Simulating real-time dynamics of hard probes in nuclear matter on a quantum computer

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arXiv: 2010.03571

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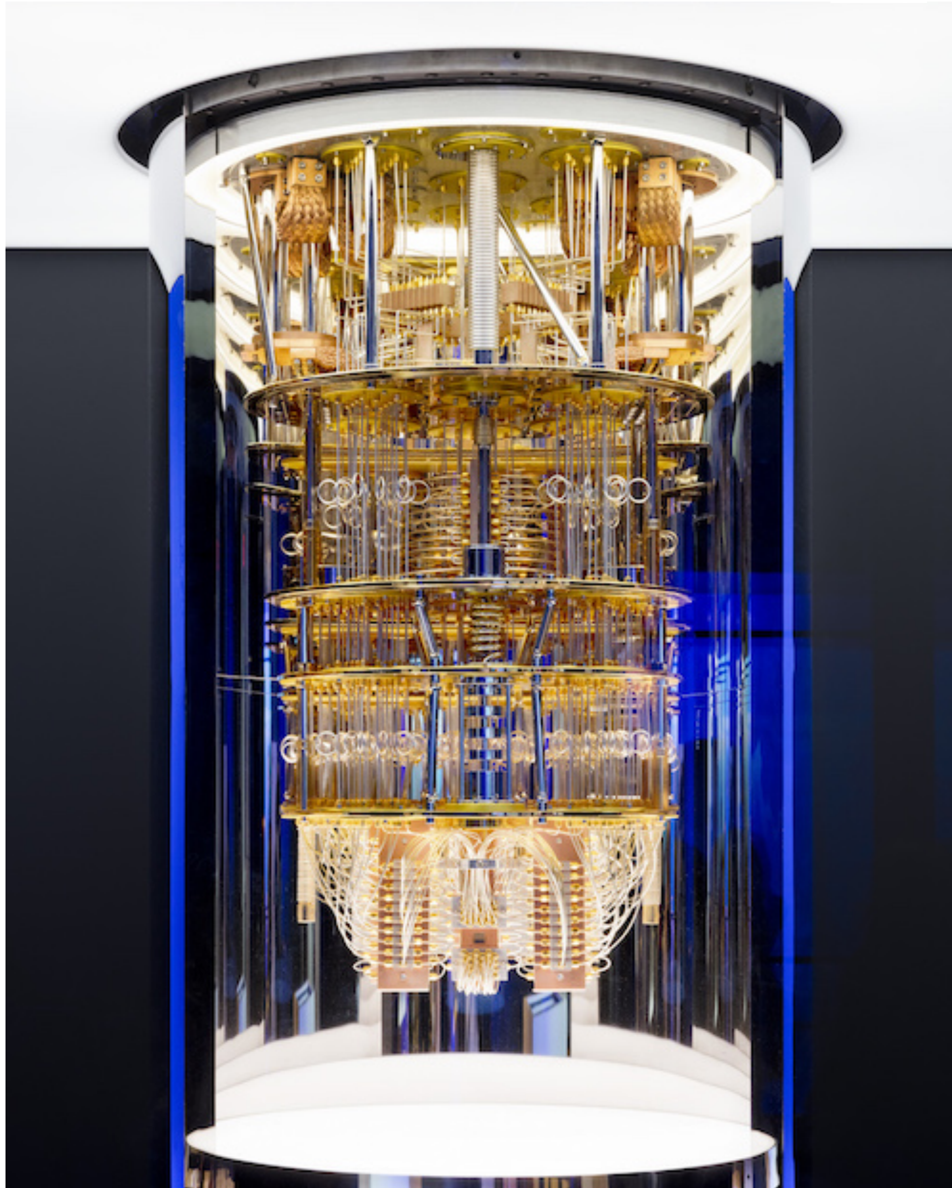
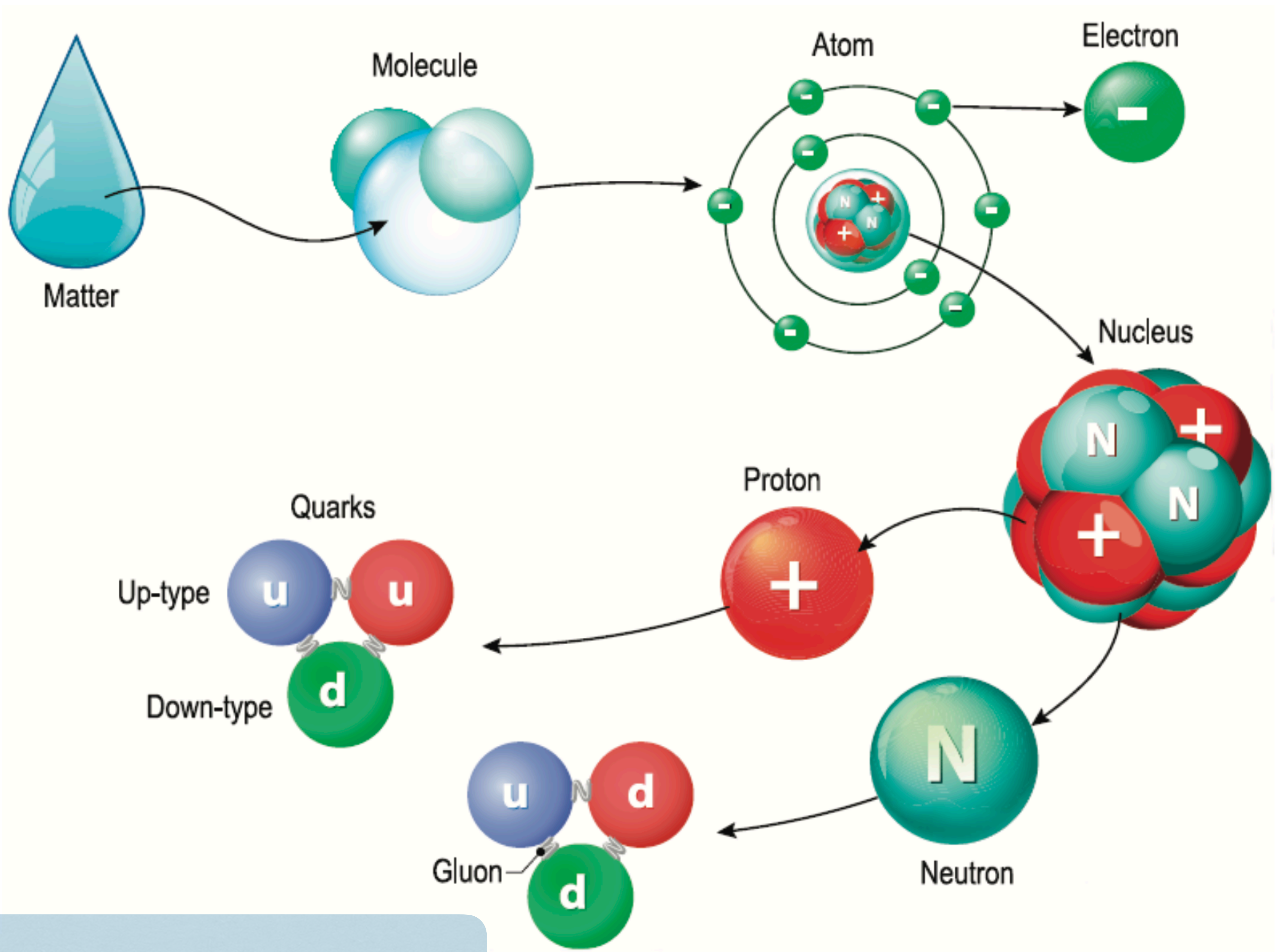
IBM Q



BERKELEY LAB

Jefferson Lab
January 25, 2021

Understanding the fundamental structure of matter

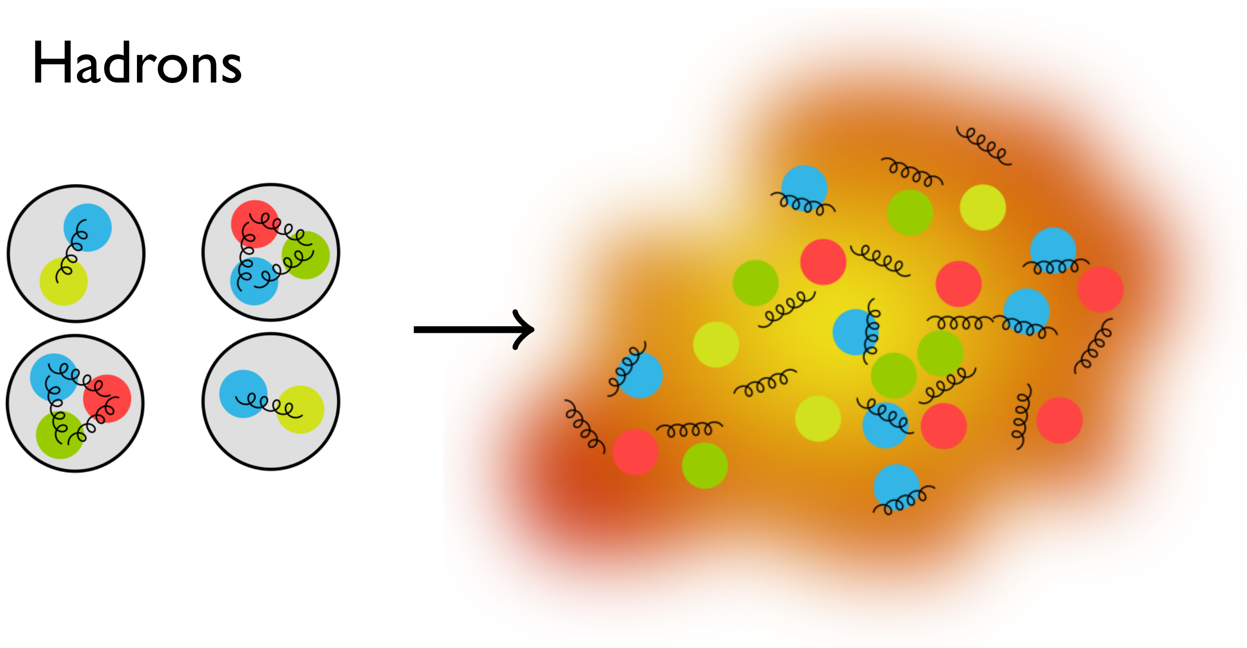


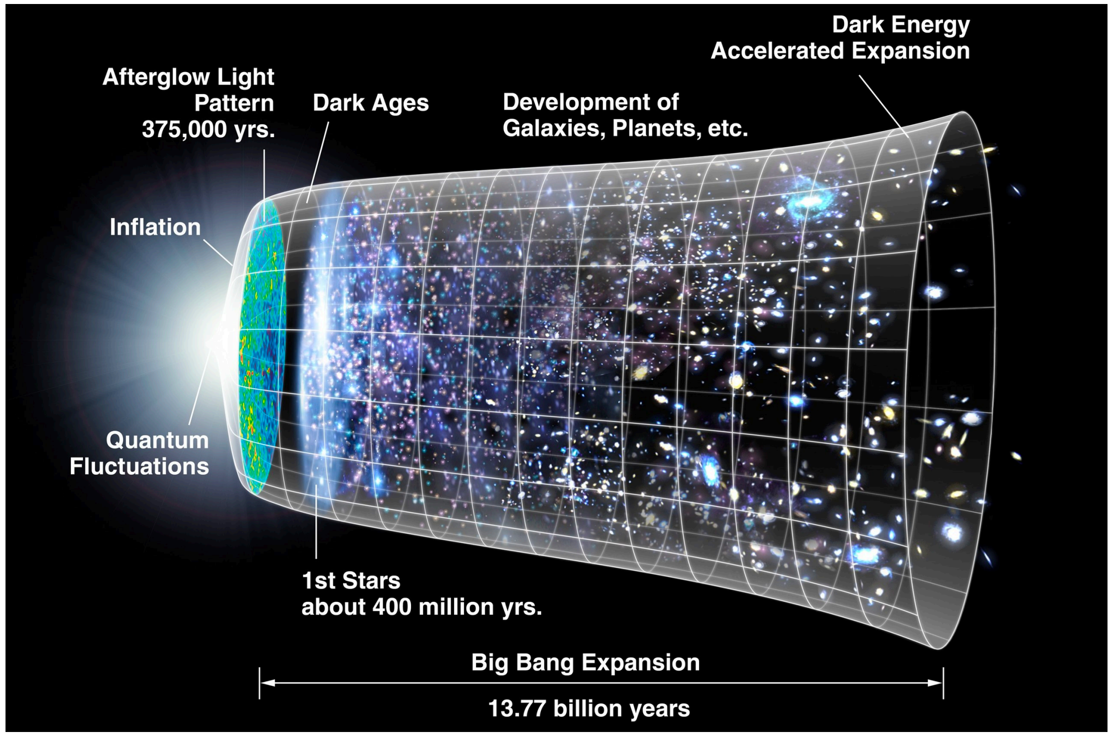
Quantum chromodynamics (QCD)

The Quark-Gluon Plasma

If we heat nuclear matter to $T = \mathcal{O}(100 \text{ MeV})$, quarks and gluons become **deconfined** into a strongly-coupled fluid

Hadrons

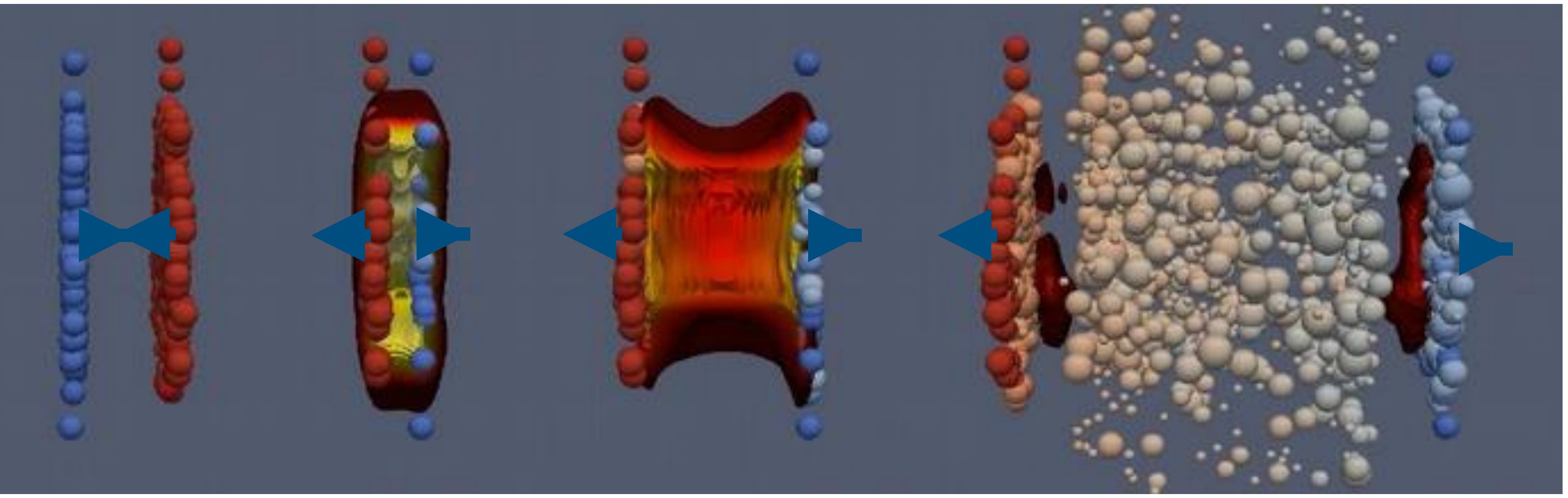




Heavy-ion collisions

We collide nuclei at

- **Large Hadron Collider (LHC)**
- **Relativistic Heavy Ion Collider (RHIC)**

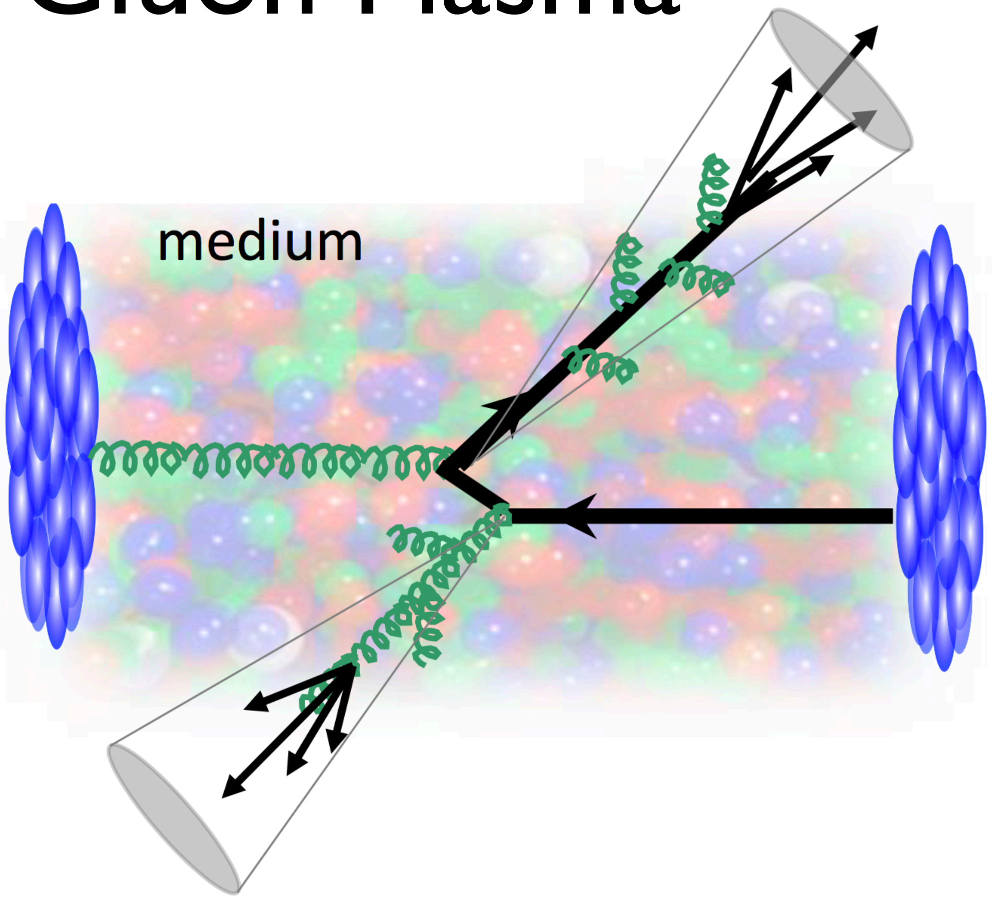


Formation and evolution of the QGP

Hard probes of the Quark Gluon Plasma

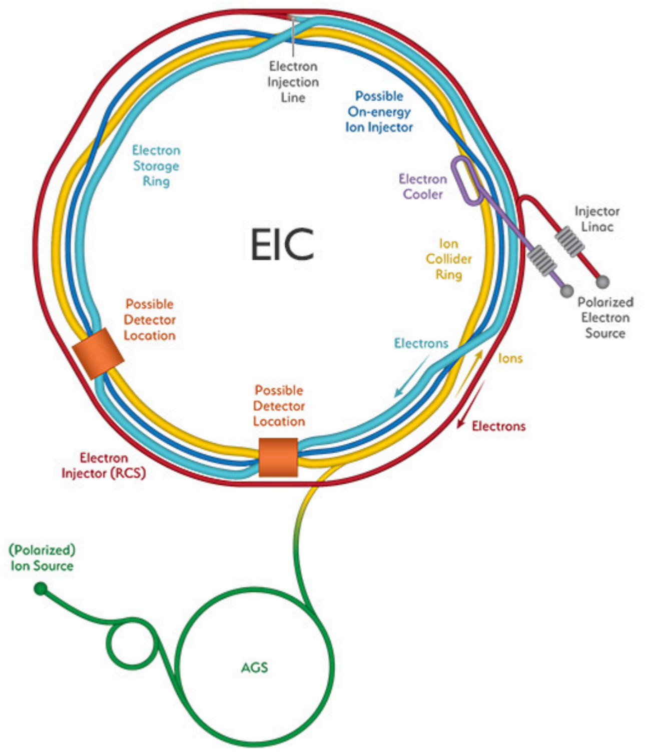
In addition to soft scatterings, there are occasional **hard scatterings** in the collisions

