pm 24$	τ
This work	$1376 \pm 6^{+18}_{-73}, 603 \pm 22^{+236}_{-141}$	$1289 \pm 8^{+52}_{-143}, 540 \pm 16^{+151}_{-111}$	τ
Reference	Model parameters $(M_{\rho''}, \Gamma_{\rho''}) [\text{MeV}]$	Pole parameters $(M_{\rho''}^{\text{pole}}, \Gamma_{\rho''}^{\text{pole}}) [\text{MeV}]$	Data
ALEPH	$= 1713, = 235$	$1700, 232$	τ
ALEPH	$1740 \pm 20, = 235$	$1728 \pm 20, 232$	τ & e^-
Belle (fixed $ F_V^\pi(o) ^2$)	$1728 \pm 17 \pm 89, 164 \pm 21^{+89}_{-26}$	$1722 \pm 18, 163 \pm 21^{+88}_{-27}$	τ
Belle (all free)	$1694 \pm 41, 135 \pm 36^{+50}_{-26}$	$1690 \pm 94, 134 \pm 36^{+49}_{-28}$	τ
Dumm et. al.	—	$1720 \pm 90, 180 \pm 90$	τ
Celis et. al.	$1685 \pm 30, 800 \pm 31$	$1494 \pm 37, 600 \pm 17$	τ
Bartos et. al.	—	$1719 \pm 65, 490 \pm 17$	e^+e^-
Bartos et. al.	—	$1767 \pm 52, 415 \pm 120$	τ
This work	$1718 \pm 4^{+57}_{-94}, 465 \pm 9^{+137}_{-53}$	$1673 \pm 4^{+68}_{-125}, 445 \pm 8^{+117}_{-49}$	τ

LOW-ENERGY OBSERVABLES

References	$\langle r^2 \rangle_V^\pi$ (GeV $^{-2}$)	c_V^π (GeV $^{-4}$)	Sum rule	s_{cut} (GeV 2)			Fit Eq. (42)
				4	10	∞	
Colangelo et al. [55]	11.07 ± 0.66	3.2 ± 1.03					
Bijnens et al. [32]	11.22 ± 0.41	3.85 ± 0.60	α_1	1.52	1.66	1.75	$1.88 \pm 0.01 \pm 0.01$
Pich et al. [6]	11.04 ± 0.30	3.79 ± 0.04	α_2	4.26	4.30	4.31	$4.34 \pm 0.01 \pm 0.03$
Bijnens et al. [33]	11.61 ± 0.33	4.49 ± 0.28					
de Troconiz et al. [56]	11.10 ± 0.03	3.84 ± 0.02					
Masjuan et al. [57]	11.43 ± 0.19	3.30 ± 0.33					
Guo et al. [58]	–	4.00 ± 0.50					
Lattice [59]	10.50 ± 1.12	3.22 ± 0.40					
Ananthanarayan et al. [60]	11.17 ± 0.53	[3.75, 3.98]					
Ananthanarayan et al. [61]	[10.79, 11.3]	[3.79, 4.00]					
Schneider et al. [48]	10.6	3.84 ± 0.03					
Dumm et al. [7]	10.86 ± 0.14	3.84 ± 0.03					
Celis et al. [8]	11.30 ± 0.07	4.11 ± 0.09					
Ananthanarayan et al. [62]	11.10 ± 0.11	–					
Hanhart et al. [63]	$11.34 \pm 0.01 \pm 0.01$	–					
Colangelo et al. [39]	11.02 ± 0.10	–					
PDG [42]	11.61 ± 0.28	–					
This work	11.28 ± 0.08	3.94 ± 0.04					

KAON VECTOR FORM FACTOR

$$\frac{d\Gamma(\tau^- \rightarrow K^- K^0 \nu_\tau)}{d\sqrt{s}} = \frac{G_F^2 |V_{ud}|^2}{768\pi^3} M_\tau^3 \left(1 - \frac{s}{M_\tau^2}\right)^2 \left(1 + \frac{2s}{M_\tau^2}\right) \sigma_K^3(s) |F_V^K(s)|^2,$$

■ Chiral Perturbation Theory $\mathcal{O}(p^4)$

$$F_{K^+K^-}(s)|_{\text{ChPT}} = 1 + \frac{2L_9^r}{F_\pi^2} - \frac{s}{192\pi^2 F_\pi^2} [A_\pi(s, \mu^2) + 2A_K(s, \mu^2)],$$

$$F_{K^0\bar{K}^0}(s)|_{\text{ChPT}} = -\frac{s}{192\pi^2 F_\pi^2} [A_\pi(s, \mu^2) - A_K(s, \mu^2)].$$

■ Extract the $I = 1$ component

$$F_V^K(s)|_{\text{ChPT}} = F_{K^+K^-}(s) - F_{K^0\bar{K}^0}(s) = 1 + \frac{2L_9^r}{F_\pi^2} - \frac{s}{96\pi^2 F_\pi^2} \left[A_\pi(s, \mu^2) + \frac{1}{2} A_K(s, \mu^2) \right]$$

■ At $\mathcal{O}(p^4)$, the pion and kaon vector form factor are the same

■ Assumption: we consider that both are also the same at higher energies

■ Different resonance mixing contribution than $F_V^\pi(s)$

$$F_V^K(s) = \frac{M_\rho^2 + s \left(\tilde{\gamma} e^{i\tilde{\phi}_1} + \tilde{\delta} e^{i\tilde{\phi}_2} \right)}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp \left\{ \operatorname{Re} \left[-\frac{s}{96\pi^2 F_\pi^2} \left(A_\pi(s) + \frac{1}{2} A_K(s) \right) \right] \right\} \\ - \tilde{\gamma} \frac{s e^{i\tilde{\phi}_1}}{M_{\rho'}^2 - s - iM_{\rho'} \Gamma_{\rho'}(s)} \exp \left\{ -\frac{s \Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)} \operatorname{Re} A_\pi(s) \right\} \\ - \tilde{\delta} \frac{s e^{i\tilde{\phi}_2}}{M_{\rho''}^2 - s - iM_{\rho''} \Gamma_{\rho''}(s)} \exp \left\{ -\frac{s \Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)} \operatorname{Re} A_\pi(s) \right\},$$

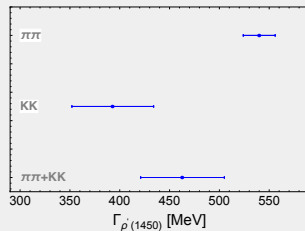
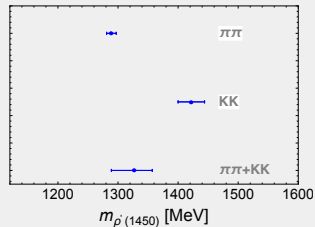
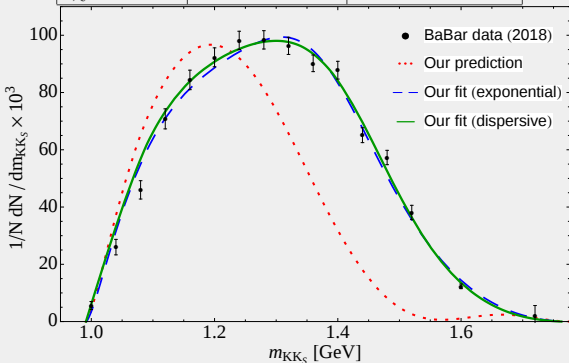
$$\Gamma_{\rho', \rho''}(s) = \Gamma_{\rho', \rho''} \frac{s}{M_{\rho', \rho''}^2} \frac{\sigma_\pi^3(s)}{\sigma_\pi^3(M_{\rho', \rho''}^2)} \theta(s - 4m_\pi^2).$$

■ Extract the phase $\tan \phi_{KK}(s) = \operatorname{Im} F_V^K(s) / \operatorname{Re} F_V^K(s)$

■ Use a three-times subtracted dispersion relation

FIT RESULTS TO BABAR $\tau^- \rightarrow K^- K_S \nu_\tau$ DATA

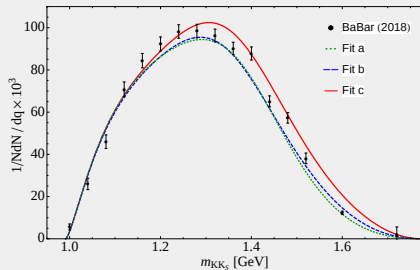
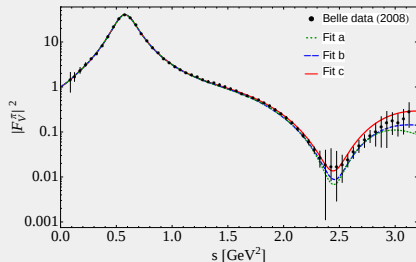
Parameter	Fit dispersive	Fit exponential
$\tilde{\alpha}_1$	$= 1.88(1)$	—
$\tilde{\alpha}_2$	$= 4.34(1)$	—
$M_{\rho'}$ [MeV]	1467(24)	1411(12)
$\Gamma_{\rho'}$ [MeV]	415(48)	394(35)
$\tilde{\gamma}$	0.10(2)	0.09(1)
ϕ_1	$-1.19(16)$	$-1.88(9)$
$\chi^2/\text{d.o.f.}$	2.9	3.3



1902.02273 [hep-ph]

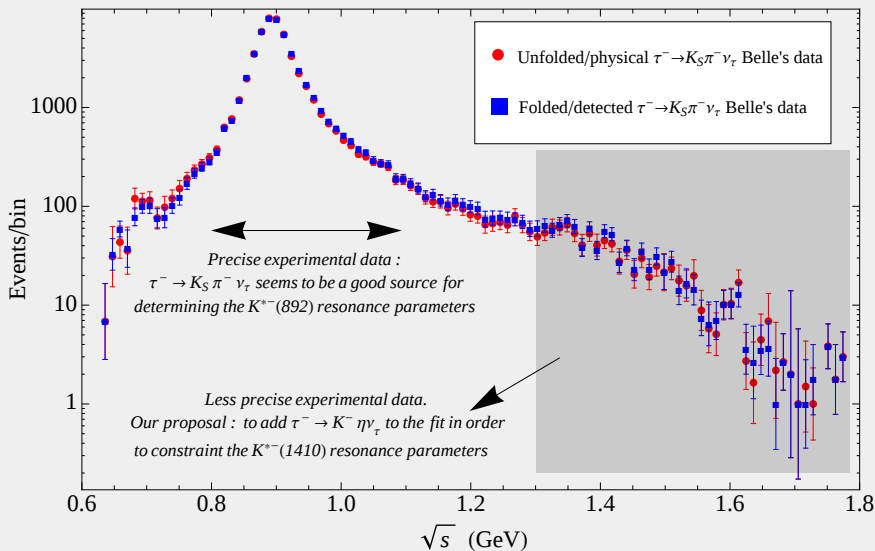
COMBINED ANALYSIS OF $F_V^\pi(s)$ AND $\tau^- \rightarrow K^- K_S \nu_\tau$

Parameter	$s_{\text{cut}} = 4 [\text{GeV}^2]$		
	Fit a	Fit b	Fit c
α_1	1.88(1)	1.89(1)	1.87(1)
α_2	4.34(2)	4.31(2)	4.38(3)
$\tilde{\alpha}_1$	$= \alpha_1$	$= \alpha_1$	1.88(24)
$\tilde{\alpha}_2$	$= \alpha_2$	$= \alpha_2$	4.38(29)
$m_\rho [\text{MeV}]$	$= 773.6(9)$	$= 773.6(9)$	$= 773.6(9)$
$M_\rho [\text{MeV}]$	$= m_\rho$	$= m_\rho$	$= m_\rho$
$M_{\rho'} [\text{MeV}]$	1396(19)	1453(19)	1406(61)
$\Gamma_{\rho'} [\text{MeV}]$	507(31)	499(51)	524(149)
$M_{\rho''} [\text{MeV}]$	1724(41)	1712(32)	1746(1)
$\Gamma_{\rho''} [\text{MeV}]$	399(126)	284(72)	413(362)
γ	0.12(3)	0.15(3)	0.11(11)
$\tilde{\gamma}$	0.11(2)	$= \gamma$	0.11(5)
ϕ_1	-0.23(26)	0.29(21)	-0.27(42)
$\tilde{\phi}_1$	-1.83(14)	-1.48(13)	-1.90(67)
δ	-0.09(2)	-0.07(2)	-0.10(5)
$\tilde{\delta}$	$= 0$	$= 0$	-0.01(4)
ϕ_2	-0.20(31)	0.27(29)	-1.15(71)
$\tilde{\phi}_2$	$= 0$	$= 0$	0.40(3)
$\chi^2/\text{d.o.f}$	1.52	1.19	1.25



BELLE $\tau^- \rightarrow K_S \pi^- \nu_\tau$ MEASUREMENT

$\tau^- \rightarrow K_S \pi^- \nu_\tau$ Belle's data [Phys. Lett. B 654 \(2007\) 65](#) [[arXiv:0706.2231](#)]



$K\pi$ VECTOR FORM FACTOR

- $R_{\chi T}$ with two resonances: $K^*(892)$ and $K^*(1410)$

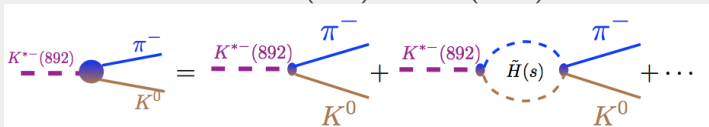


figure courtesy of D. Boito

$$\tilde{F}_V^{K\pi}(s) = \frac{m_{K^*}^2 - \kappa_{K^*} \tilde{H}_{K\pi}(0) + \gamma s}{D(m_{K^*}, \gamma_{K^*})} - \frac{\gamma s}{D(m_{K^{*'}}, \gamma_{K^{*'}})},$$

$$D(m_n, \gamma_n) \equiv m_n^2 - s - \kappa_n \text{Re}[H_{K\pi}(s)] - im_n \Gamma_n(s),$$

$$\kappa_n = \frac{192\pi F_K F_\pi}{\sigma_{K\pi}(m_{K^*}^2)} \frac{\gamma_{K^*}}{m_{K^*}}, \quad \Gamma_n(s) = \Gamma_n \frac{s}{m_n^2} \frac{\sigma_{K\pi}^3(s)}{\sigma_{K\pi}^3(m_n^2)}$$

- We then have a phase with two resonances

$$\delta^{K\pi}(s) = \tan^{-1} \left[\frac{\text{Im} F_V^{K\pi}(s)}{\text{Re} F_V^{K\pi}(s)} \right]$$

VECTOR FORM FACTOR: DISPERSIVE REPRESENTATION

- **Three subtractions**: helps the convergence of the form factor and **suppresses the the high-energy region of the integral**

$$F_V^{K\pi}(s) = P(s) \exp \left[\alpha_1 \frac{s}{m_{\pi^-}^2} + \frac{1}{2} \alpha_2 \frac{s^2}{m_{\pi^-}^4} + \frac{s^3}{\pi} \int_{s_{K\pi}}^{s_{\text{cut}}} ds' \frac{\delta^{K\pi}(s')}{(s')^3 (s' - s - i0)} \right]$$

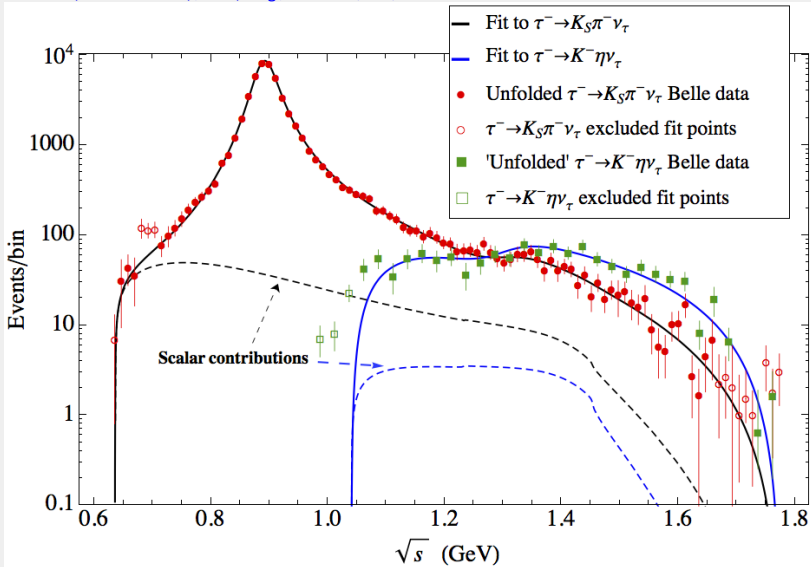
- $\alpha_1 = \lambda'_+$ and $\alpha_1^2 + \alpha_2 = \lambda''_+$ low energies parameters

$$F_V^{K\pi}(t) = 1 + \frac{\lambda'_+}{M_{\pi^-}^2} t + \frac{1}{2} \frac{\lambda''_+}{M_{\pi^-}^4} t^2$$