- Highly energetic particles: **jets**
- Large mass particles: **heavy quarks**



These "hard probes" interact with the QGP as they traverse it

Similar physics relevant in eA collisions at the EIC

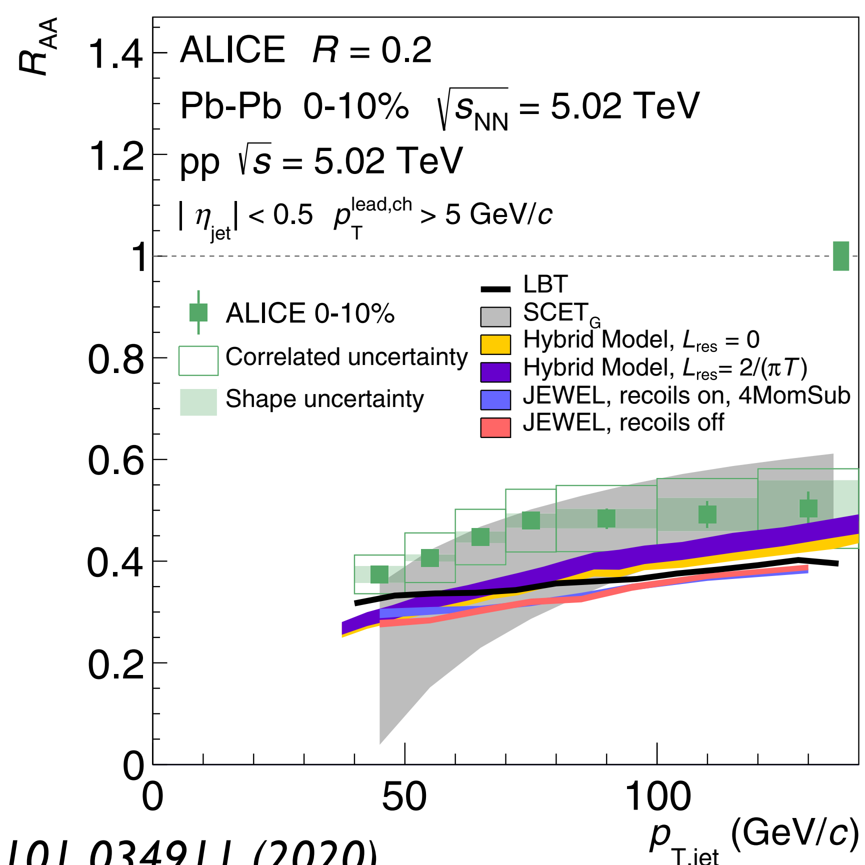


Hard probes — experiment

Experiments measure how cross-sections of hard probes are modified in heavy-ion collisions compared to proton-proton collisions

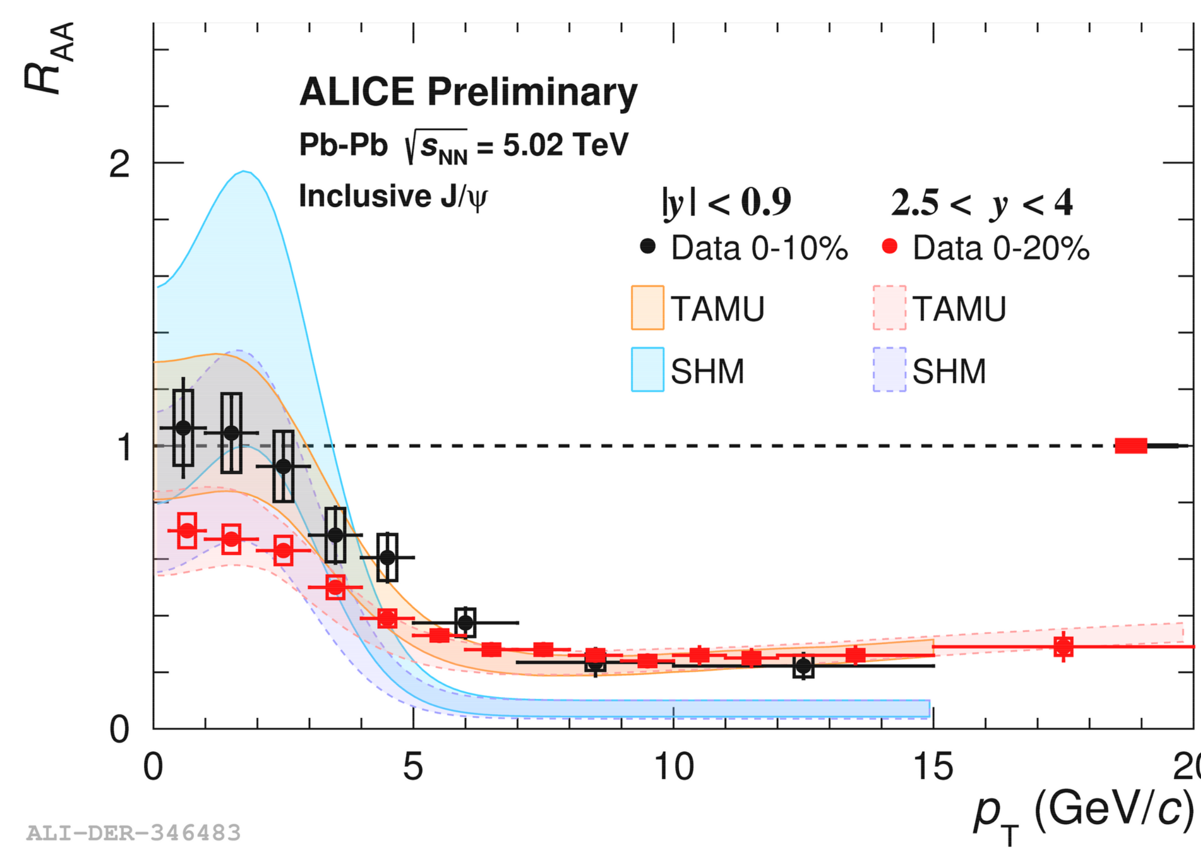
$$R_{\mathrm{AA}} = \frac{\mathrm{d}\sigma^{\mathrm{PbPb}}}{\langle T_{\mathrm{AA}} \rangle \, \mathrm{d}\sigma^{pp}}$$

Jets



PRC 101 034911 (2020)

Heavy quarks

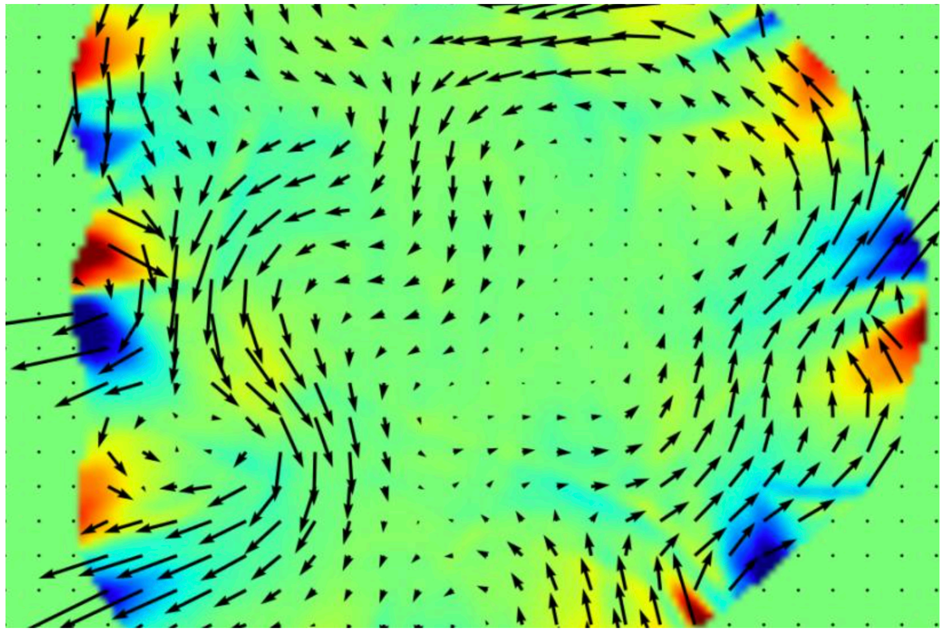


Hard probes — theory

- In vacuum: calculate scattering of asymptotic states using perturbative QCD

  Note that there is no sense of "real-time evolution"

- In medium: must combine probe evolution with hydrodynamic evolution of the QGP

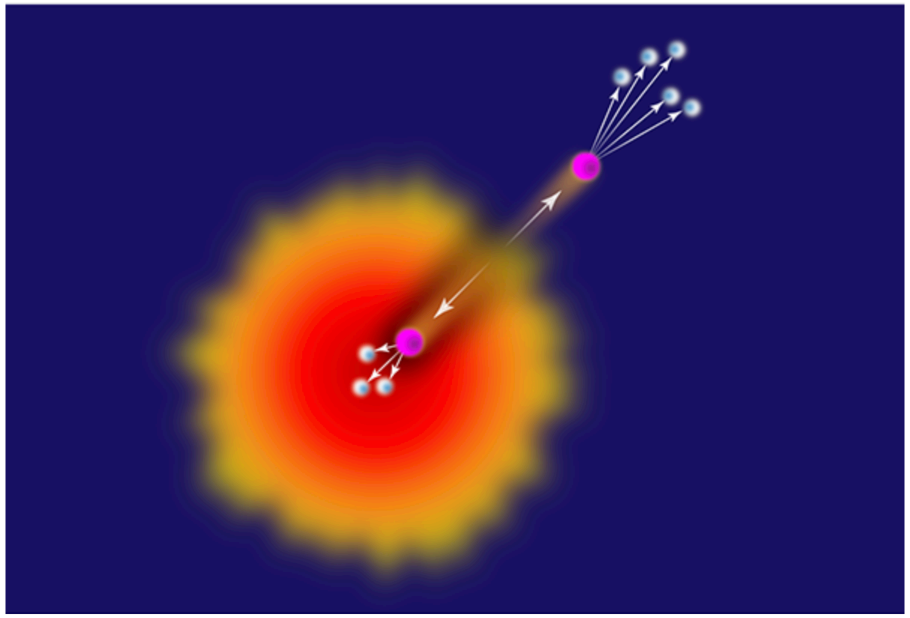


X.N.Wang

Need real-time evolution

Current medium-modified parton shower put in time evolution "by hand"

see e.g. Jetscape



Can we solve the real-time dynamics of QCD?

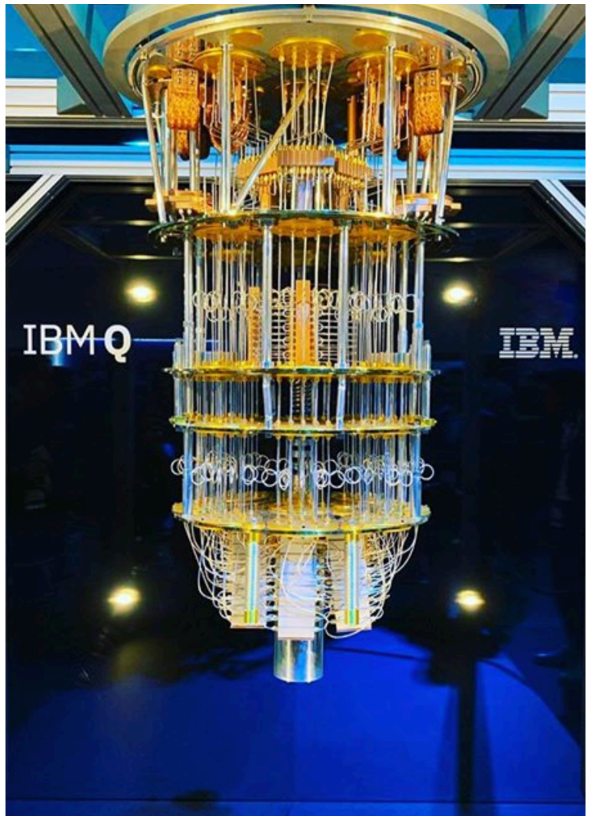
- Typical methods in **lattice QCD** have a sign problem and use imaginary- instead of real time

$$\int e^{i\mathscr{L}t} \qquad t \to it$$

- Can use the **Hamiltonian formulation of QCD** *see e.g. Kogut, Susskind 70s, Preskill `18*
  - Theoretical formulation ongoing
  - Gauge, color
  - Large Hilbert space

→ **Quantum computing** may allow for a solution of the real-time dynamics of QCD!

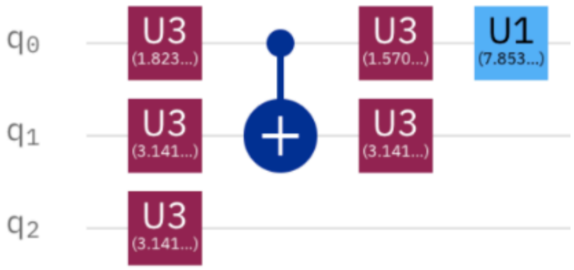
Quantum computing



- Significant in process in recent years
- Digital circuit-based e.g. IBMQ, Rigetti
- Noisy Intermediate Scale Quantum (NISQ) era
- Can achieve exponential speedup

- Holds great promise for nuclear physics
- Address computationally expensive problems
- Solve the real-time dynamics of QCD

e.g. Preskill `18, Klco, Savage et al.`18-`20, Cloet, Dietrich et al. `19



IBM Q rigetti

Google

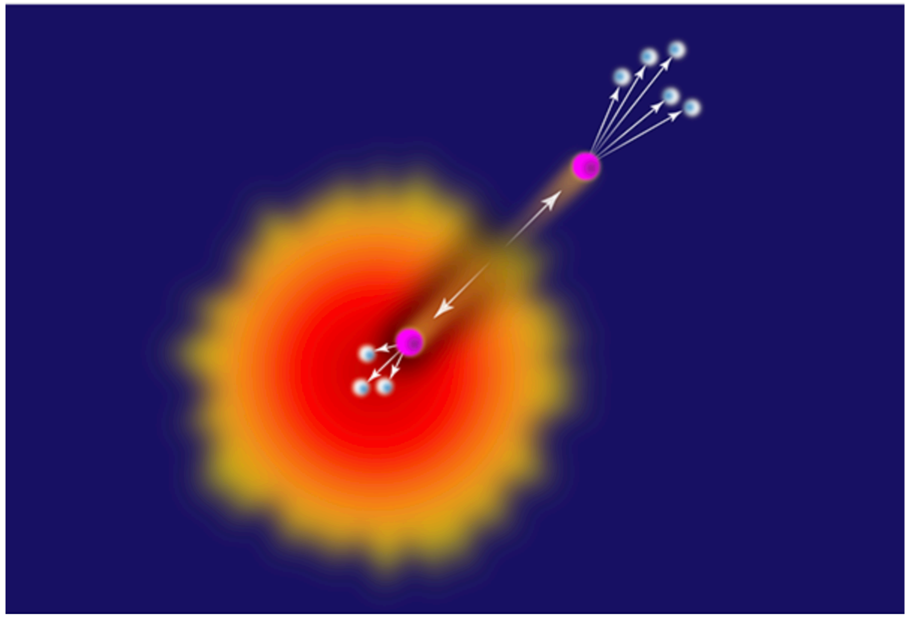
Outline

Open quantum systems in heavy-ion collisions

Quantum simulation with IBM Q

Open quantum systems and the nuclear medium

Study the real time dynamics of the quantum evolution of probes in the nuclear medium (LHC/RHIC/EIC)



System - Jet/heavy-flavor

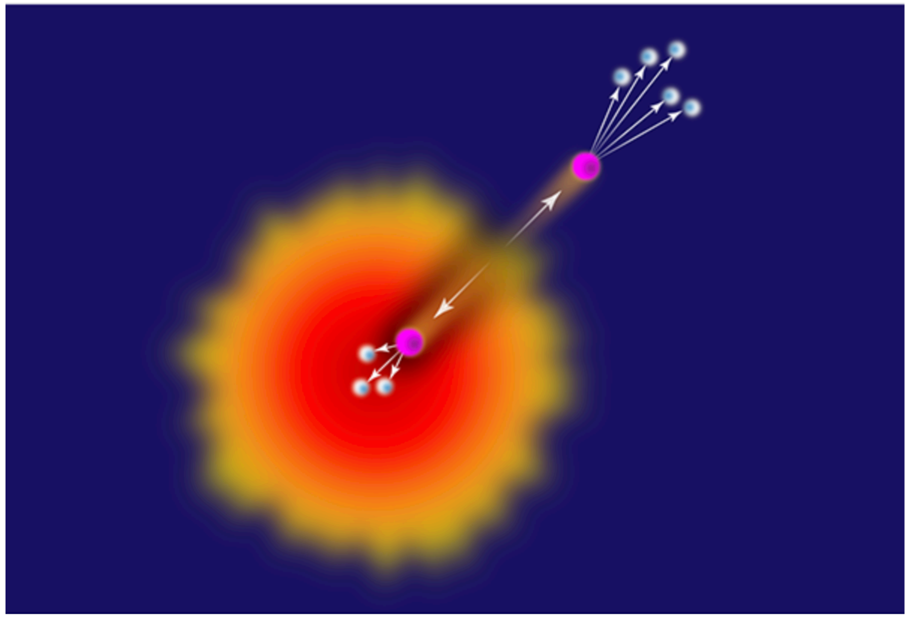
Environment - Nuclear matter

$$H(t) = H_S(t) + H_E(t) + H_I(t)$$

Akamatsu, Rothkopf `12-`20, Müller et al `18, Mehen, Yao `18, Qiu, Ringer, Sato, Zurita `19, Vaidya, Yao `20

Open quantum systems and the nuclear medium

Study the real time dynamics of the quantum evolution of probes in the nuclear medium (LHC/RHIC/EIC)



System - Jet/heavy-flavor

Environment - Nuclear matter

$$H(t) = H_S(t) + H_E(t) + H_I(t)$$

The time evolution is governed by the von Neumann equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho^{(\mathrm{int})}(t) = -i\left[H_I^{(\mathrm{int})}(t), \rho^{(\mathrm{int})}(t)\right]$$

where $$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

Akamatsu, Rothkopf `12-`20, Müller et al `18, Mehen, Yao `18, Qiu, Ringer, Sato, Zurita `19, Vaidya, Yao `20

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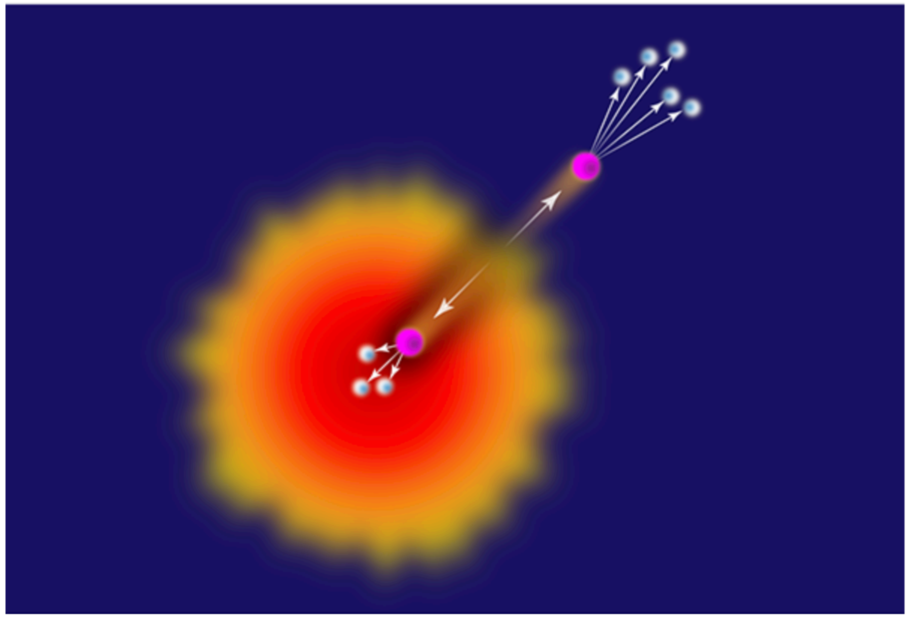


System - Jet/heavy-flavor

Environment - Nuclear matter

$$H(t) = H_S(t) + H_E(t) + H_I(t)$$

In the Markovian limit, the subsystem is described by a **Lindblad equation**

$$\rho_S = \mathrm{tr}_E[\rho]$$

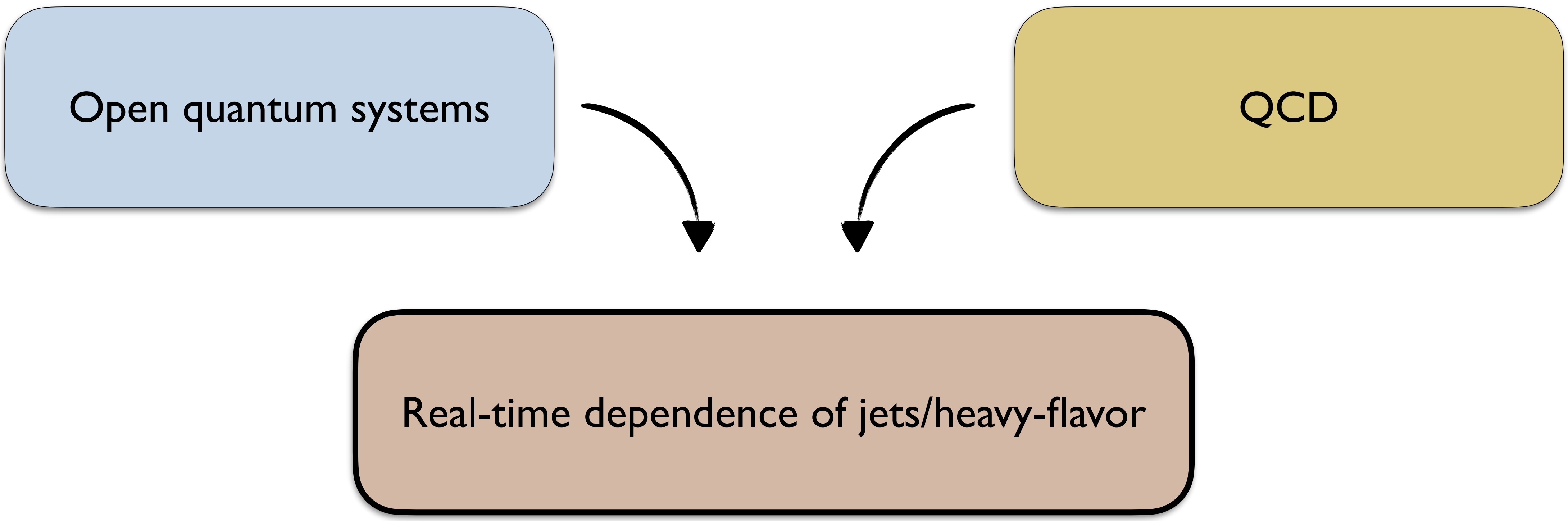
$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_S = -i\,[H_S, \rho_S] + \sum_{j=1}^{m}\left(L_j\rho_S L_j^\dagger - \frac{1}{2}L_j^\dagger L_j\rho_S - \frac{1}{2}\rho_S L_j^\dagger L_j\right)$$

See also e.g. non-global logarithms and CGC

Neill `15, Armesto et al. `19, Li, Kovner `20

Akamatsu, Rothkopf `12-`20, Brambilla et al. `18, Mehen, Yao `18, Vaidya, Yao `20

Open quantum systems and the nuclear medium



- Currently various approximations are considered *Blaizot, Escobedo `18, Yao, Mehen `18, `20*
  - Markovian limit
  - Small coupling of system and environment
  - Semi-classical transport

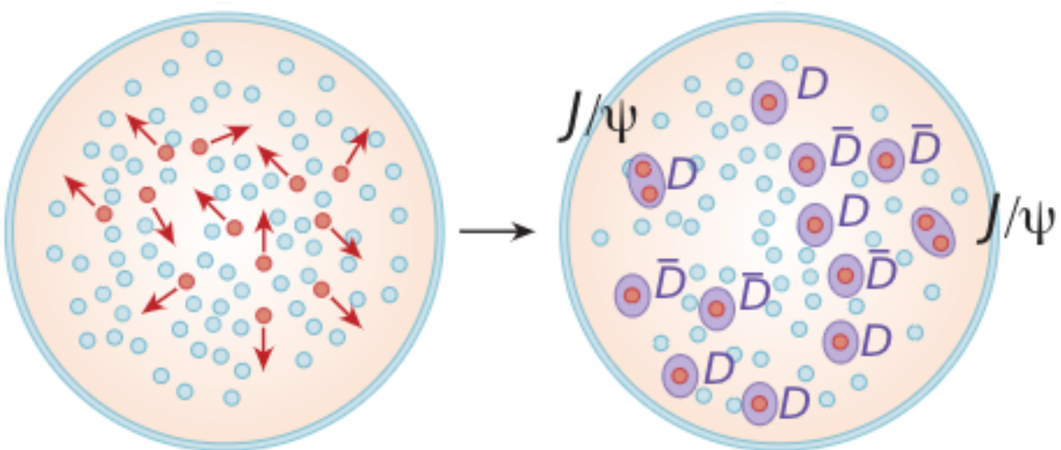
Akamatsu, Rothkopf et al. `12-`20, Brambilla et al. `17-`20
Yao, Mueller, Mehen `18-`20, Sharma,Tiwari `20
Yao,Vaidya `19,Vaidya `20

Quarkonium suppression

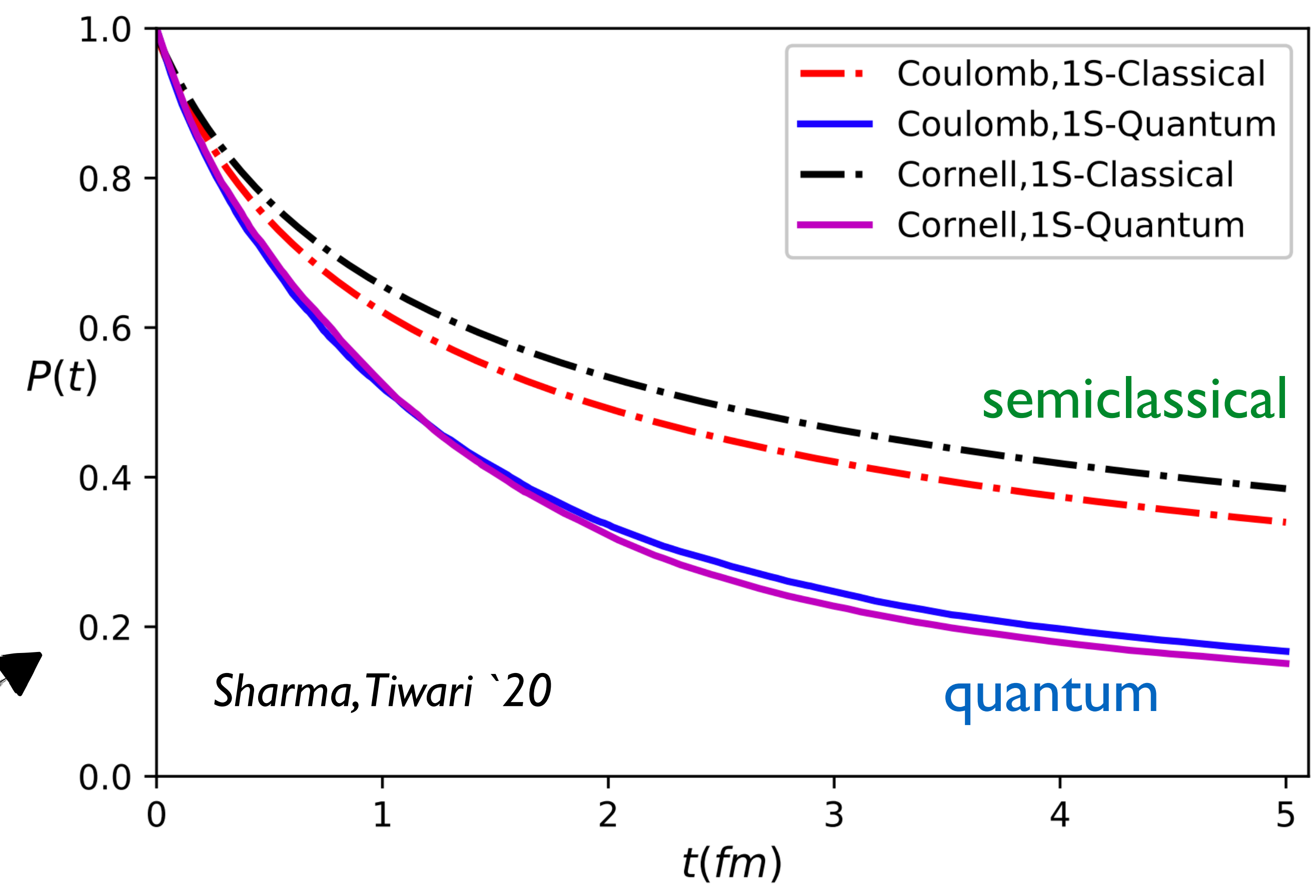
Open quantum system formalism for quarkonia

Akamatsu, Rothkopf et al. `12-`20, Brambilla et al. `17-`20
Yao, Mueller, Mehen `18-`20, Sharma, Tiwari `20

Quarkonium production in heavy-ion collisions



Survival probability of the vacuum state



NRQCD + semiclassical approach compared to quantum evolution

Bjorken expanding QGP $T_0 = 475\ \mathrm{MeV}$

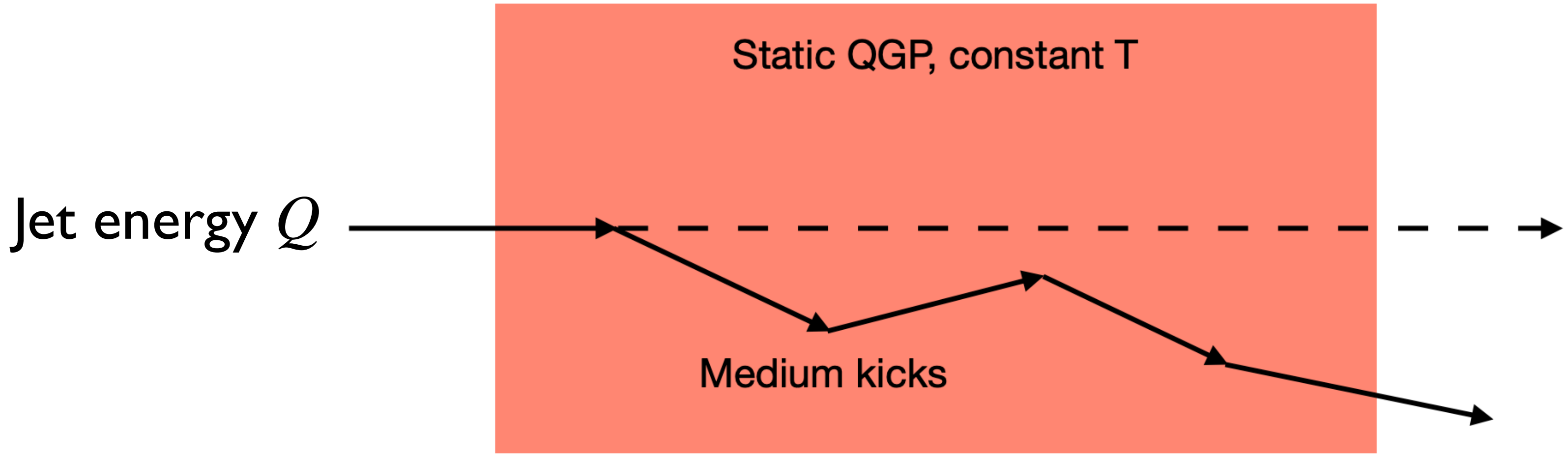
see also Miura, Akamatsu, Asakawa, Rothkopf `19

Jet broadening

Yao, Vaidya `20

Open quantum system formalism for jets

First steps in the direction of jet physics



Soft Collinear Effective Theory

- Forward scattering, Glauber gluon exchange

Markovian master equation describes evolution of jet density matrix:

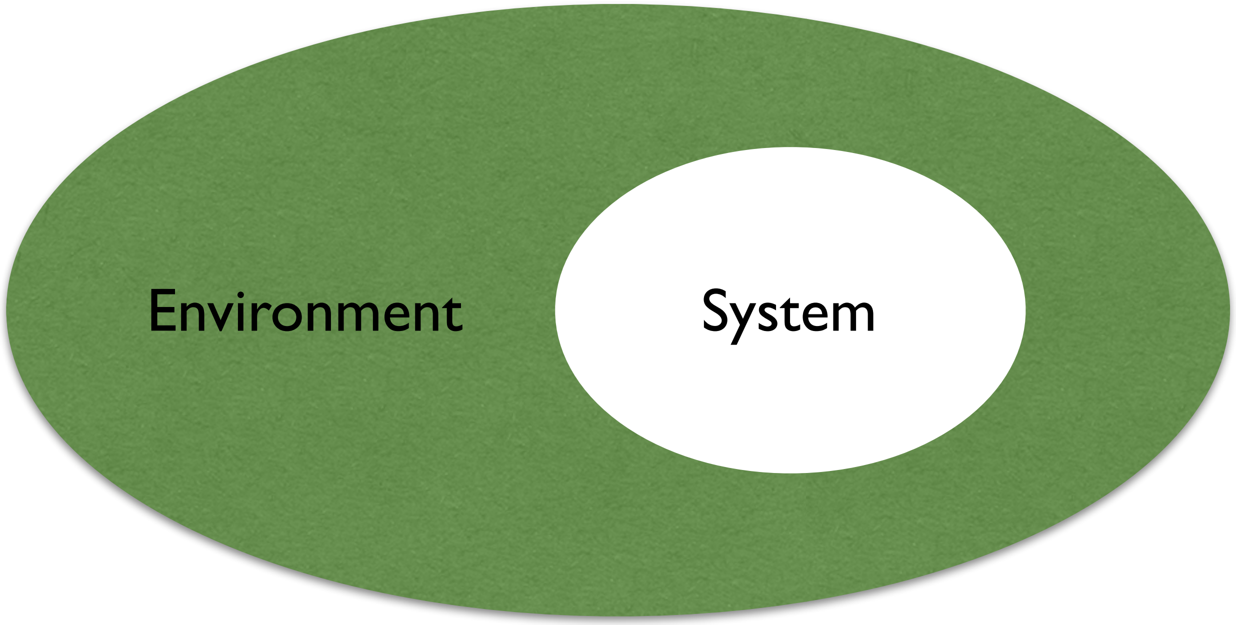
$$\partial_t P(Q,t) = -R(Q)P(Q,t) + \int \widetilde{\mathrm{d}q} K(Q,q)P(q,t)$$

where the probability to be in a given momentum state is:

$$P(Q,t) = \langle Q|\rho_S(t)|Q\rangle$$

Open quantum systems … more generally

- **Basics of Quantum Mechanics/Collapse of the wave function (measurement theory)**
- **Cosmology/Inflation**
- **Qubits**



Outline

Open quantum systems in heavy-ion collisions

Quantum simulation with IBM Q

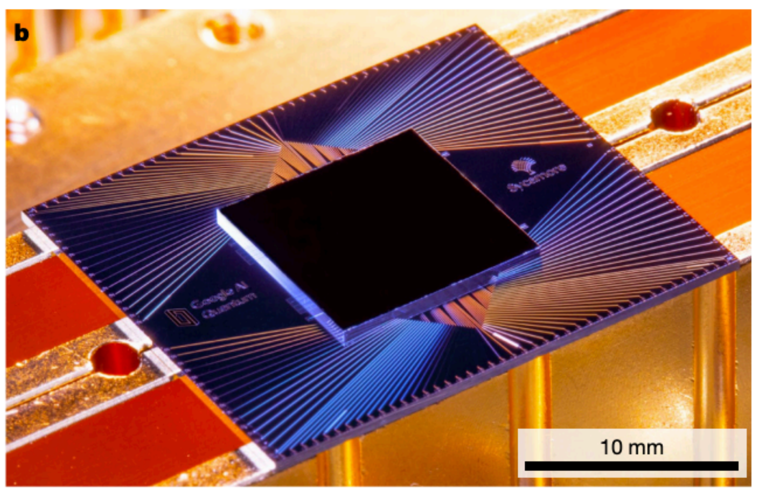
Quantum advantage

Article

Quantum supremacy using a programmable superconducting processor

Google

Martinis et al. (2019)