- s_{cut} : cut-off to check stability
- Parameters to Fit: $\lambda'_+, \lambda''_+, m_{K^*}, \gamma_{K^*}, m_{K^{*'}}, \gamma_{K^{*'}}$

RESULTS OF THE COMBINED $\tau^- \rightarrow K_S \pi^- \nu_\tau$ AND $\tau^- \rightarrow K^- \eta \nu_\tau$ ANALYSIS

Escubano, González-Solís, Jamin, Roig JHEP 1409 (2014) 042



RESULTS OF THE COMBINED $\tau^- \rightarrow K_S \pi^- \nu_\tau$ AND $\tau^- \rightarrow K^- \eta \nu_\tau$ ANALYSIS

- Different choices regarding linear slopes and resonance mixing parameters ($s_{cut} = 4 \text{ GeV}^2$)

Fitted value	Reference Fit	Fit A	Fit B	Fit C
$\bar{B}_{K\pi}(\%)$	0.404 ± 0.012	0.400 ± 0.012	0.404 ± 0.012	0.397 ± 0.012
$(B_{K\pi}^{th})(\%)$	(0.402)	(0.394)	(0.400)	(0.394)
M_{K^*}	892.03 ± 0.19	892.04 ± 0.19	892.03 ± 0.19	892.07 ± 0.19
Γ_{K^*}	46.18 ± 0.42	46.11 ± 0.42	46.15 ± 0.42	46.13 ± 0.42
$M_{K^{*'}}$	1305^{+15}_{-18}	1308^{+16}_{-19}	1305^{+15}_{-18}	1310^{+14}_{-17}
$\Gamma_{K^{*'}}$	168^{+52}_{-44}	212^{+66}_{-54}	174^{+58}_{-47}	184^{+56}_{-46}
$\gamma_{K\pi} \times 10^2$	$= \gamma_{K\eta}$	$-3.6^{+1.1}_{-1.5}$	$-3.3^{+1.0}_{-1.3}$	$= \gamma_{K\eta}$
$\lambda'_{K\pi} \times 10^3$	23.9 ± 0.7	23.6 ± 0.7	23.8 ± 0.7	23.6 ± 0.7
$\lambda''_{K\pi} \times 10^4$	11.8 ± 0.2	11.7 ± 0.2	11.7 ± 0.2	11.6 ± 0.2
$\bar{B}_{K\eta} \times 10^4$	1.58 ± 0.10	1.62 ± 0.10	1.57 ± 0.10	1.66 ± 0.09
$(B_{K\eta}^{th}) \times 10^4$	(1.45)	(1.51)	(1.44)	(1.58)
$\gamma_{K\eta} \times 10^2$	$-3.4^{+1.0}_{-1.3}$	$-5.4^{+1.8}_{-2.6}$	$-3.9^{+1.4}_{-2.1}$	$-3.7^{+1.0}_{-1.4}$
$\lambda'_{K\eta} \times 10^3$	20.9 ± 1.5	$= \lambda'_{K\pi}$	21.2 ± 1.7	$= \lambda'_{K\pi}$
$\lambda''_{K\eta} \times 10^4$	11.1 ± 0.4	11.7 ± 0.2	11.1 ± 0.4	11.8 ± 0.2
$\chi^2/\text{n.d.f.}$	$108.1/105 \sim 1.03$	$109.9/105 \sim 1.05$	$107.8/104 \sim 1.04$	$111.9/106 \sim 1.06$

RESULTS OF THE COMBINED $\tau^- \rightarrow K_S \pi^- \nu_\tau$ AND $\tau^- \rightarrow K^- \eta \nu_\tau$ ANALYSIS

■ Reference fit results obtained for different values of S_{cut}

Parameter	3.24	4	9	∞
$\bar{B}_{K\pi}(\%)$	0.402 ± 0.013	0.404 ± 0.012	0.405 ± 0.012	0.405 ± 0.012
$(B_{K\pi}^{th})(\%)$	(0.399)	(0.402)	(0.403)	(0.403)
M_{K^*}	892.01 ± 0.19	892.03 ± 0.19	892.05 ± 0.19	892.05 ± 0.19
Γ_{K^*}	46.04 ± 0.43	46.18 ± 0.42	46.27 ± 0.42	46.27 ± 0.41
$M_{K^{*'}}$	1301^{+17}_{-22}	1305^{+15}_{-18}	1306^{+14}_{-17}	1306^{+14}_{-17}
$\Gamma_{K^{*'}}$	207^{+73}_{-58}	168^{+52}_{-44}	155^{+48}_{-41}	155^{+47}_{-40}
$\gamma_{K\pi}$	$= \gamma_{K\eta}$	$= \gamma_{K\eta}$	$= \gamma_{K\eta}$	$= \gamma_{K\eta}$
$\lambda'_{K\pi} \times 10^3$	23.3 ± 0.8	23.9 ± 0.7	24.3 ± 0.7	24.3 ± 0.7
$\lambda''_{K\pi} \times 10^4$	11.8 ± 0.2	11.8 ± 0.2	11.7 ± 0.2	11.7 ± 0.2
$\bar{B}_{K\eta} \times 10^4$	1.57 ± 0.10	1.58 ± 0.10	1.58 ± 0.10	1.58 ± 0.10
$(B_{K\eta}^{th}) \times 10^4$	(1.43)	(1.45)	(1.46)	(1.46)
$\gamma_{K\eta} \times 10^2$	$-4.0^{+1.3}_{-1.9}$	$-3.4^{+1.0}_{-1.3}$	$-3.2^{+0.9}_{-1.1}$	$-3.2^{+0.9}_{-1.1}$
$\lambda'_{K\eta} \times 10^3$	18.6 ± 1.7	20.9 ± 1.5	22.1 ± 1.4	22.1 ± 1.4
$\lambda''_{K\eta} \times 10^4$	10.8 ± 0.3	11.1 ± 0.4	11.2 ± 0.4	11.2 ± 0.4
$\chi^2/\text{n.d.f.}$	105.8/105	108.1/105	111.0/105	111.1/105

RESULTS OF THE COMBINED $\tau^- \rightarrow K_S \pi^- \nu_\tau$ AND $\tau^- \rightarrow K^- \eta \nu_\tau$ ANALYSIS

■ Central results including the largest variation of S_{cut}

$$M_{K^*(892)} = 892.03 \pm 0.19 \text{ MeV}$$

$$\Gamma_{K^*(892)} = 46.18 \pm 0.44 \text{ MeV}$$

$$M_{K^*(1410)} = 1305^{+16}_{-18} \text{ MeV}$$

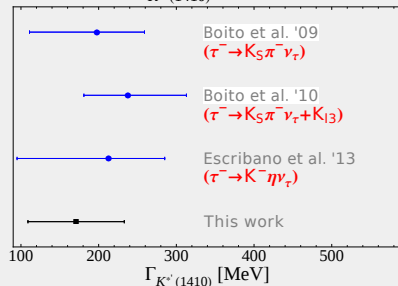
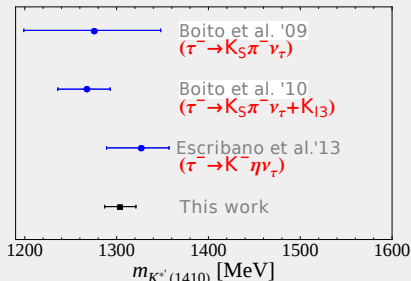
$$\Gamma_{K^*(1410)} = 168^{+65}_{-59} \text{ MeV}$$

$$\gamma_{K\pi} = \gamma_{K\eta} = -3.4^{+1.2}_{-1.4} \cdot 10^{-2}$$

$$\bar{B}_{K\pi} = (0.0404 \pm 0.012)\%$$

$$\bar{B}_{K\eta} = (1.58 \pm 0.10) \cdot 10^{-4}$$

$$\chi^2/d.o.f = 108.1/105 = 1.03$$



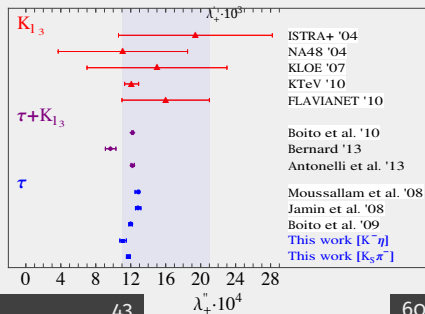
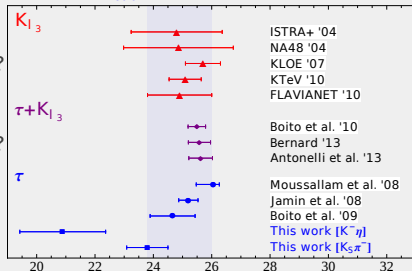
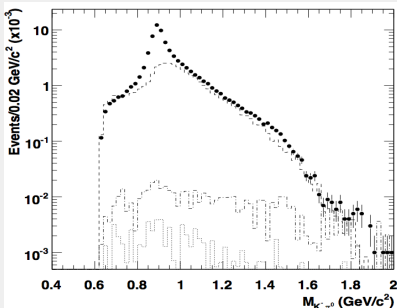
RESULTS OF THE COMBINED $\tau^- \rightarrow K_S \pi^- \nu_\tau$ AND $\tau^- \rightarrow K^- \eta \nu_\tau$ ANALYSIS