53-qubit superconducting circuit device

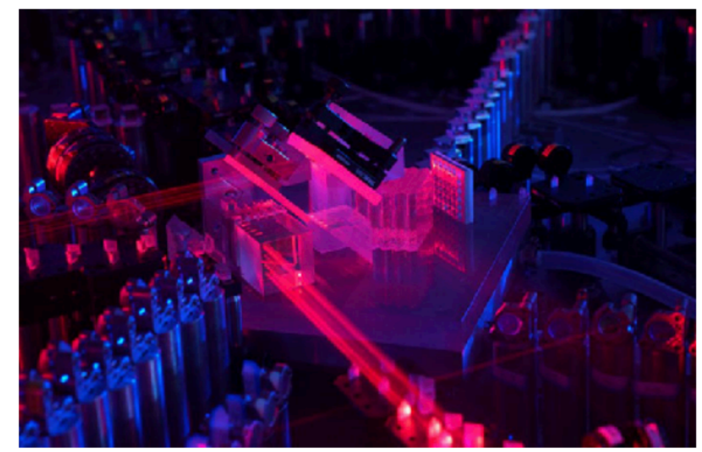
Algorithm: sampling of random circuits

$\mathcal{O}\left(10^{3}\right)$ **times faster than best classical supercomputers**

Quantum computational advantage using photons

Han-Sen Zhong[1,2*], Hui Wang[1,2*], Yu-Hao Deng[1,2*], Ming-Cheng Chen[1,2*], Li-Chao Peng[1,2], Yi-Han Luo[1,2], Jian Qin[1,2], Dian Wu[1,2], Xing Ding[1,2], Yi Hu[1,2], Peng Hu[3], Xiao-Yan Yang[3], Wei-Jun Zhang[3], Hao Li[3], Yuxuan Li[4], Xiao Jiang[1,2], Lin Gan[4], Guangwen Yang[4], Lixing You[3], Zhen Wang[3], Li Li[1,2], Nai-Le Liu[1,2], Chao-Yang Lu[1,2], Jian-Wei Pan[1,2†]

Science (2020)



Photonic device — special-purpose

Algorithm: boson sampling

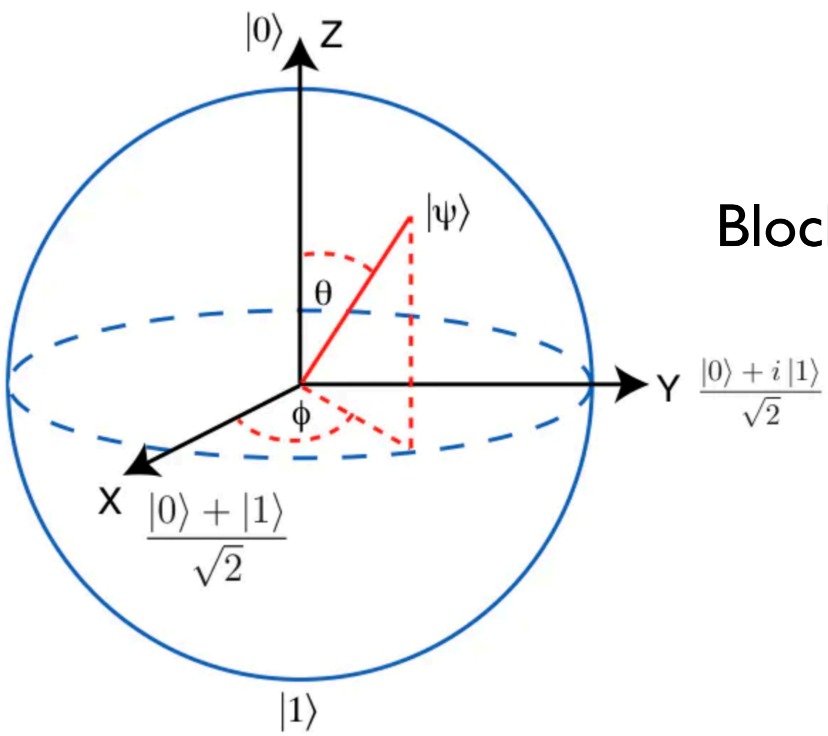
$\mathcal{O}\left(10^{14}\right)$ **times faster than best classical supercomputers**

Quantum computing

Qubits

Computational basis $|0\rangle, |1\rangle$

- Can be in superposition
- Multiple qubits can be entangled



Bloch sphere

Multi-qubit state vector $|\psi\rangle = \sum_{i=1}^{2^N} a_i |\psi_i\rangle$ For N qubits, there are 2^N amplitudes

3-qubit example

$$|\psi\rangle = a_1|000\rangle + a_2|001\rangle + a_3|010\rangle + a_4|011\rangle + a_5|100\rangle + a_6|101\rangle + a_7|110\rangle + a_8|111\rangle$$

⟶ Potential for exponential speedup

Quantum computing

Qubits

Multi-qubit state vector $$|\psi\rangle = \sum_{i=1}^{2^N} a_i \, |\psi_i\rangle$$ For N qubits, there are 2^N amplitudes

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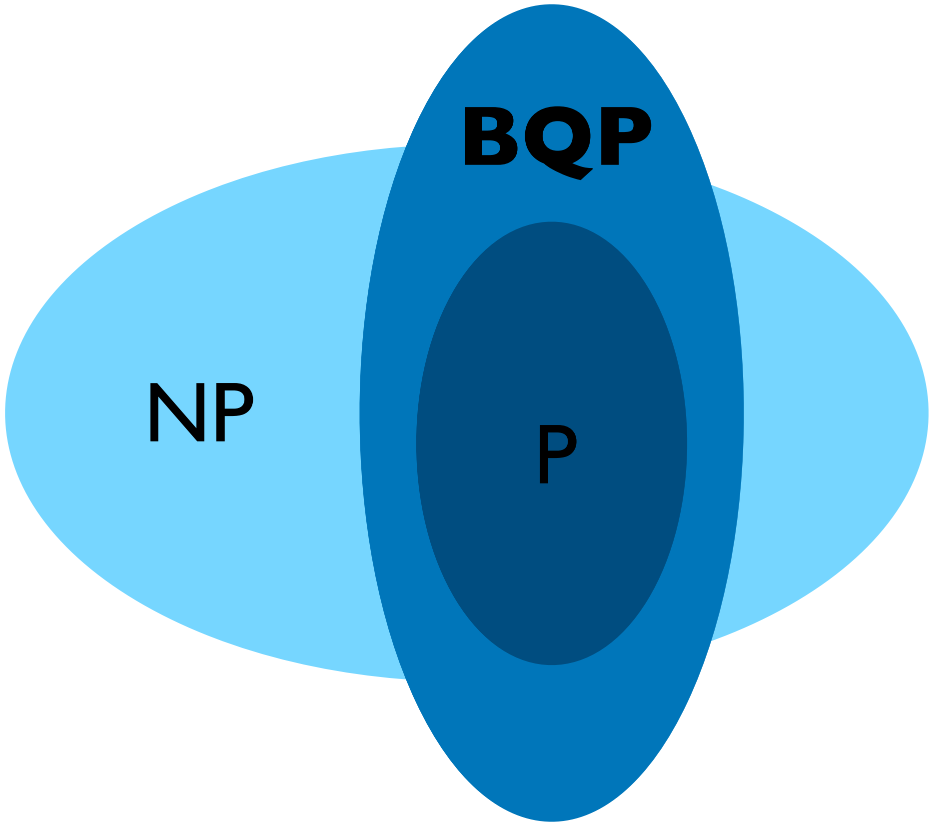
Complexity classes

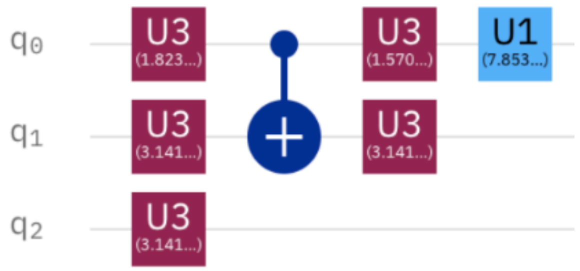


It is expected that quantum computers can solve *some* classically hard problems with exponential speedup

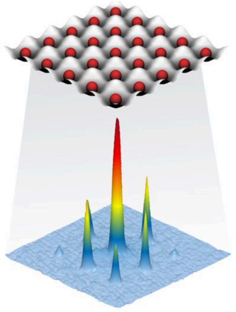
These include a number of highly impactful problems such as quantum simulation

Quantum computing

Digital quantum computers
Universal



Analog quantum computers
Application-specific

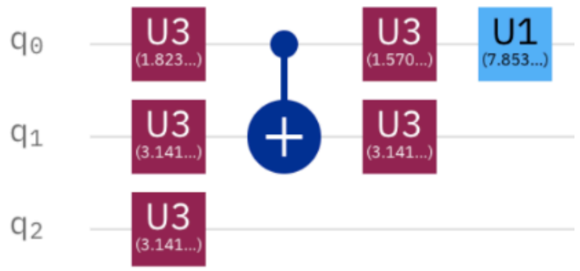


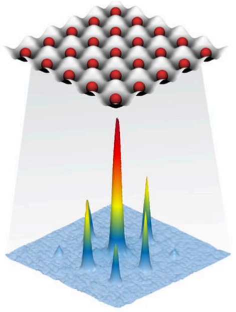
Both will likely be useful in the "near"-term

Quantum computing

Digital quantum computers
Universal

Analog quantum computers
Application-specific





Both will likely be useful in the "near"-term

- **The dream: universal, fault-tolerant digital quantum computer**

  Shor's and Grover's algorithm, quantum error correction

- **Noisy Intermediate Scale Quantum (NISQ) era**

  Decoherence, limited number of qubits, imperfect gates

  Aim: achieve quantum advantage without full quantum error correction

  Experimentation and data analysis

Shor, Preskill, Kitaev, Zoller …

Quantum devices

Superconducting circuits

IBM Q

Google rigetti

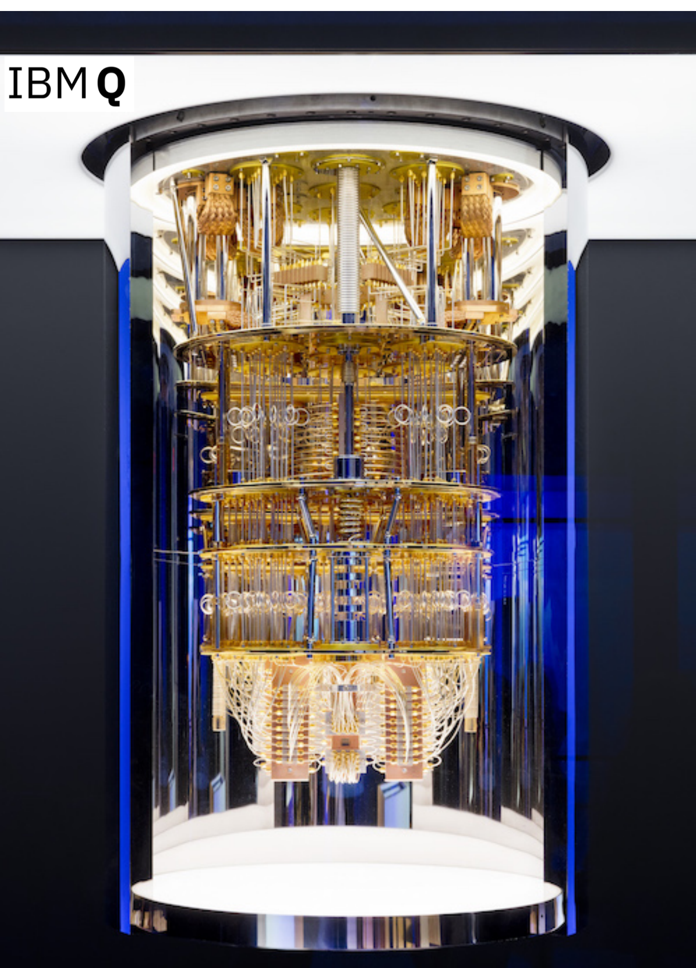
...

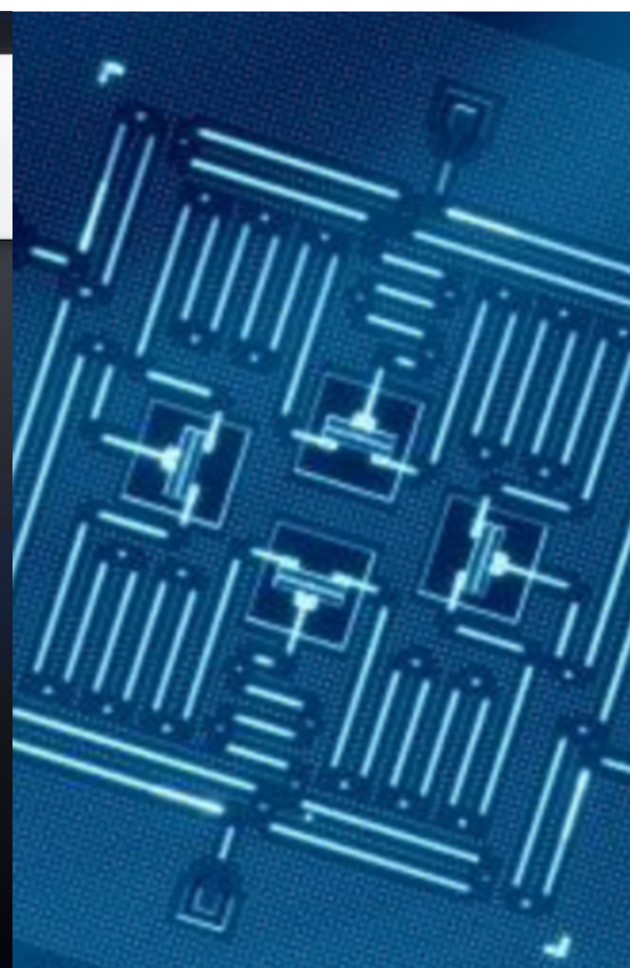
And a variety of others...

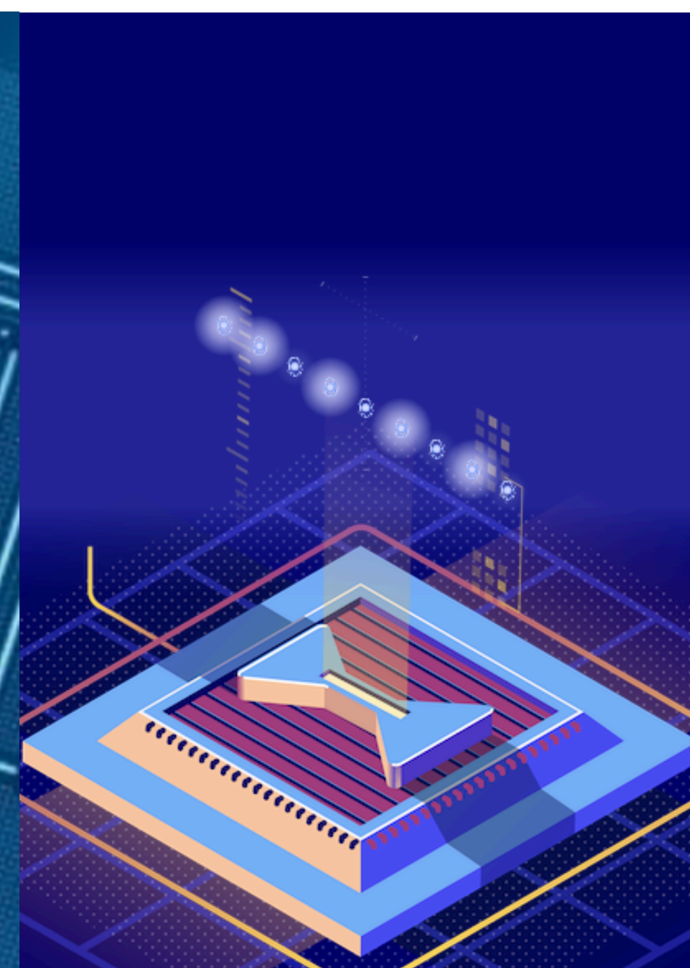
Trapped ions
Optical lattice
Photonics
Topological
...

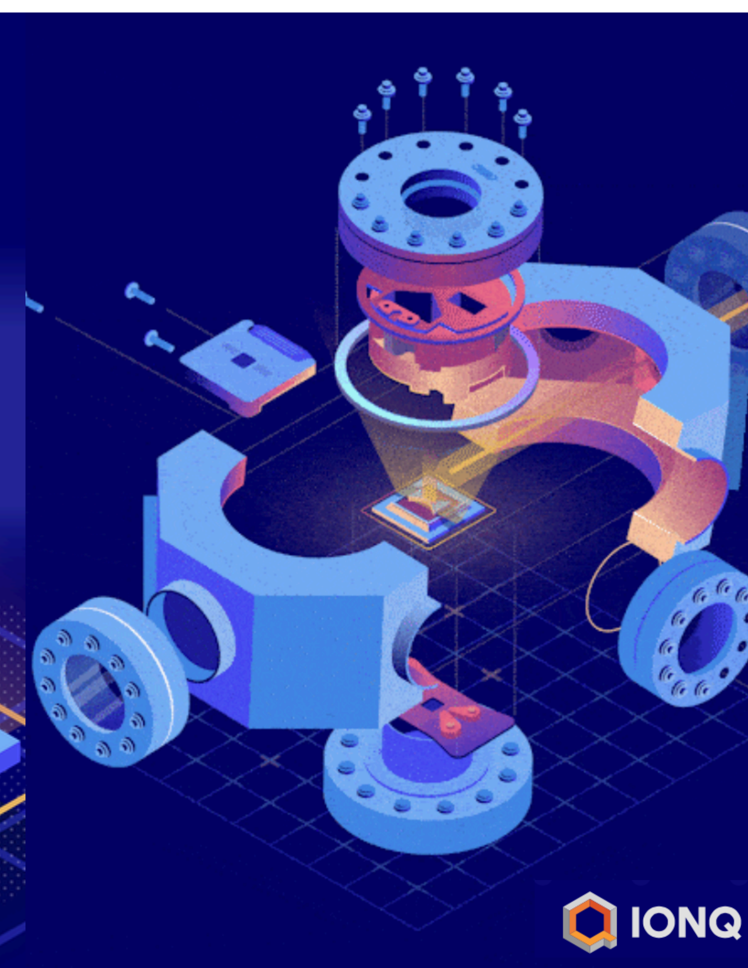
IONQ
Honeywell
Ψ PsiQuantum

...