Central results including the largest variation of S_{cut}

$$\left. \begin{aligned} \lambda'_{K\pi} &= (23.9 \pm 0.9) \cdot 10^{-3} \\ \lambda'_{K\eta} &= (20.9 \pm 2.7) \cdot 10^{-3} \end{aligned} \right\} \text{isospin violation?}$$

$$\left. \begin{aligned} \lambda''_{K\pi} &= (11.8 \pm 0.2) \cdot 10^{-4} \\ \lambda''_{K\eta} &= (11.1 \pm 0.5) \cdot 10^{-4} \end{aligned} \right\} \text{isospin violation?}$$

$$\tau^- \rightarrow K^- \pi^0 \nu_\tau \text{ (PRD 76 (2007) 051104)}$$



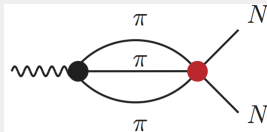
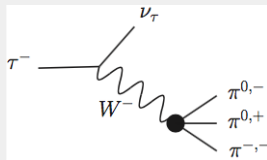
$\tau \rightarrow 3\pi\nu_\tau$ (**PRELIMINARY RESULTS**)

$\tau \rightarrow 3\pi\nu_\tau$ DECAYS

■ Motivation:

- ▶ To investigate the axial-vector weak hadronic current (J_A^μ)
- ▶ Dynamics of the 3π system
- ▶ Properties of the $a_1(1260)$

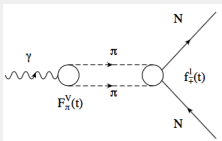
- Applications: input for the axial-vector form factor of the nucleon



ELECTROMAGNETIC FORM FACTORS OF THE NUCLEON

■ Dispersive parametrizations

(Belushkin'06, Lorenz'12, Hoferichter'16, Leupold'17, Alarcon'18)



$$G_{E,M}^{p,n}(t) = \frac{1}{\pi} \int_{s_{th}}^{\infty} ds \frac{\text{Im} G_{E,M}^{p,n}(s)}{s - t - i\epsilon},$$

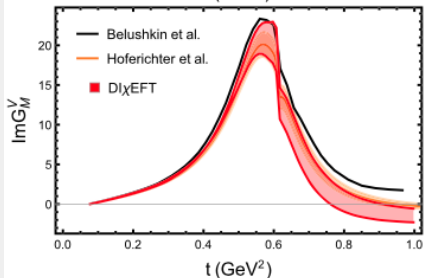
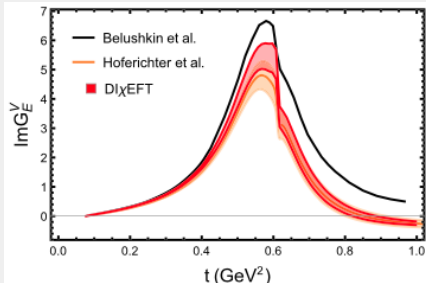
$$G_{E,M}^{V,S} = \frac{1}{2} \left(G_{E,M}^p - G_{E,M}^n \right),$$

$$\text{Im} G_E^V(t) = \frac{k_{cm}^3}{m_N \sqrt{t}} f_+^1(t) F_{\pi}^{V*}(t)$$

$$\text{Im} G_M^V(t) = \frac{k_{cm}^3}{\sqrt{2t}} f_-^1(t) F_{\pi}^{V*}(t)$$

where $k_{cm} = \sqrt{t/4 - m_{\pi}^2}$

Alarcón, Weiss, Phys.Lett. B784 (2018)



$\tau^- \rightarrow (PPP)^- \nu_\tau$: BASICS

■ Generic Amplitude for a 3-meson decay of the τ

$$\mathcal{M}(\tau^- \rightarrow (PPP)^- \nu_\tau) = \frac{G_F}{\sqrt{2}} |V_{ij}| \bar{u}_{\nu_\tau} \gamma_\mu (1 - \gamma_5) u_\tau \langle (PPP)^- | (V - A)^\mu | 0 \rangle ,$$

■ Hadronic matrix element in terms of four form factors

$$\begin{aligned} \langle (P(p_1)P(p_2)P(p_3))^- | (V - A)^\mu | 0 \rangle &= V_1^\mu F_1^A(Q^2, s_1, s_2) + V_2^\mu F_2^A(Q^2, s_1, s_2) , \\ &+ Q^\mu F_3^A(Q^2, s_1, s_2) + i V_4^\mu F_4^V(Q^2, s_1, s_2) , \end{aligned}$$

where

$$V_1^\mu = \left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} \right) (p_1 - p_3)_\nu , \quad V_2^\mu = \left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} \right) (p_2 - p_3)_\nu ,$$

$$V_4^\mu = \varepsilon^{\mu\alpha\beta\gamma} p_{1\alpha} p_{2\beta} p_{3\gamma} , \quad Q^\mu = (p_1 + p_2 + p_3)^\mu , \quad s_i = (Q - p_i)^2 ,$$

$\tau^- \rightarrow (PPP)^- \nu_\tau$: BASICS

- Generic Amplitude for a 3-meson decay of the τ

$$\mathcal{M}(\tau^- \rightarrow (PPP)^- \nu_\tau) = \frac{G_F}{\sqrt{2}} |V_{ij}| \bar{u}_{\nu_\tau} \gamma_\mu (1 - \gamma_5) u_\tau \langle (PPP)^- | (V - A)^\mu | 0 \rangle,$$

- hadronic matrix element in terms of four form factors

$$\begin{aligned} \langle (P(p_1)P(p_2)P(p_3))^- | (V - A)^\mu | 0 \rangle &= V_1^\mu F_1^A(Q^2, s_1, s_2) + V_2^\mu F_2^A(Q^2, s_1, s_2), \\ &+ Q^\mu F_3^A(Q^2, s_1, s_2) + iV_4^\mu F_4^V(Q^2, s_1, s_2), \end{aligned}$$

- ▶ $F_{1,2}^A(Q^2, s_1, s_2)$: $J^P = 1^+$ transition (axial-vector form factors)
- ▶ $F_3^A(Q^2, s_1, s_2)$: $J^P = 0^-$ transition (pseudoscalar form factor)
- ▶ $F_4^V(Q^2, s_1, s_2)$: $J^P = 1^-$ transition (vector form factor)

$$\tau \rightarrow \pi^- \pi^+ \pi^- \nu_\tau$$

- Bose symmetry: $F_1^A(Q^2, s_1, s_2) = F_2^A(Q^2, s_2, s_1) \equiv F_A(Q^2, s_1, s_2)$
- Conservation of J_A^μ in the chiral limit: $F_3^A(Q^2, s_1, s_2)$ must vanish with m_π^2
- G-parity conservation: $F_4^V(Q^2, s_1, s_2) = 0$
- The axial-vector hadronic current takes the form

$$J_A^\mu = F_A(Q^2, s_1, s_2) V_1^\mu + F_A(Q^2, s_2, s_1) V_2^\mu,$$

- Decay rate

$$\frac{d\Gamma(\tau^- \rightarrow \pi^- \pi^- \pi^+ \nu_\tau)}{dQ^2} = \frac{G_F^2 |V_{ud}|^2}{32\pi^2 M_\tau} (M_\tau^2 - Q^2)^2 \left(1 + \frac{2Q^2}{M_\tau^2}\right) a_1(Q^2),$$

$$\tau \rightarrow \pi^- \pi^+ \pi^- \nu_\tau$$

■ Spectral function

$$a_1(Q^2) = \frac{1}{768\pi^3} \frac{1}{Q^4} \int_{s_{1,\min}}^{s_1^{\max}} ds_1 \int_{s_{2,\min}}^{s_2^{\max}} ds_2 W_A .$$

where

$$W_A = - \left[V_1^\mu F_A(Q^2, s_1, s_2) + V_2^\mu F_A(Q^2, s_2, s_1) \right] \times \\ \left[V_{1\mu} F_A(Q^2, s_1, s_2) + V_{2\mu} F_A(Q^2, s_2, s_1) \right] ,$$