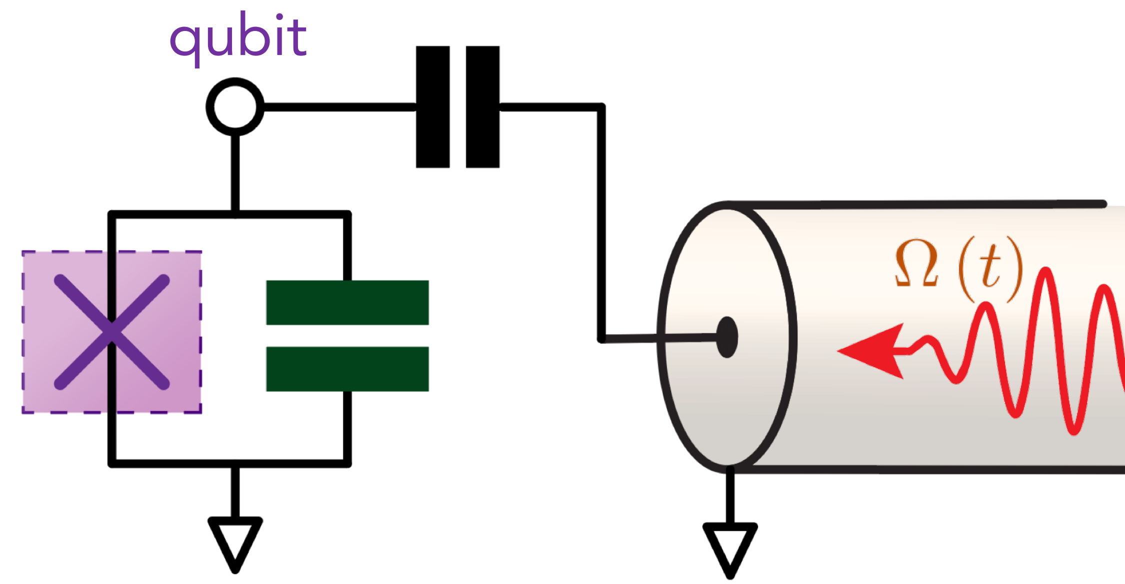


Quantum devices

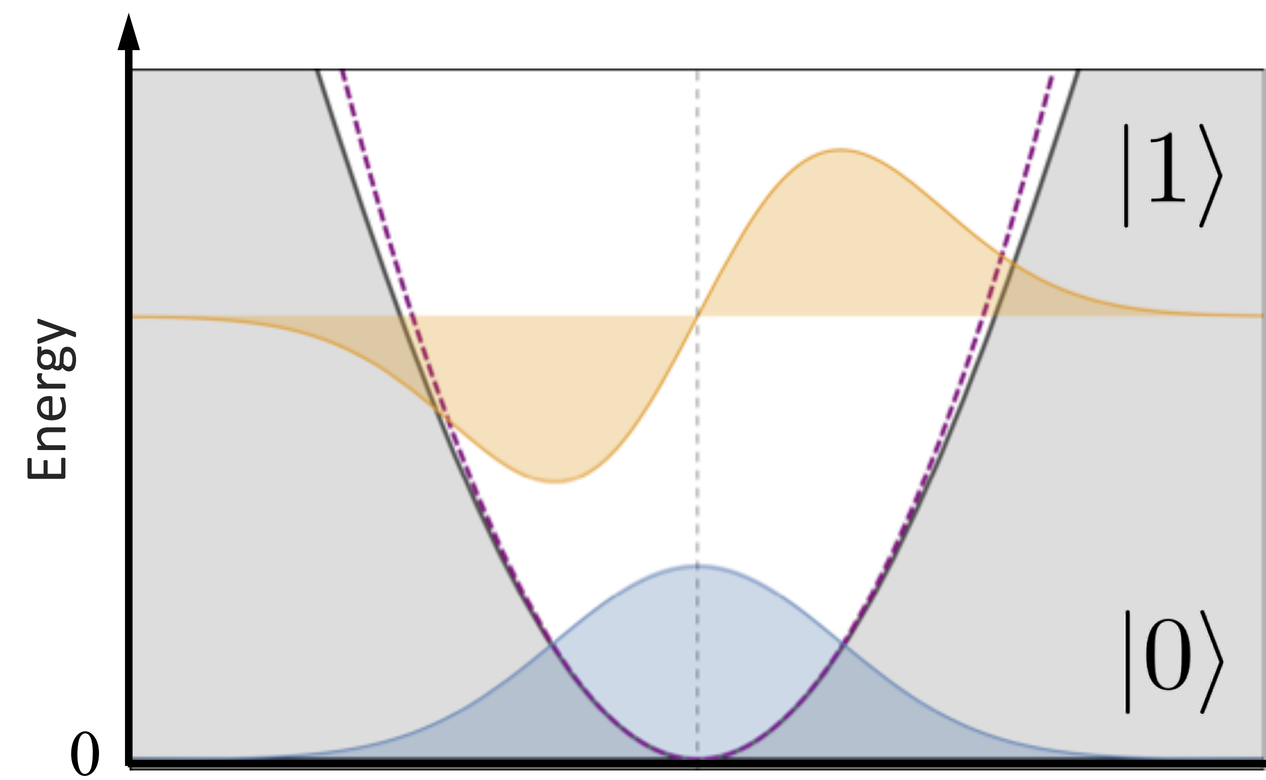
Superconducting circuits

IBM Q

Google rigetti



Qubits: Nonlinear quantum oscillator
Gates: coupled microwave pulses



Zlatko Minev — Qiskit Global Summer School 2020

Quantum devices

Superconducting circuits

IBM Q

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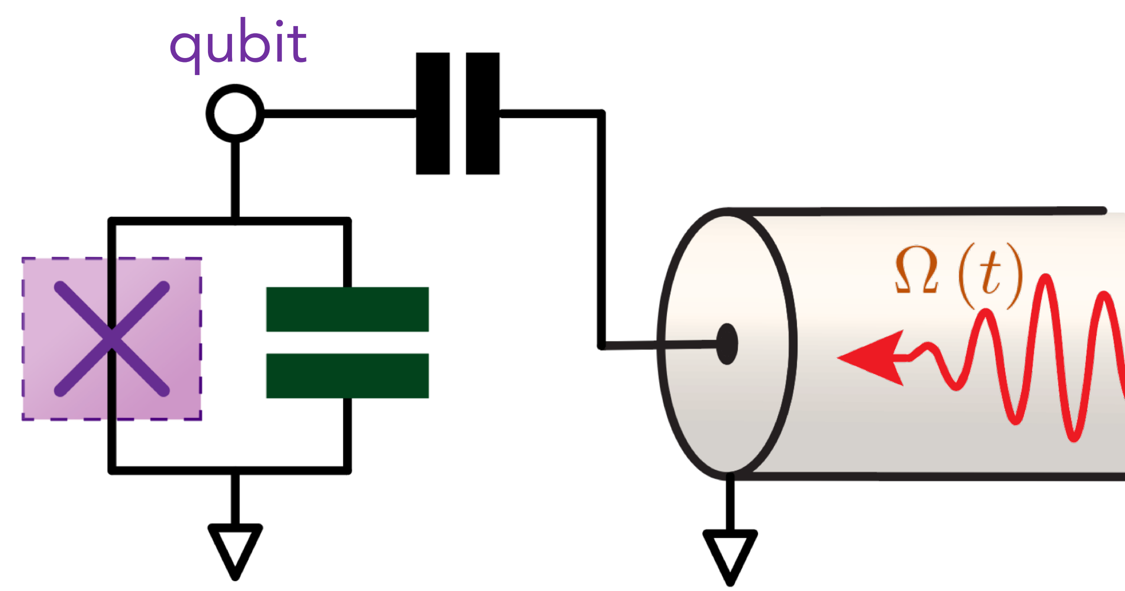
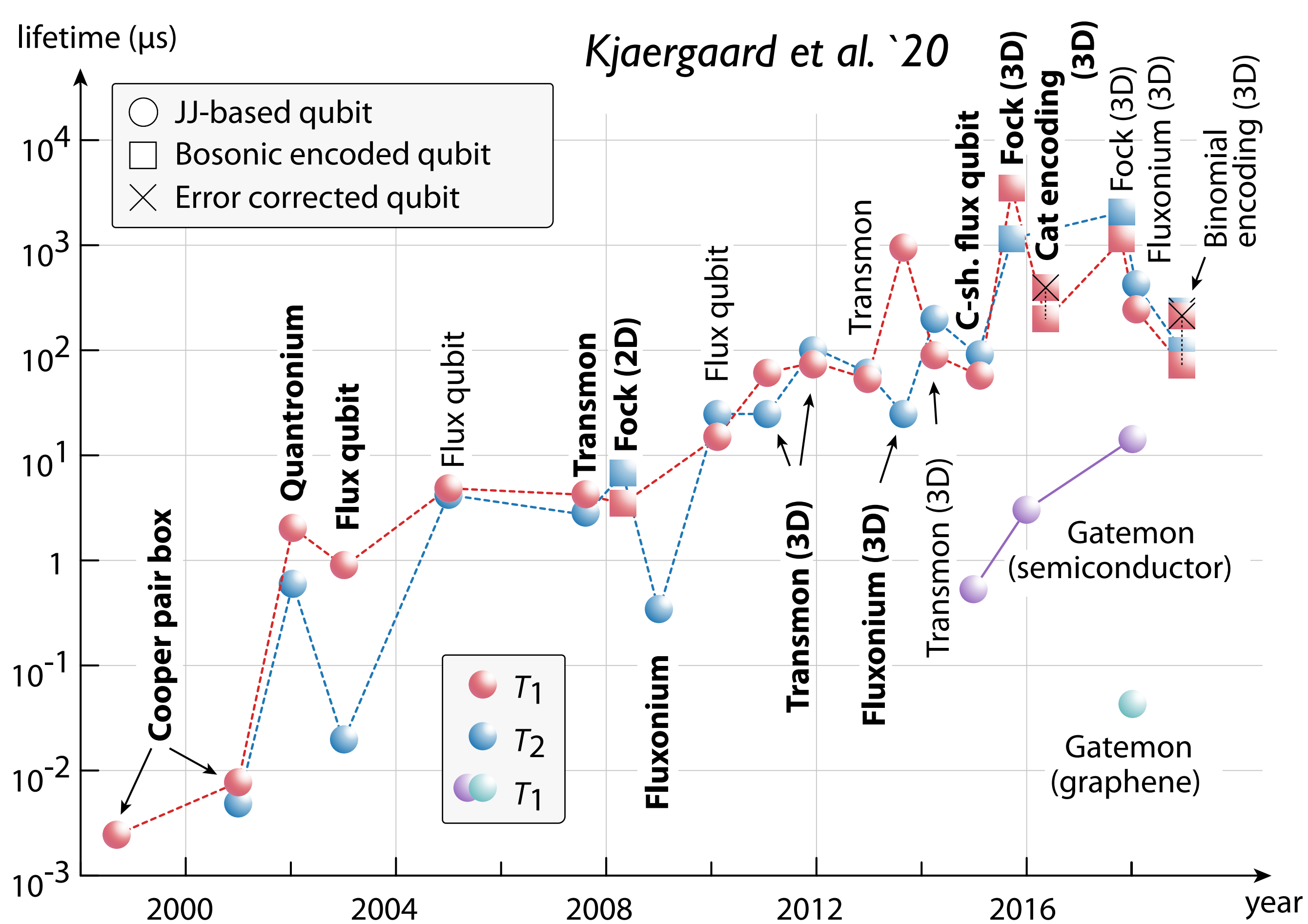


Qubit coherence times have become $\mathcal{O}(100\ \mu s)$, long enough to perform $\mathcal{O}(10-100)$ two-qubit operations



Qubits: Nonlinear quantum oscillator
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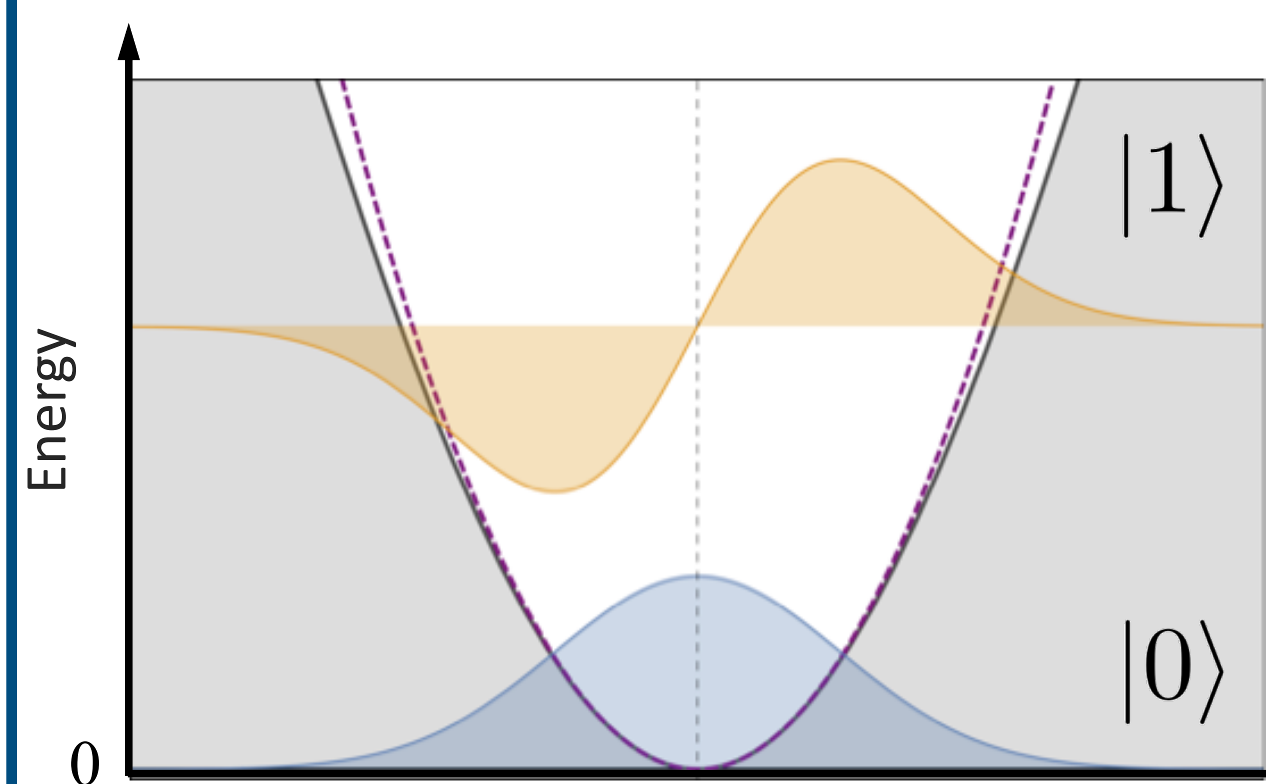
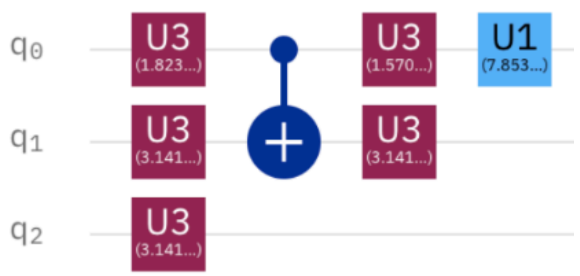
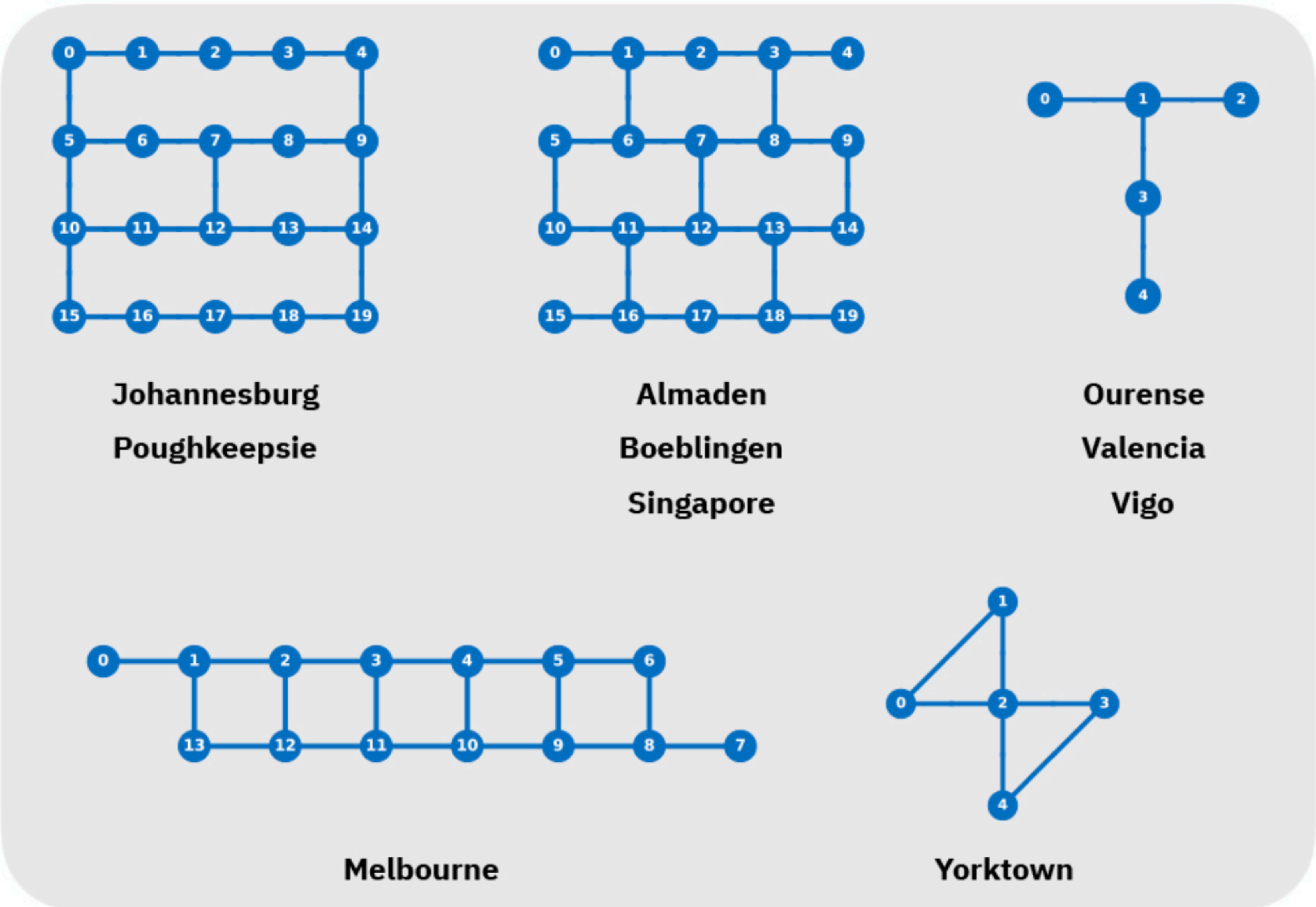


The IBM Q platform

- up to 65 qubits
- single qubit and CNOT



- Qubits connected to at most 3 others
- Take into account decoherence time, gate errors, read-out error …