■ For comparison of theory and experiment

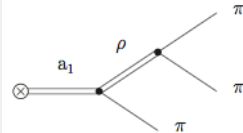
$$\frac{\Gamma(\tau^- \rightarrow (3\pi)^- \nu_\tau)}{\Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)} \frac{1}{N_{\text{events}}} \frac{dN_{\text{events}}}{dQ^2} = \frac{6\pi |V_{ub}|^2 S_{EW}}{M_\tau^2} \left(1 - \frac{Q^2}{M_\tau^2}\right)^2 \left(1 + 2 \frac{Q^2}{M_\tau^2}\right) a_1(Q^2)$$

EVALUATION OF THE AXIAL-VECTOR FORM FACTOR

Kühn, Santamaria, Z.Phys.C 48 (1990)

■ Factorization ansatz: $\tau \rightarrow \nu_\tau a_1 \rightarrow \nu_\tau \rho \pi \rightarrow 3\pi$

$$F_A(Q^2, s_1, s_2) = F_{a_1}(Q^2) F_\rho(s_2),$$



- ▶ $F_{a_1}(Q^2)$ accounts for the $a_1(1260)$ -resonance production
- ▶ $F_\rho(s_i) = \Omega(s_i)$ (Omnès): line-shape of the ρ -meson ($\rho \rightarrow \pi\pi$)

$$F_\rho(s_i) \rightarrow 1, \quad s_i \rightarrow 0.$$

- ▶ Axial-vector current

$$J_A^\mu = F_{a_1}(Q^2) [F_\rho(s_2) V_{1\mu} + F_\rho(s_1) V_{2\mu}],$$

- ▶ Isospin relation: same predictions for both modes (in the isospin limit)

$$F^A(Q^2, s_1, s_2) = -F_{\pi^0 \pi^0 \pi^-}^A(Q^2, s_1, s_2),$$

EVALUATION OF THE AXIAL-VECTOR FORM FACTOR

- ChPT prediction at $\mathcal{O}(p^2)$ (Fischer, Wess, Z. Phys. C 3 (1980))

$$J_A^\mu|_{\text{ChPT}} = -\frac{2\sqrt{2}}{3F_\pi} (V_{1\mu} + V_{2\mu}) ,$$

- Normalization of $F_{a_1}(Q^2)$

$$F_{a_1}(Q^2) = -\frac{2\sqrt{2}}{3F_\pi} f_{a_1}(Q^2), \quad f_{a_1}(Q^2 \rightarrow 0) \rightarrow 1 ,$$

- In this framework, the axial spectral function reads:

$$a_1(Q^2) = \frac{1}{768\pi^3} \left(-\frac{2\sqrt{2}}{3F_\pi} \right)^2 |f_{a_1}(Q^2)|^2 \frac{g(Q^2)}{Q^2} ,$$

where

$$g(Q^2) = \frac{1}{Q^2} \int_{s_{1,\min}}^{s_1^{\max}} ds_1 \int_{s_{2,\min}}^{s_2^{\max}} ds_2 \left\{ \begin{aligned} & - V_1^2 |F_\rho(s_2)|^2 - V_2^2 |F_\rho(s_1)|^2 \\ & - 2V_1 V_2 \text{Re} [F_\rho(s_1)(F_\rho(s_2))^*] \end{aligned} \right\} ,$$

BREIT-WIGNER EVALUATION OF THE AXIAL-VECTOR F.F.

■ Breit-Wigner with $a_1(1260)$:

$$f_{a_1}(Q^2)|_{\text{BW}}^{1\text{res}} = \frac{m_{a_1}^2 + \text{Re}\Pi_{a_1}(0)}{m_{a_1}^2 - Q^2 + \text{Re}\Pi_{a_1}(Q^2) - im_{a_1}\Gamma_{a_1}(Q^2)}, \quad \Gamma_{a_1}(Q^2) = \gamma_{a_1} \frac{g(Q^2)}{g(m_{a_1}^2)}$$

$$\text{Re}\Pi_{a_1}(Q^2) = \mathcal{H}_{a_1}(Q^2) - \mathcal{H}_{a_1}(m_{a_1}^2), \quad \mathcal{H}_{a_1}(Q^2) = -\frac{Q^2}{\pi} \int_{9m_\pi^2}^{s_{\text{cut}}} ds' \frac{m_{a_1}\Gamma_{a_1}(s')}{(s')(s' - Q^2)},$$

■ Breit-Wigner with $a_1(1260) + a_1(1640)$:

$$f_{a_1}(Q^2)|_{\text{BW}}^{2\text{res}} = \frac{1}{1 + |\kappa|e^{i\phi}} \left[\frac{m_{a_1}^2 + \text{Re}\Pi_{a_1}(0)}{m_{a_1}^2 - Q^2 + \text{Re}\Pi_{a_1}(Q^2) - im_{a_1}\Gamma_{a_1}(Q^2)} \right. \\ \left. + |\kappa|e^{i\phi} \frac{m_{a_1'}^2 + \text{Re}\Pi_{a_1'}(0)}{m_{a_1'}^2 - Q^2 + \text{Re}\Pi_{a_1'}(Q^2) - im_{a_1'}\Gamma_{a_1'}(Q^2)} \right],$$

■ Dispersive description:

$$f_{a_1}(Q^2) = \exp \left[\alpha_1 Q^2 + \frac{Q^4}{\pi} \int_{9m_\pi^2}^{\infty} ds' \frac{\delta(s')}{(s')^2 (s' - Q^2 - i\varepsilon)} \right],$$

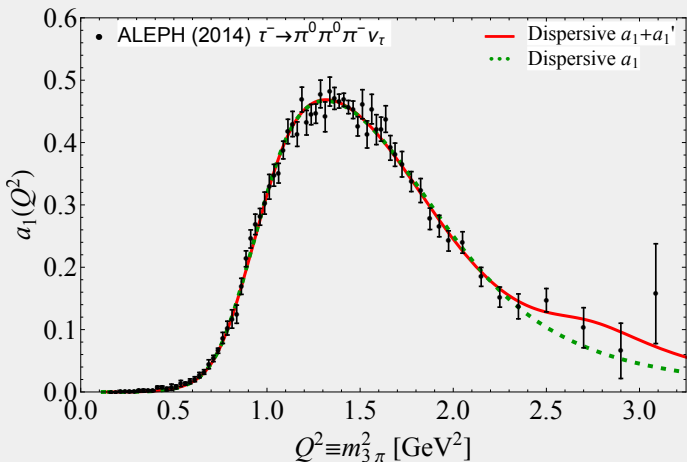
$$\tan \delta(Q^2) = \frac{\text{Im} f_{a_1}(Q^2)|_{\text{BW}}^{1(2)\text{res}}}{\text{Re} f_{a_1}(Q^2)|_{\text{BW}}^{1(2)\text{res}}},$$

$$\alpha_k^{\text{s.r.}} = \frac{k!}{\pi} \int_{9m_\pi^2}^{\infty} ds' \frac{\delta(s')}{s'^{k+1}}.$$

- α_1 : fit parameter that absorbs other production mechanism

FITS TO THE $\tau \rightarrow \pi^0 \pi^0 \pi^- \nu_\tau$ AXIAL SPECTRAL FUNCTION