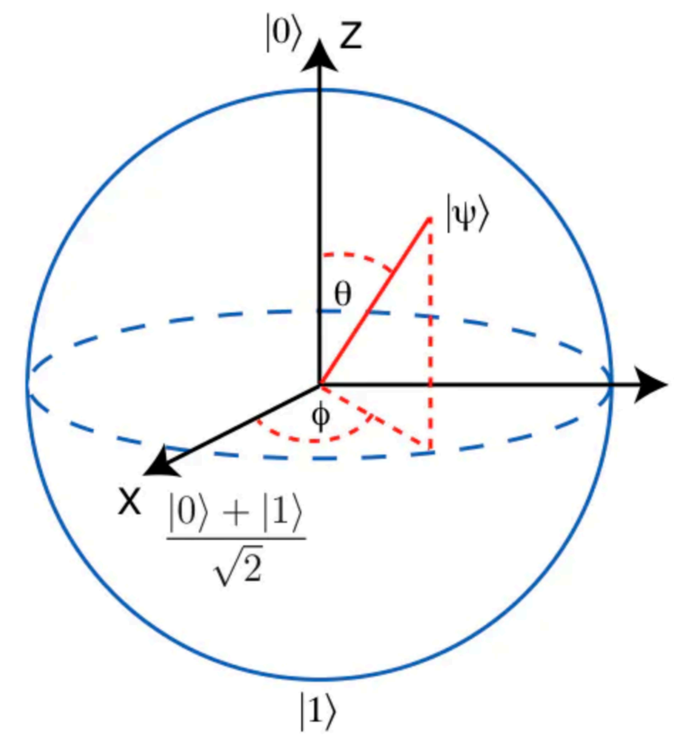
IBM Quantum Hub

OAK RIDGE National Laboratory | LEADERSHIP COMPUTING FACILITY

Single- and two-qubit gates

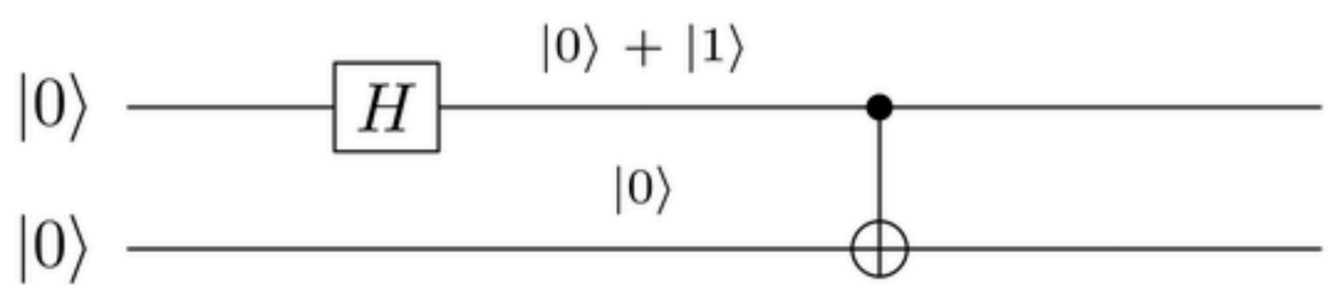
- **Single qubit rotations**

$$U_3(\theta,\phi,\lambda) = \begin{bmatrix} \cos(\theta/2) & -e^{i\lambda}\sin(\theta/2) \\ e^{i\phi}\sin(\theta/2) & e^{i\lambda+i\phi}\cos(\theta/2) \end{bmatrix}$$



$$\frac{|0\rangle + i|1\rangle}{\sqrt{2}} \qquad \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

- **CNOT gate**

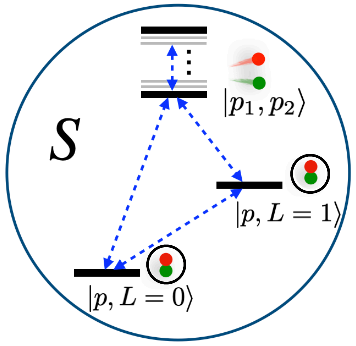


$|00\rangle$ $\quad |00\rangle + |10\rangle \quad$ $|00\rangle + |11\rangle$ → Entangled

→ Complete basis of gates

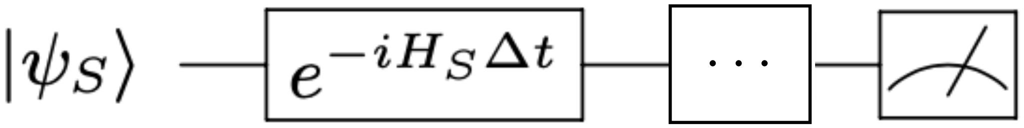
Closed quantum systems

Feynman `81
Lloyd `96



Time evolution of closed systems

- Quantum simulation of the Schrödinger equation



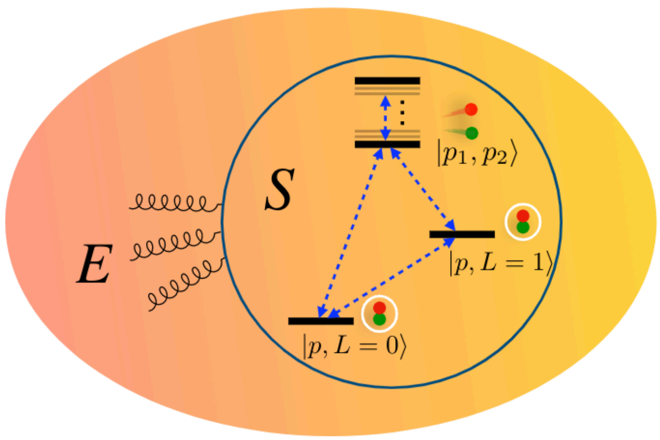
Evolution in time steps $\Delta t = t/N_{\text{cycle}}$

- The evolution is unitary and time reversible

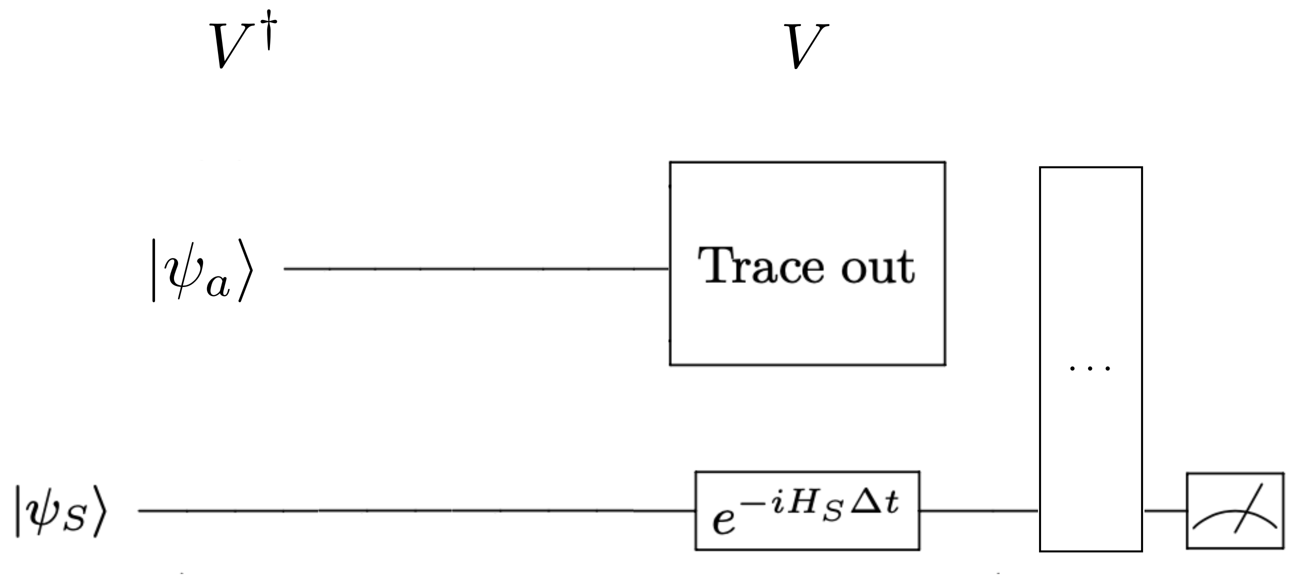
→ For open quantum systems we need to introduce a non-unitarity part

Non-unitarity and time irreversible evolution

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_S = -i\left[H_S, \rho_S\right] + \sum_{j=1}^{m}\left(L_j \rho_S L_j^\dagger - \frac{1}{2}L_j^\dagger L_j \rho_S - \frac{1}{2}\rho_S L_j^\dagger L_j\right)$$



- **The Stinespring dilation theorem**

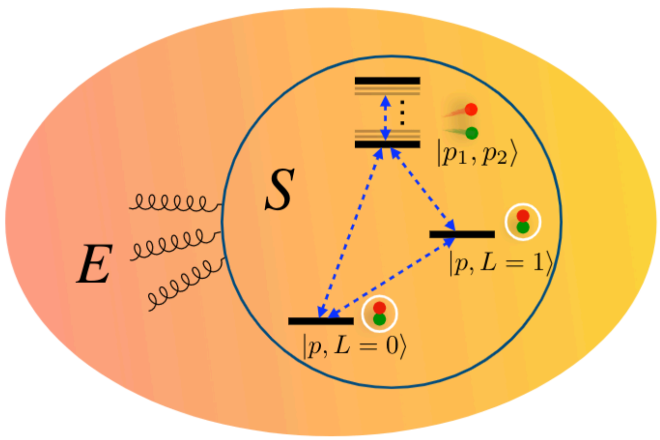


- Introducing and tracing out an ancillary system is not a unitary operation

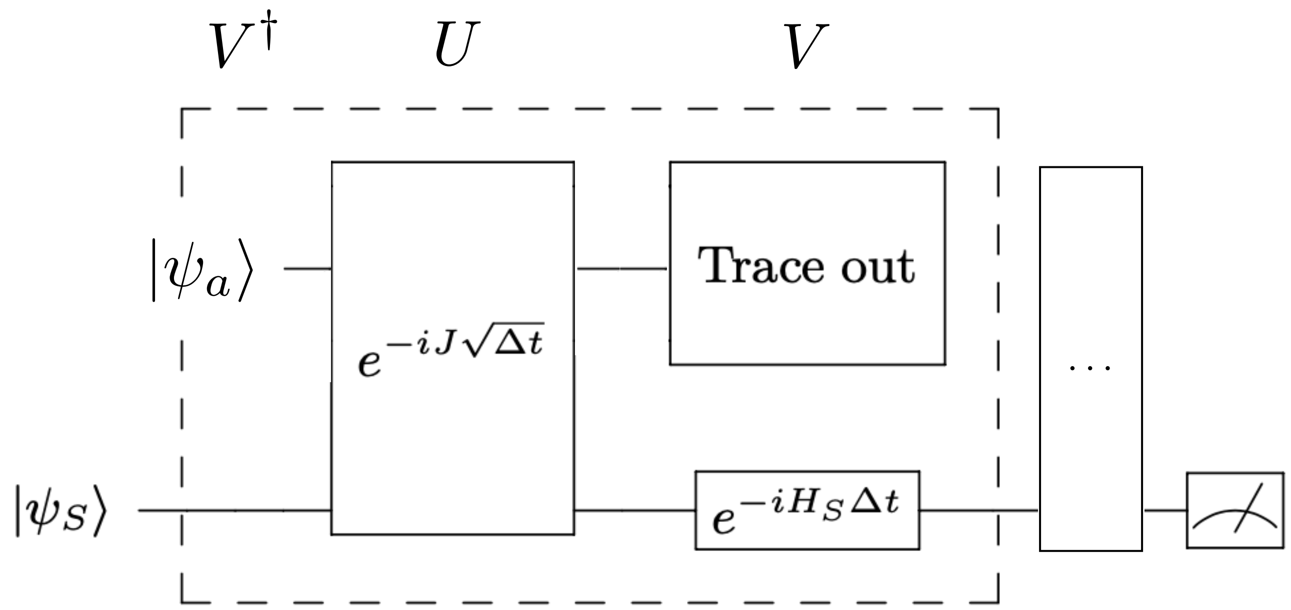
$$V^\dagger V = 1 \qquad V V^\dagger \neq 1$$

Non-unitarity and time irreversible evolution

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_S = -i\left[H_S, \rho_S\right] + \sum_{j=1}^{m}\left(L_j \rho_S L_j^\dagger - \frac{1}{2} L_j^\dagger L_j \rho_S - \frac{1}{2}\rho_S L_j^\dagger L_j\right)$$



- **The Stinespring dilation theorem**



- Introducing and tracing out an ancillary system is not a unitary operation

$$V^\dagger V = 1 \qquad V V^\dagger \neq 1$$

- Sandwich in between a unitary evolution step
- Evolve in time steps $\Delta t = t/N_{\mathrm{cycle}}$

$$J = \begin{pmatrix} 0 & L_1^\dagger & \dots & L_m^\dagger \\ L_1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ L_m & 0 & \dots & 0 \end{pmatrix}$$

Quantum simulation of open quantum systems

Toy model setup

Two-level system in a thermal environment

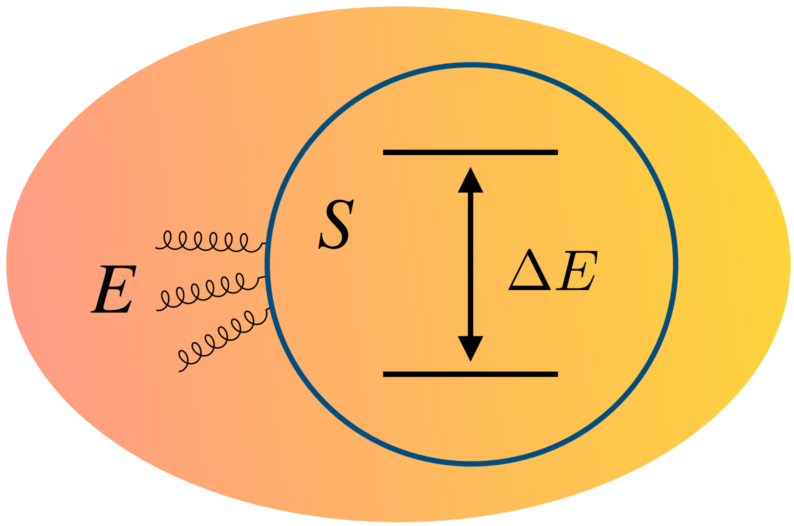
e.g. bound/unbound $J/\psi,\ c\bar{c}$

$$H_S = -\frac{\Delta E}{2} Z$$

$$H_E = \int \mathrm{d}^3x \left[\frac{1}{2}\Pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{4!}\lambda\phi^4\right]$$

$$H_I = gX \otimes \phi(x=0)$$

Pauli matrices $X, Y, Z,$ interaction strength g



$$\rho(0) = \rho_S(0) \otimes \rho_E$$

$$\rho_E = \frac{e^{-\beta H_E}}{\mathrm{Tr}(e^{-\beta H_E})}$$

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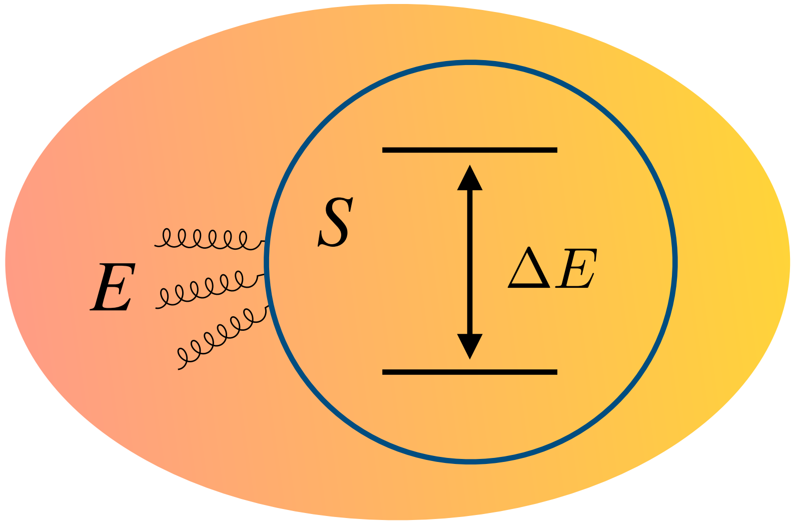

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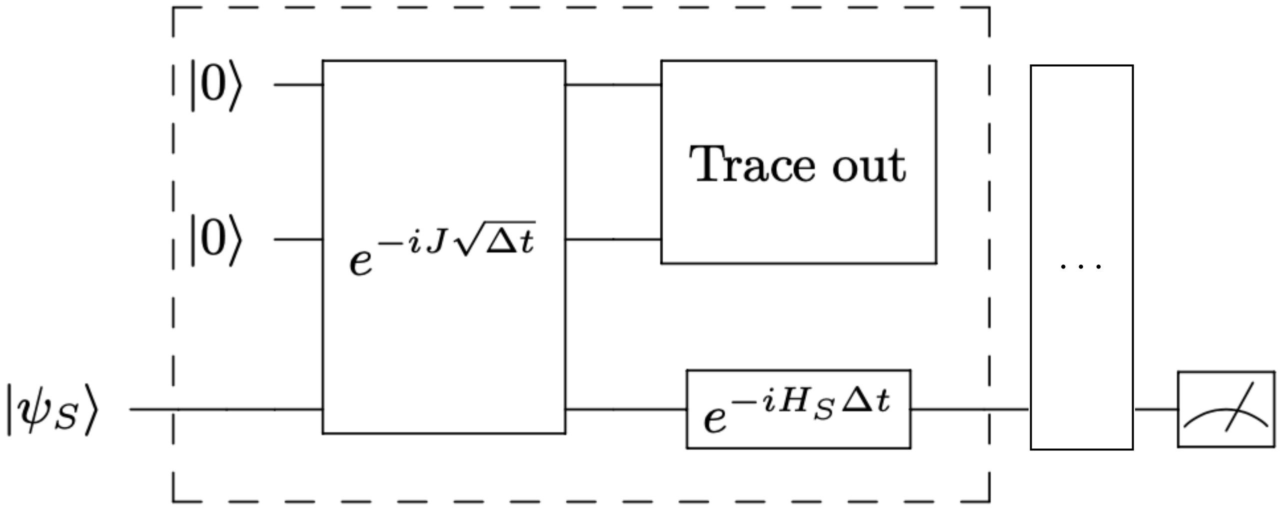
Lindblad operators

$$L_j \sim g(X \mp iY) \qquad j = 0, 1$$

$$J = \begin{pmatrix} 0 & L_0^\dagger & L_1^\dagger & 0 \\ L_0 & 0 & 0 & 0 \\ L_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