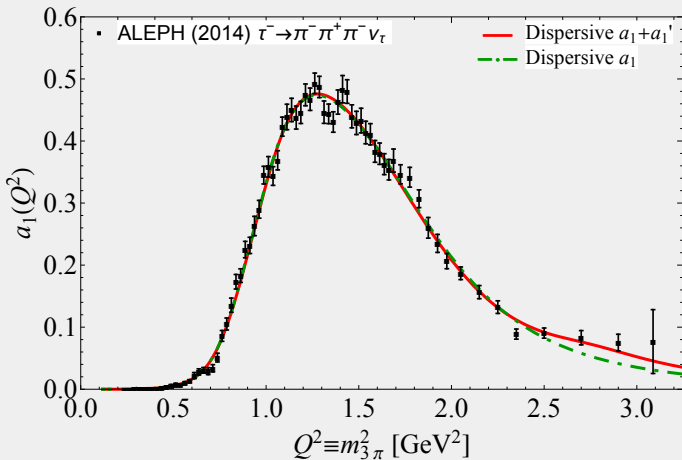
Fit results: $m_{a_1} = 1302(8) \text{ MeV}, \gamma_{a_1} = 493(11) \text{ MeV}, \alpha_1 = 0.59(1), \alpha_1^{\text{S.F.}} = 0.64(1), \chi^2_{\text{dof}} = 0.96$
 $m_{a_1} = 1296(6) \text{ MeV}, \gamma_{a_1} = 483(10) \text{ MeV}, \alpha_1 = 0.60(1), \alpha_1^{\text{S.F.}} = 0.62(1), |\kappa| = 0.10(4), \chi^2_{\text{dof}} = 0.88$



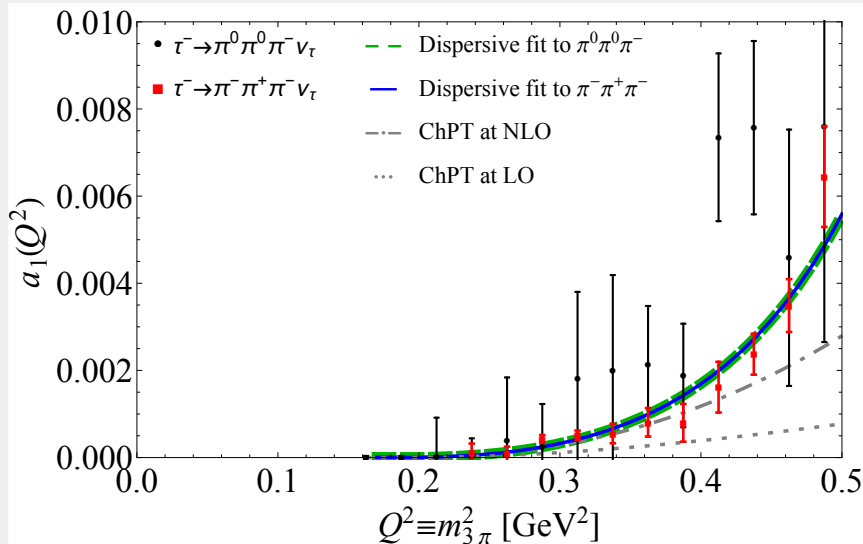
FITS TO THE $\tau \rightarrow \pi^+ \pi^- \pi^- \nu_\tau$ AXIAL SPECTRAL FUNCTION

Fit results: $m_{a_1} = 1277(5) \text{ MeV}$, $\gamma_{a_1} = 475(8) \text{ MeV}$, $\alpha_1 = 0.58(1)$, $\alpha_1 = 0.66(1)$, $\chi^2_{\text{dof}} = 1.49$

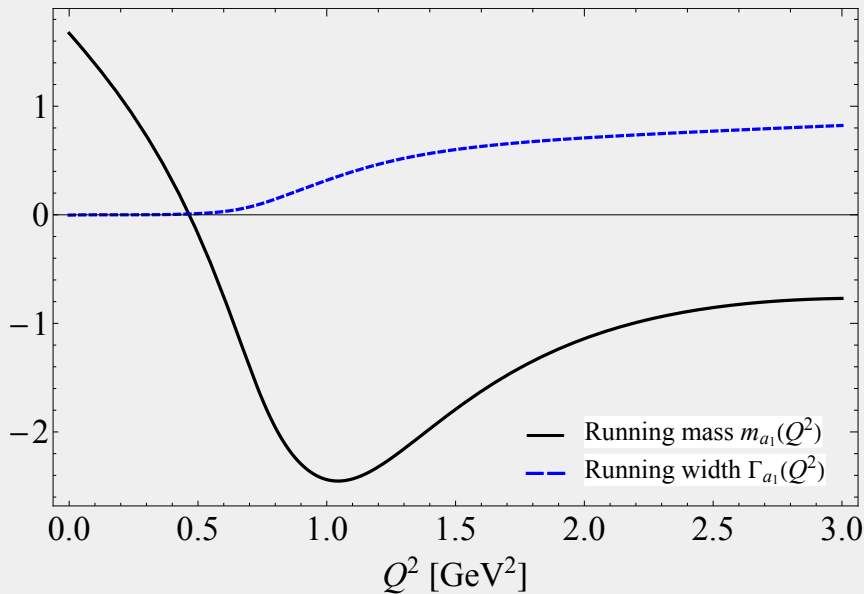
$m_{a_1} = 1273(5) \text{ MeV}$, $\gamma_{a_1} = 466(8) \text{ MeV}$, $\alpha_1 = 0.59(1)$, $\alpha_1^{\text{S.T.}} = 0.64(1)$, $|\kappa| = 0.06(2)$, $\chi^2_{\text{dof}} = 1.39$



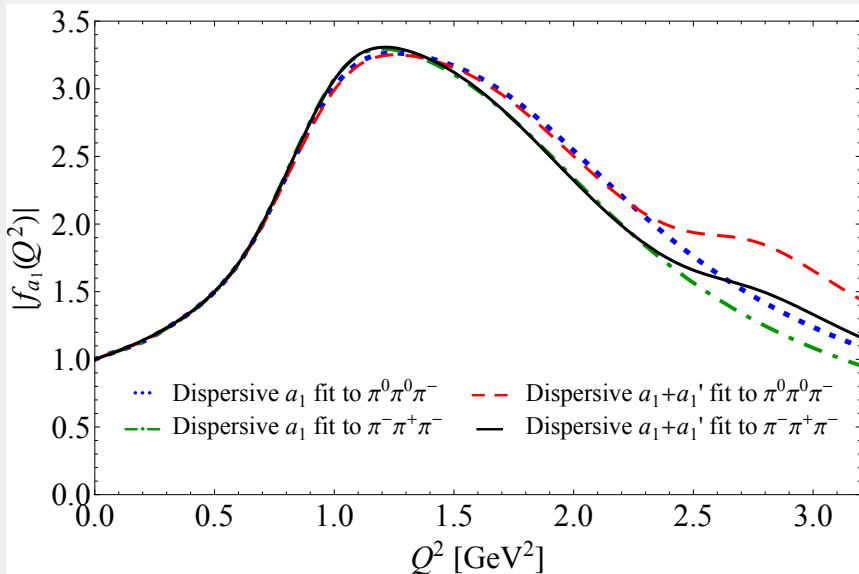
FITS TO $\tau \rightarrow 3\pi\nu_\tau$: LOW- Q^2 REGION



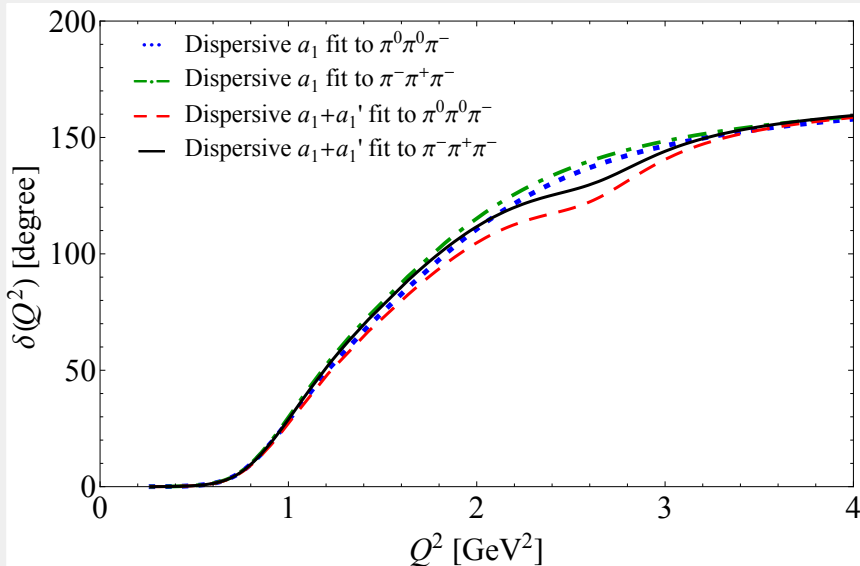
RUNNING $a_1(1260)$ MASS AND WIDTH



AXIAL-VECTOR FORM FACTOR



AXIAL-VECTOR FORM FACTOR



OUTLOOK

OUTLOOK

- Tau physics is a very rich field to test QCD and EW
- Important experimental activities: Belle (II), BaBar, LHCb, BESIII
- Form Factors from dispersion relations with subtractions
 - ▶ $F_V^\pi(s)$: important for testing QCD dynamics and the SM
 - ▶ Determination of the $\rho(1450)$ and $\rho(1700)$ limited by th. errors
 - ▶ Higher-quality data on $\tau^- \rightarrow K_S K^- \nu_\tau$
 - ▶ $K\pi$ form factor: extraction of the K^* resonance parameters
 - ▶ $\tau \rightarrow 3\pi \nu_\tau$: input for the axial form factor of the nucleon
- A lot of interesting physics to be done in the tau sector

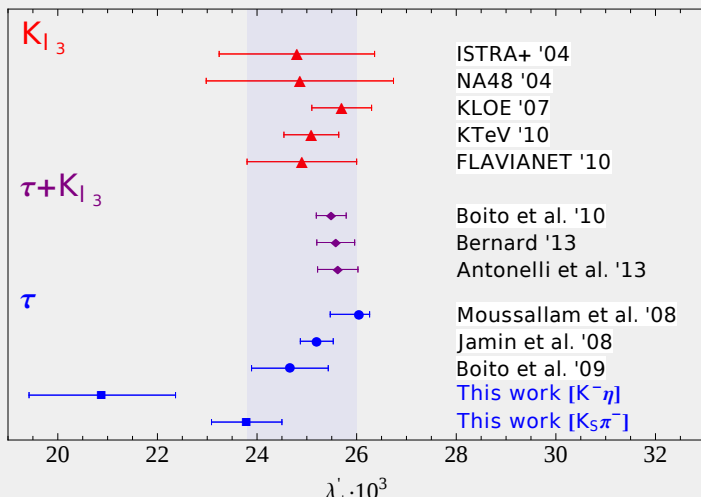
VARIANT (IV)

■ Fits for different s_{cut} and allowing the ρ -mass to float

Fits	Parameter	$s_{\text{cut}} [\text{GeV}^2]$			
		m_τ^2	4 (reference fit)	10	∞
Fit 1- ρ	$\alpha_1 [\text{GeV}^{-2}]$	1.88(1)	1.88(1)	1.89(1)	1.88(1)
	$\alpha_2 [\text{GeV}^{-4}]$	4.37(3)	4.34(1)	4.31(3)	4.34(1)
	$m_\rho [\text{MeV}]$	773.9(3)	773.8(3)	773.9(3)	773.9(3)
	$M_\rho [\text{MeV}]$	$= m_\rho$	$= m_\rho$	$= m_\rho$	$= m_\rho$
	$M_{\rho'} [\text{MeV}]$	1382(71)	1375(11)	1316(9)	1312(8)
	$\Gamma_{\rho'} [\text{MeV}]$	516(165)	608(35)	728(92)	726(26)
	$M_{\rho''} [\text{MeV}]$	1723(1)	1715(22)	1655(1)	1656(8)
	$\Gamma_{\rho''} [\text{MeV}]$	315(271)	455(16)	569(160)	571(13)
	γ	0.12(13)	0.16(1)	0.18(2)	0.17(1)
	ϕ_1	-0.56(35)	-0.69(1)	-1.40(19)	-1.41(8)
	δ	-0.09(3)	-0.13(1)	-0.17(4)	-0.17(3)
	ϕ_2	-0.19(69)	-0.45(12)	-1.06(10)	-1.05(11)
	$\chi^2/\text{d.o.f}$	1.09	0.70	0.63	0.66

RESULTS OF THE COMBINED $\tau^- \rightarrow K_S \pi^- \nu_\tau$ AND $\tau^- \rightarrow K^- \eta \nu_\tau$ ANALYSIS

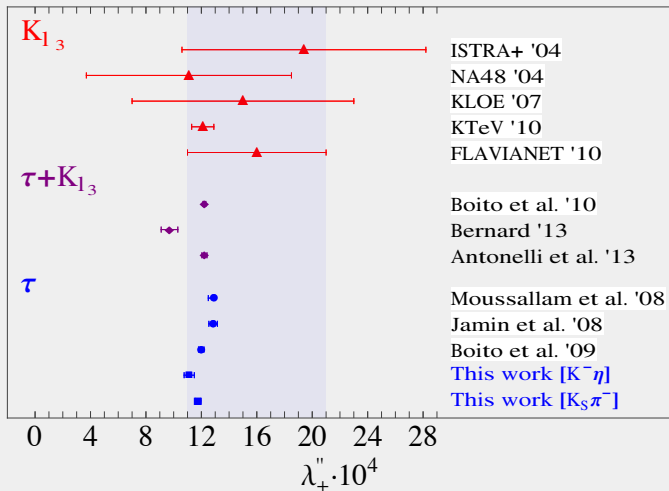
$$\left. \begin{aligned} \lambda'_{K\pi} &= (23.9 \pm 0.9) \cdot 10^{-3} \\ \lambda'_{K\eta} &= (20.9 \pm 2.7) \cdot 10^{-3} \end{aligned} \right\} \text{isospin violation?}$$



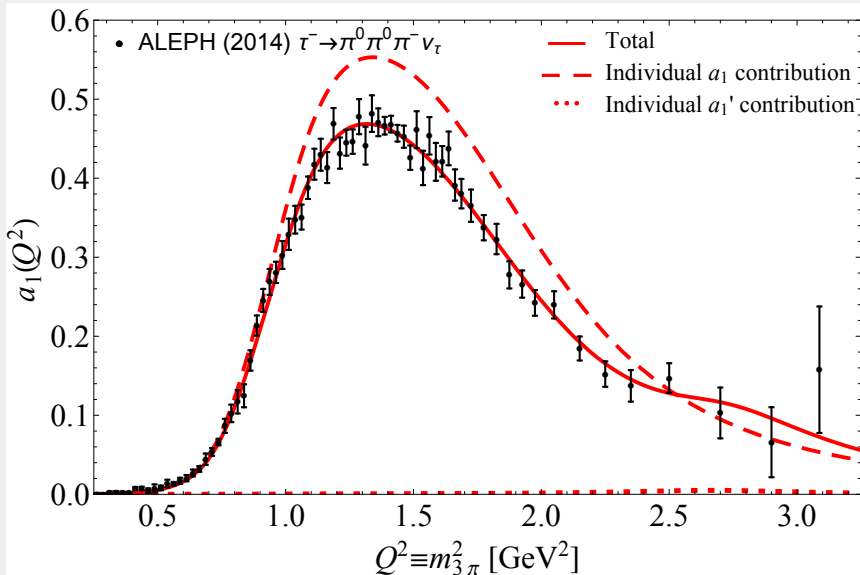
RESULTS OF THE COMBINED $\tau^- \rightarrow K_S \pi^- \nu_\tau$ AND $\tau^- \rightarrow K^- \eta \nu_\tau$ ANALYSIS

$$\lambda''_{K\pi} = (11.8 \pm 0.2) \cdot 10^{-4}$$

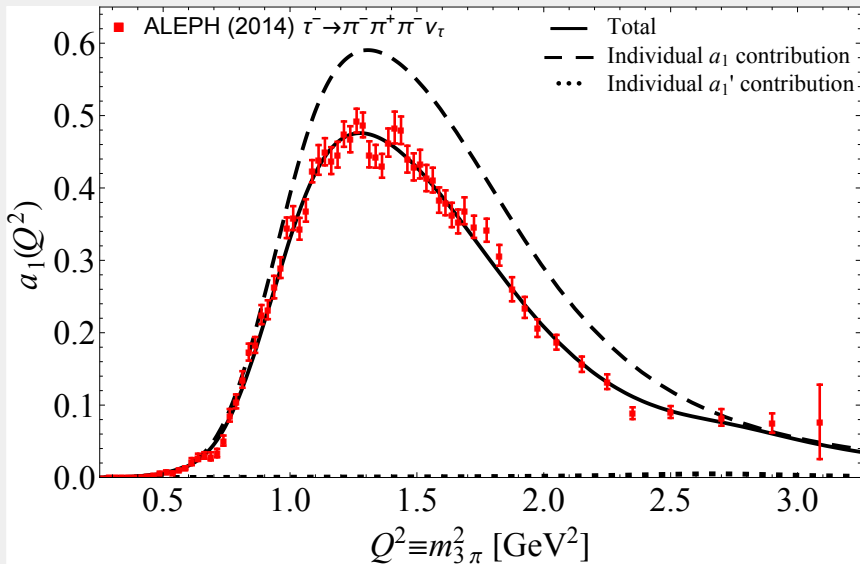
$$\lambda''_{K\eta} = (11.1 \pm 0.5) \cdot 10^{-4}$$