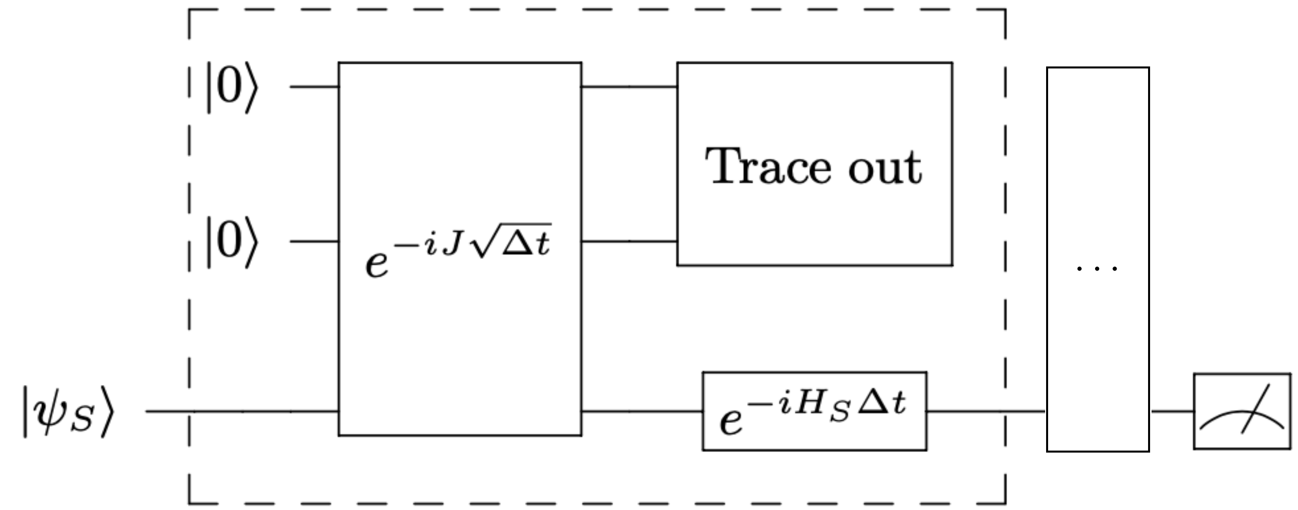


Quantum circuit synthesis

Approximate unitary operations with a compiled circuit of one- and two-qubit gates

Optimization problem w/unitary loss function

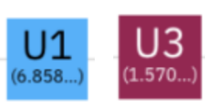
`qsearch` *Siddiqi et al. `20*



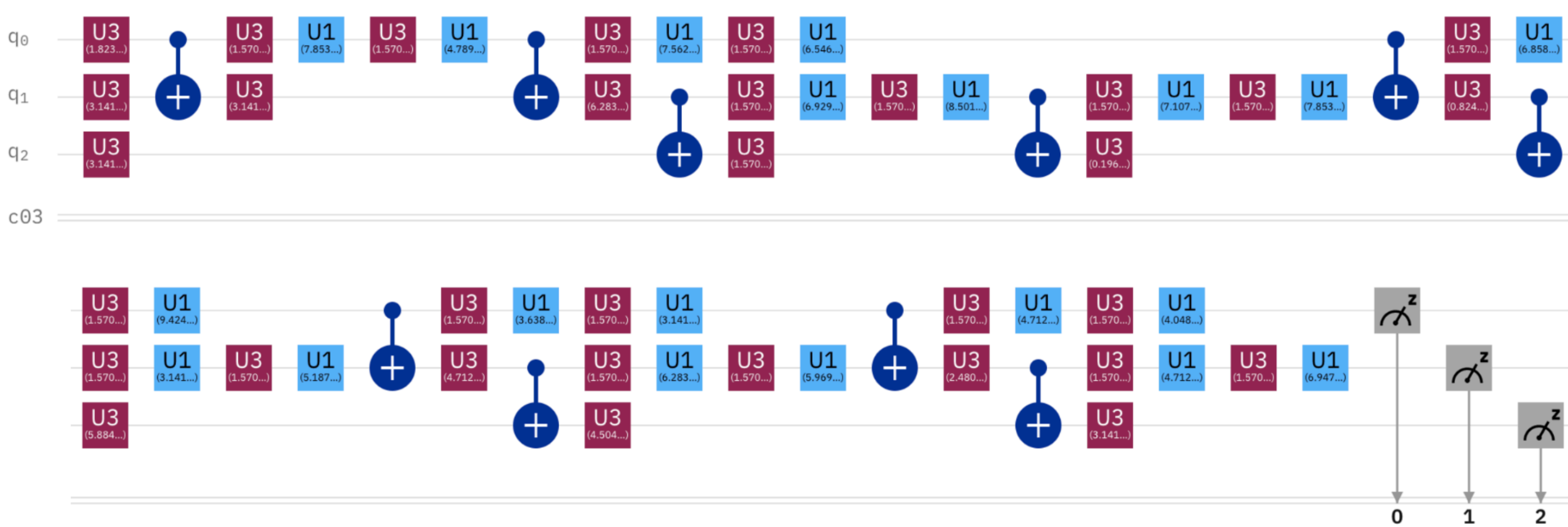
10 CNOT gates/cycle

IBM **Q**

Single qubit



CNOT



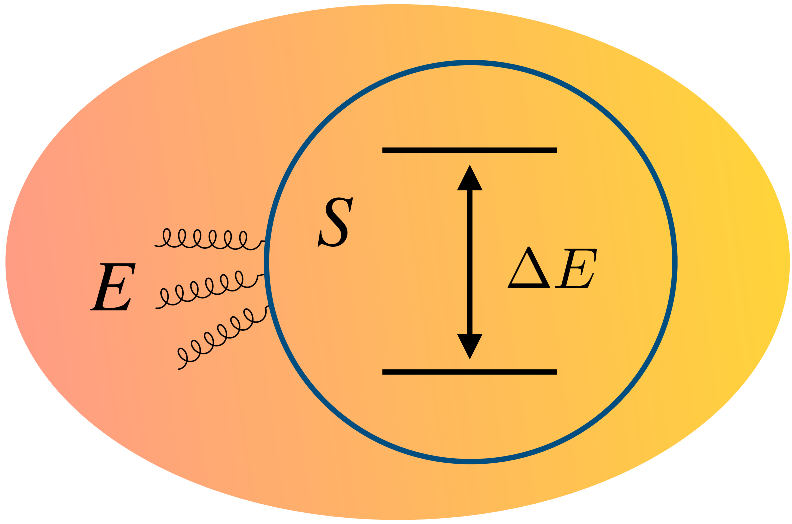
Quantum simulation of open quantum systems

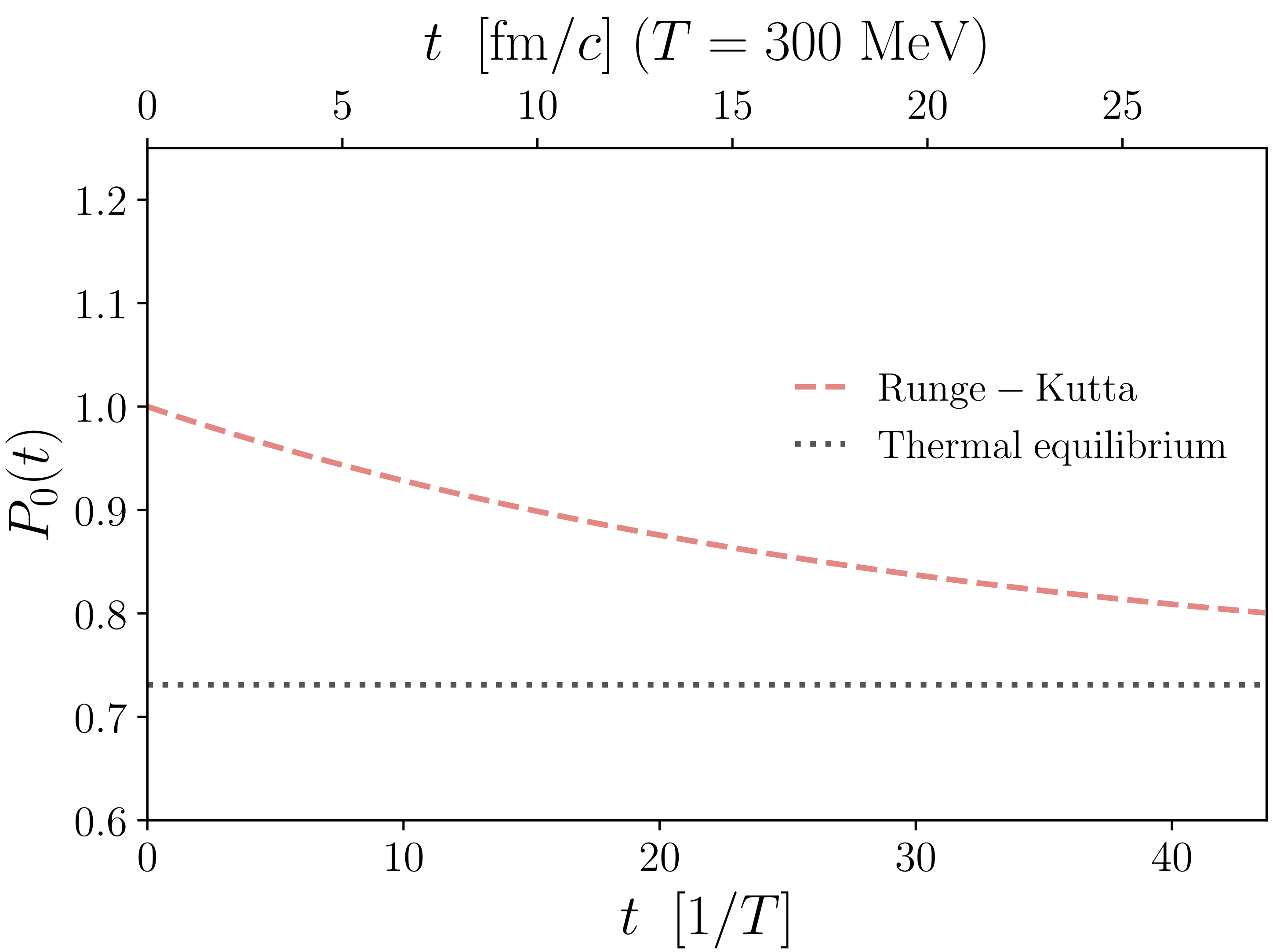
arXiv: 2010.03571

Real-time evolution

$P_0(t)$ describes fraction that remains in "bound state"

Similar to *t*-dependent $R_{AA} = \frac{\mathrm{d}\sigma_{AA}}{\langle N_{\rm coll}\rangle\, \mathrm{d}\sigma_{pp}}$





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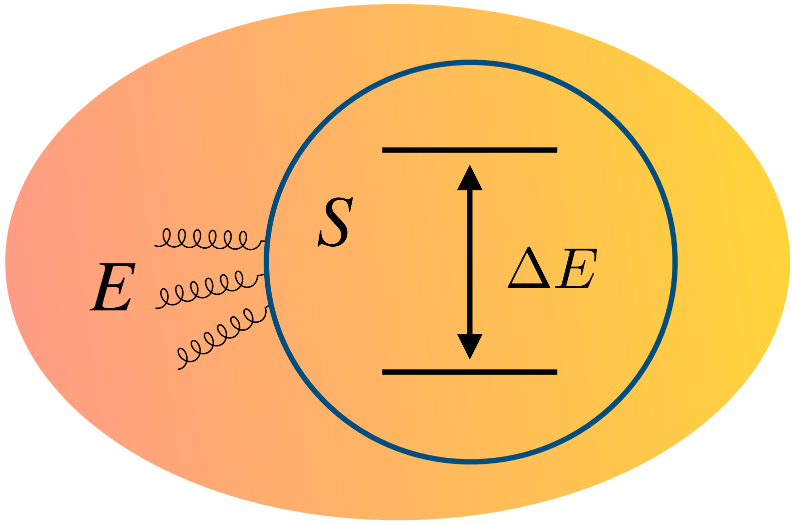

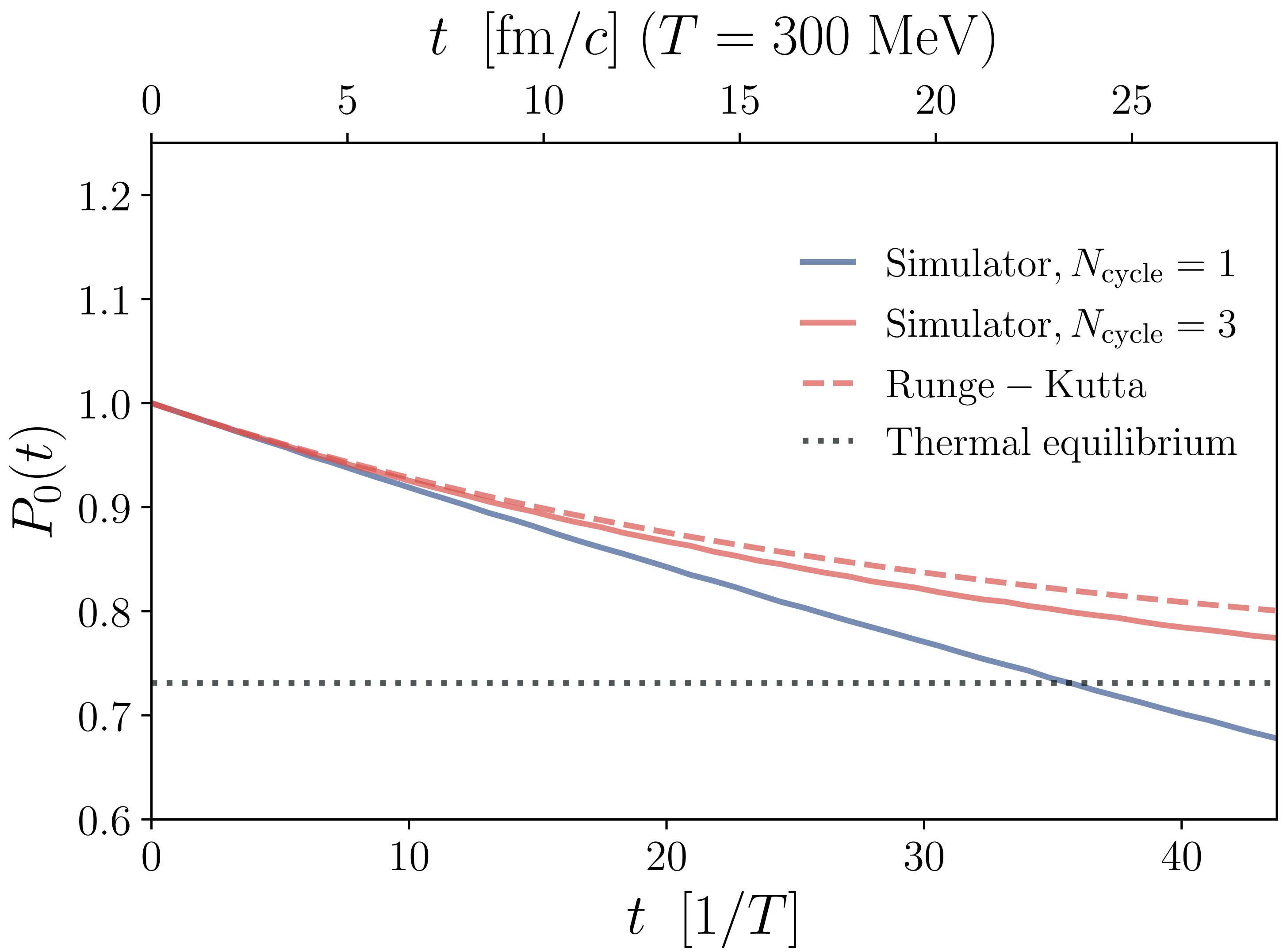

The algorithm converges to Lindblad evolution with a small number of cycles

Quantum simulation of open quantum systems

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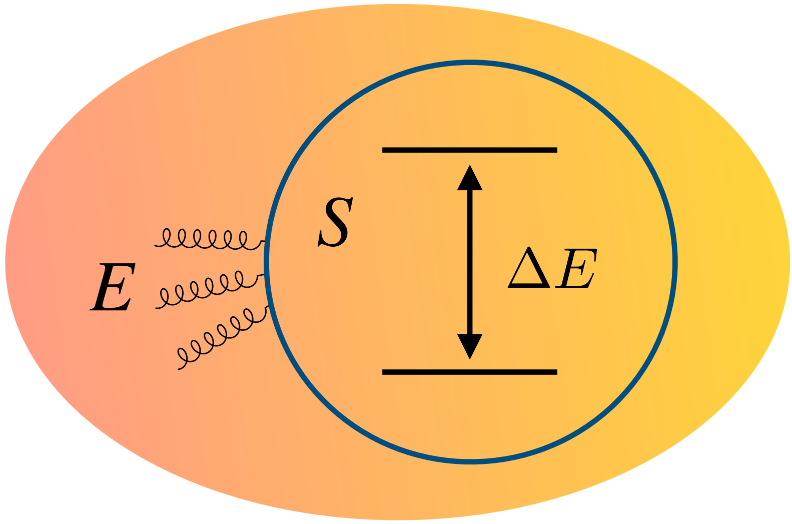
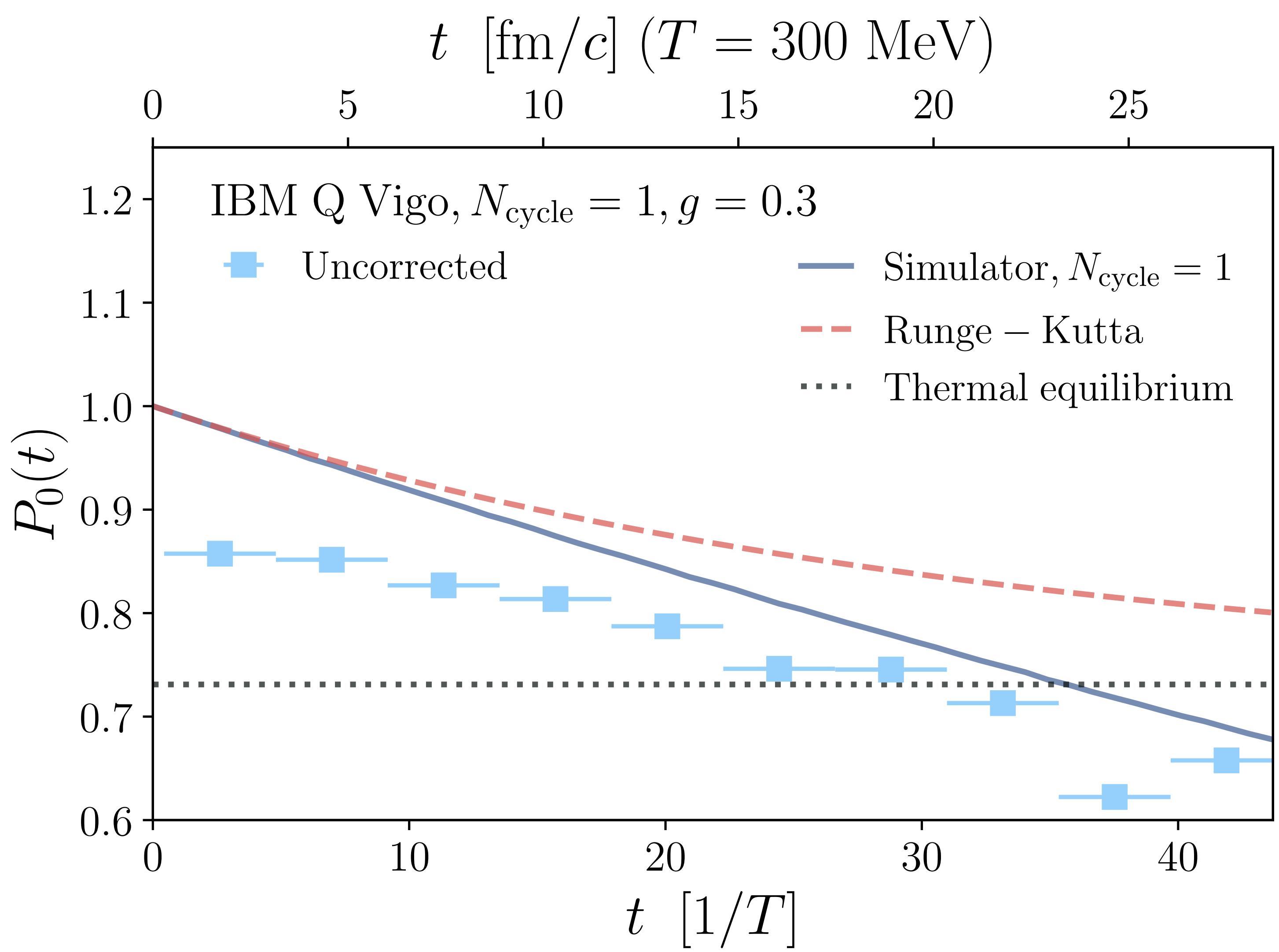
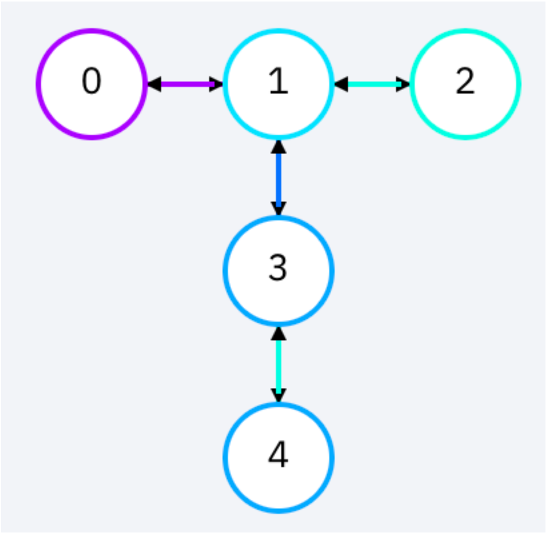