SPECTRAL FUNCTION: INDIVIDUAL CONTRIBUTIONS



SPECTRAL FUNCTION: INDIVIDUAL CONTRIBUTIONS



FORM FACTORS OF THE NUCLEON

■ Nucleon matrix element

$$\langle N(p') | J_{em}^\mu | N(p) \rangle = \bar{u}(p') \left[\gamma^\mu F_1(t) + \frac{i}{2m_N} \sigma^{\mu\nu} (p' - p)_\nu F_2(t) \right] u(p),$$

$$\langle N(p') | J_A^{\mu a} | N(p) \rangle = \bar{u}(p') \frac{\tau^a}{2} \gamma_5 \left[\gamma^\mu F_A(t) + \frac{(p' - p)^\mu}{2m_N} F_P(t) \right] u(p),$$

■ Four Form Factor to determine ($t = (p' - p)^2$)

- ▶ $F_1(t)$ and $F_2(t)$: Dirac and Pauli Form Factors
- ▶ $G_E(t)$ and $G_M(t)$: electric and magnetic (Sachs) FFs, well-known

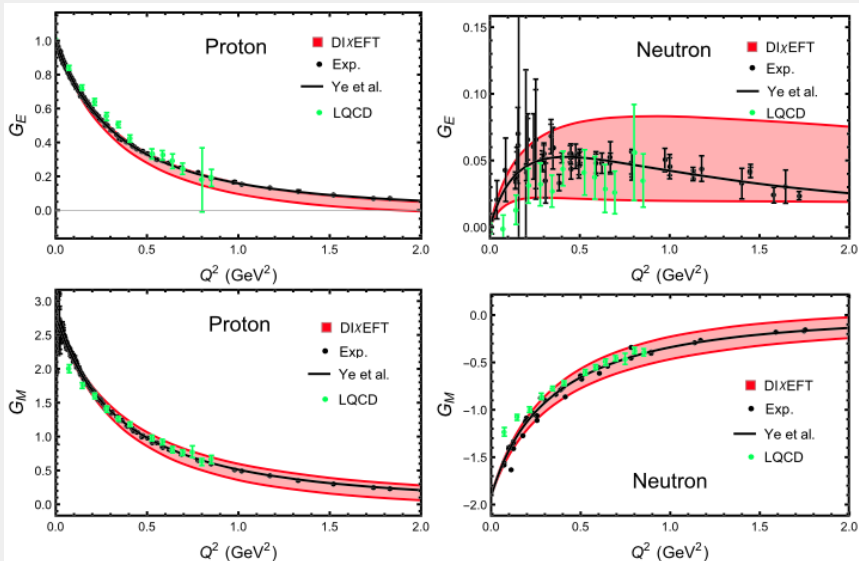
$$G_E(t) = F_1(t) + \frac{t}{4m_N^2} F_2(t), \quad G_M(t) = F_1(t) + F_2(t)$$

- ▶ $F_A(t)$: Axial Form Factor: main unknown
- ▶ $F_P(t)$: Pseudo-scalar Form Factor. It can be related to the Axial FF using PCAC or pion-pole approximation

$$F_P(t) = \frac{2m_N^2}{m_\pi^2 - t} F_A(t)$$

ELECTROMAGNETIC FORM FACTS: SPACE-LIKE

$$Q^2 \equiv -t \geq 0$$

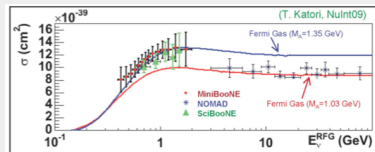
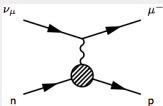


AXIAL VECTOR FORM FACTOR

■ $F_A(t)$: Main Unknown

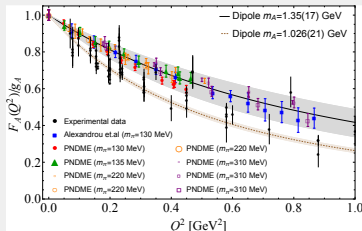
■ Motivation: How to determine $F_A(t)$ experimentally?

► $\nu_\mu n \rightarrow \mu p: F_A(Q^2) = \frac{F_A(0)}{(1+Q^2/M_A^2)^2}$

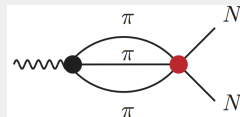


■ Objective: Theoretical effort to improve $F_A(t)$

► Lattice



► Analytically: 3π intermediate state

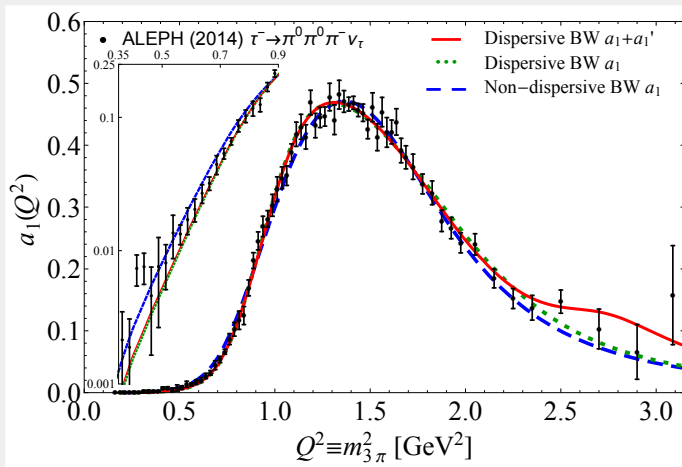


FITS TO THE $\tau \rightarrow \pi^0 \pi^0 \pi^- \nu_\tau$ AXIAL SPECTRAL FUNCTION

Breit-Wigner $m_{a_1} = 1271(13) \text{ MeV}, \gamma_{a_1} = 523(18) \text{ MeV}, \mathcal{N} = 1.59(7), \chi^2_{\text{dof}} = 1.16$

$m_{a_1} = 1293(10) \text{ MeV}, \gamma_{a_1} = 501(11) \text{ MeV}, \mathcal{N} = 0.88(2), \chi^2_{\text{dof}} = 1.01$

$m_{a_1} = 1293(6) \text{ MeV}, \gamma_{a_1} = 485(10) \text{ MeV}, |\kappa| = 0.12(4), \mathcal{N} = 0.97(5), \chi^2_{\text{dof}} = 0.86$

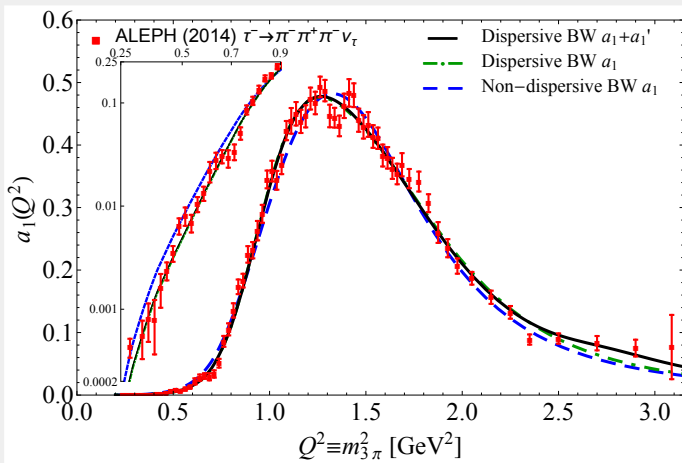


FITS TO THE $\tau \rightarrow \pi^+ \pi^- \pi^- \nu_\tau$ AXIAL SPECTRAL FUNCTION

Breit-Wigner: $m_{a_1} = 1243(6) \text{ MeV}$, $\gamma_{a_1} = 480(8) \text{ MeV}$, $\mathcal{N} = 1.43(3)$, $\chi^2_{\text{dof}} = 3.11$

$m_{a_1} = 1259(6) \text{ MeV}$, $\gamma_{a_1} = 474(8) \text{ MeV}$, $\mathcal{N} = 0.81(2)$, $\chi^2_{\text{dof}} = 1.51$

$m_{a_1} = 1260(5) \text{ MeV}$, $\gamma_{a_1} = 467(8) \text{ MeV}$, $|\kappa| = 0.05(2)$, $\mathcal{N} = 0.85(2)$, $\chi^2_{\text{dof}} = 1.43$



$K\pi$ PHASE