ibmq_vigo device



Error mitigation

Readout error

Constrained matrix inversion

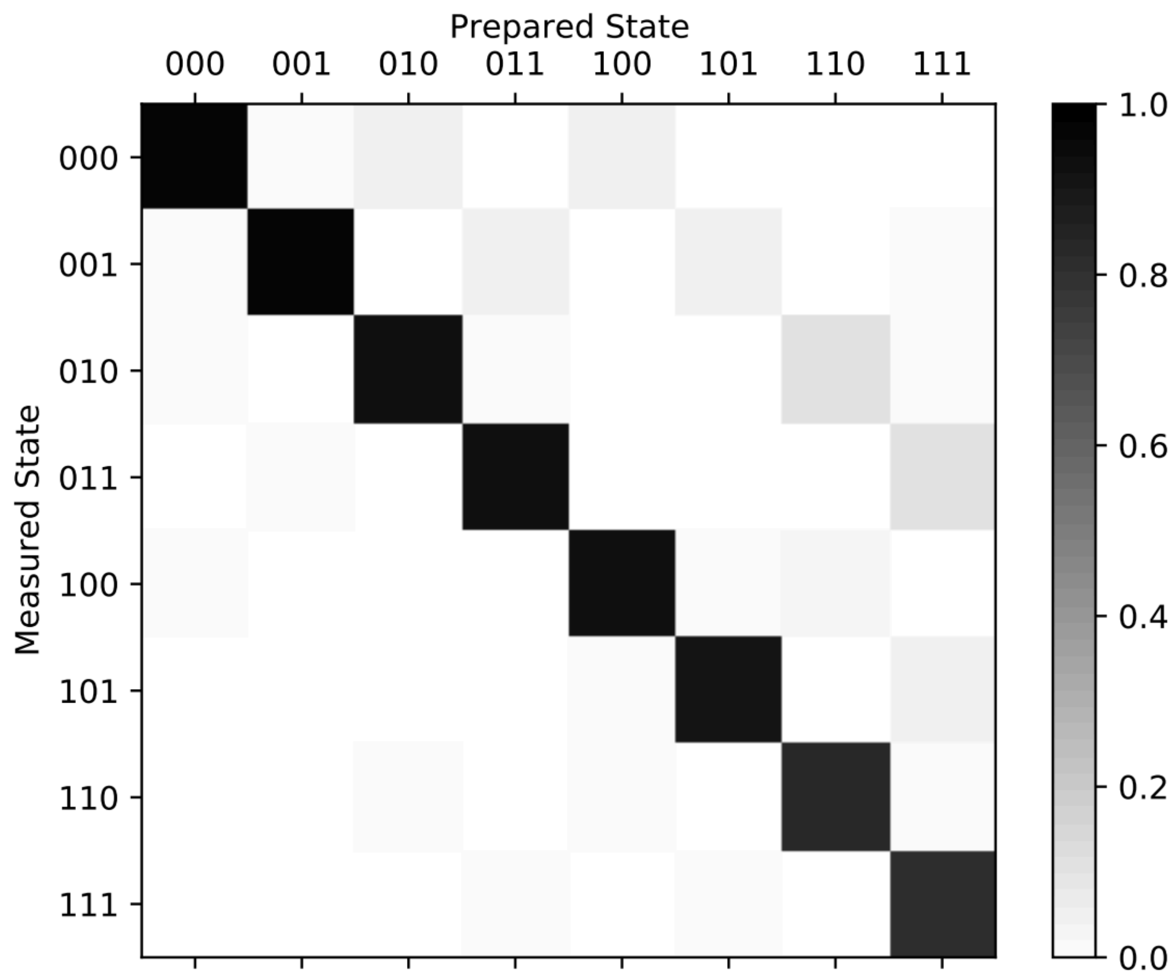
IBM Q qiskit-ignis

Unfolding

Nachman, Urbanek, de Jong, Bauer `19

Prepare states by applying bit-flip X gates and read out

ibmq_vigo device



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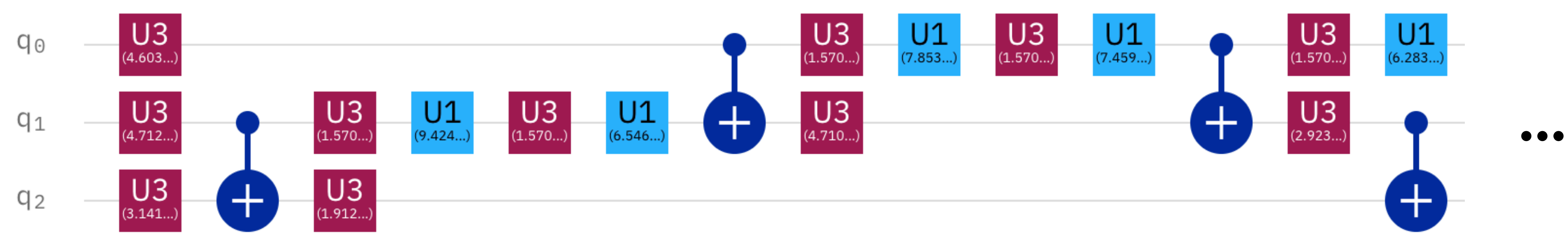
Gate error

Zero-noise extrapolation of CNOT noise using Random Identity Insertions

He, Nachman, de Jong, Bauer `20

Circuit 1



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Constrained matrix inversion — IBM Q `qiskit-ignis`

Unfolding — *Nachman, Urbanek, de Jong, Bauer `19*

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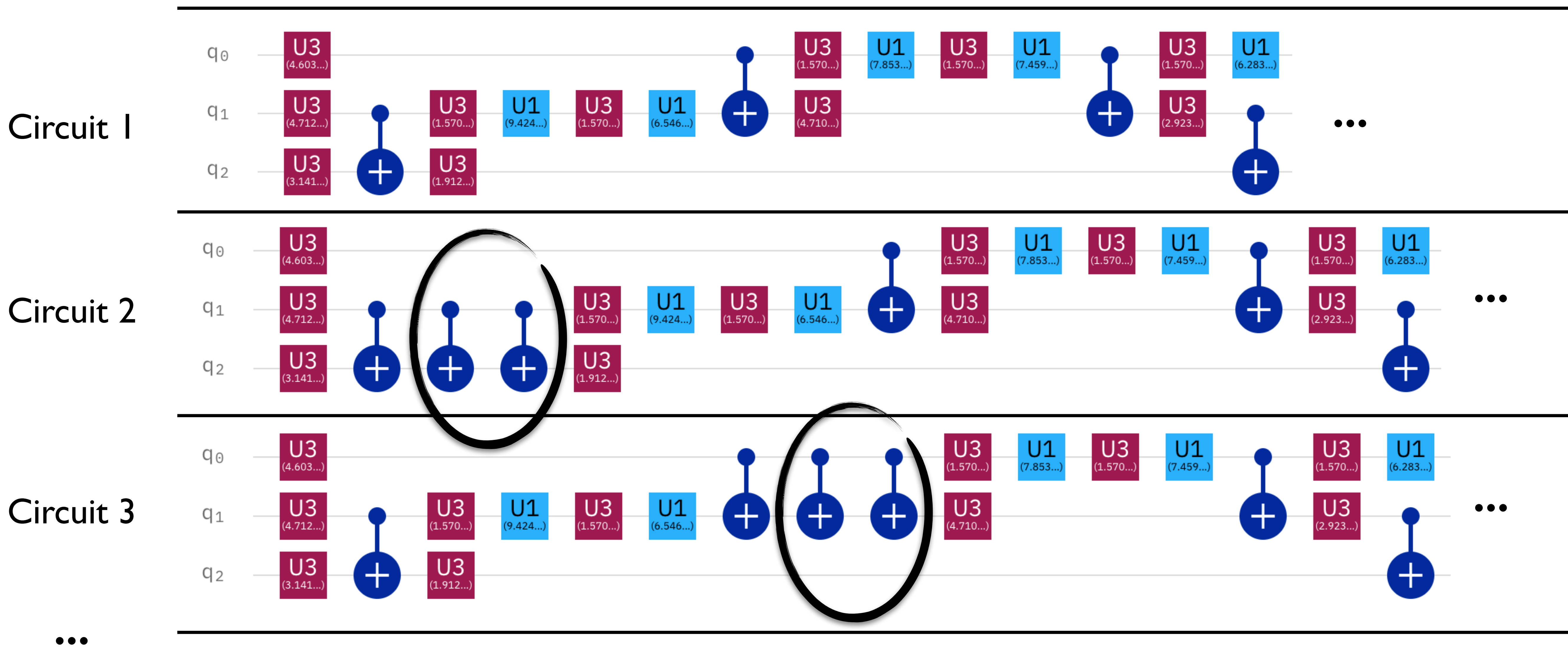

Quantum simulation of open quantum systems

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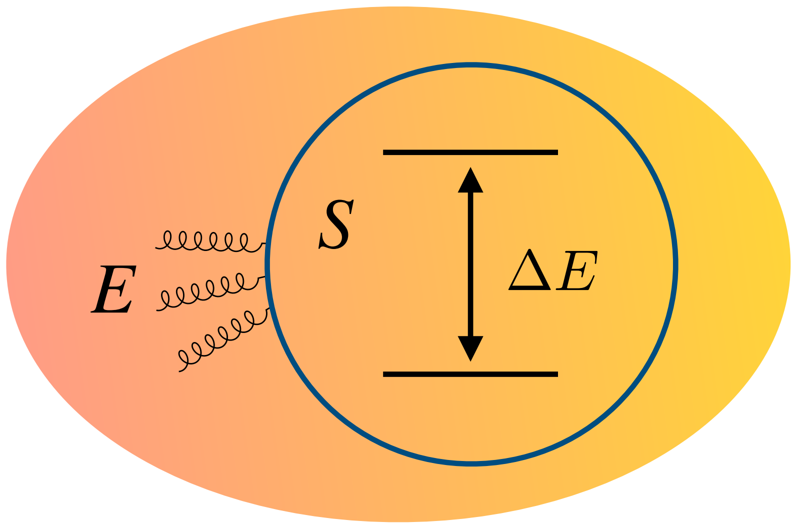
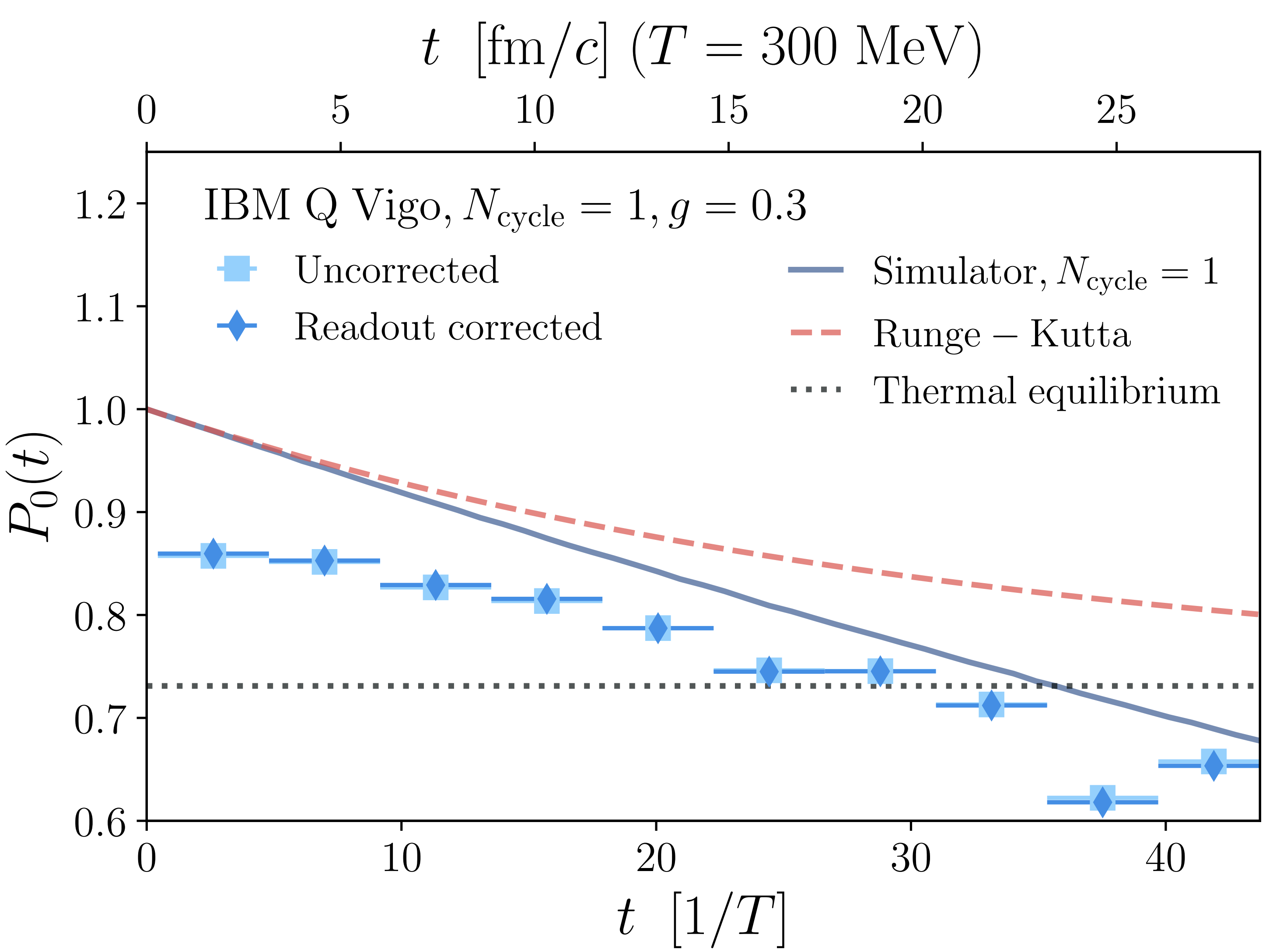
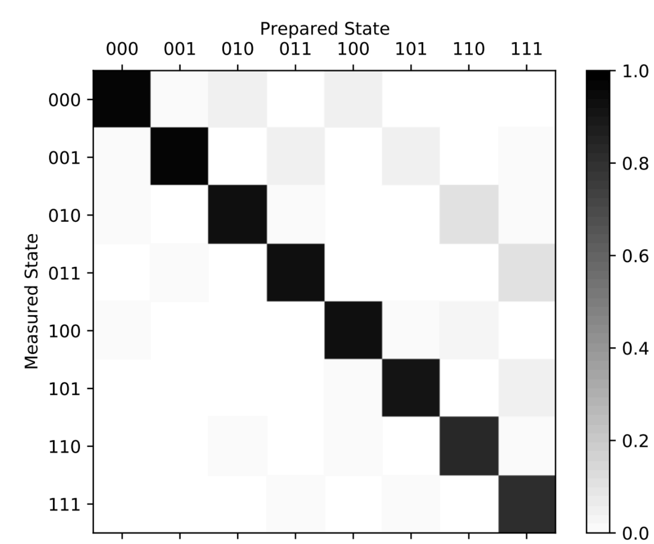




ibmq_vigo device

Readout correction small



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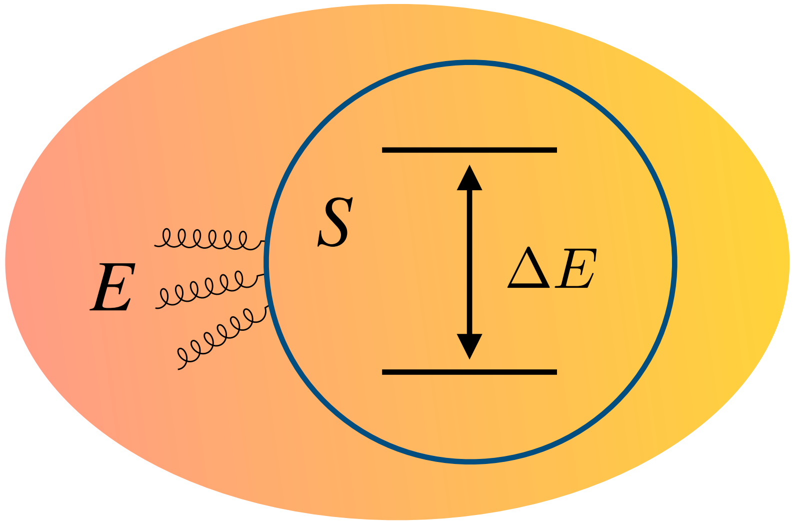

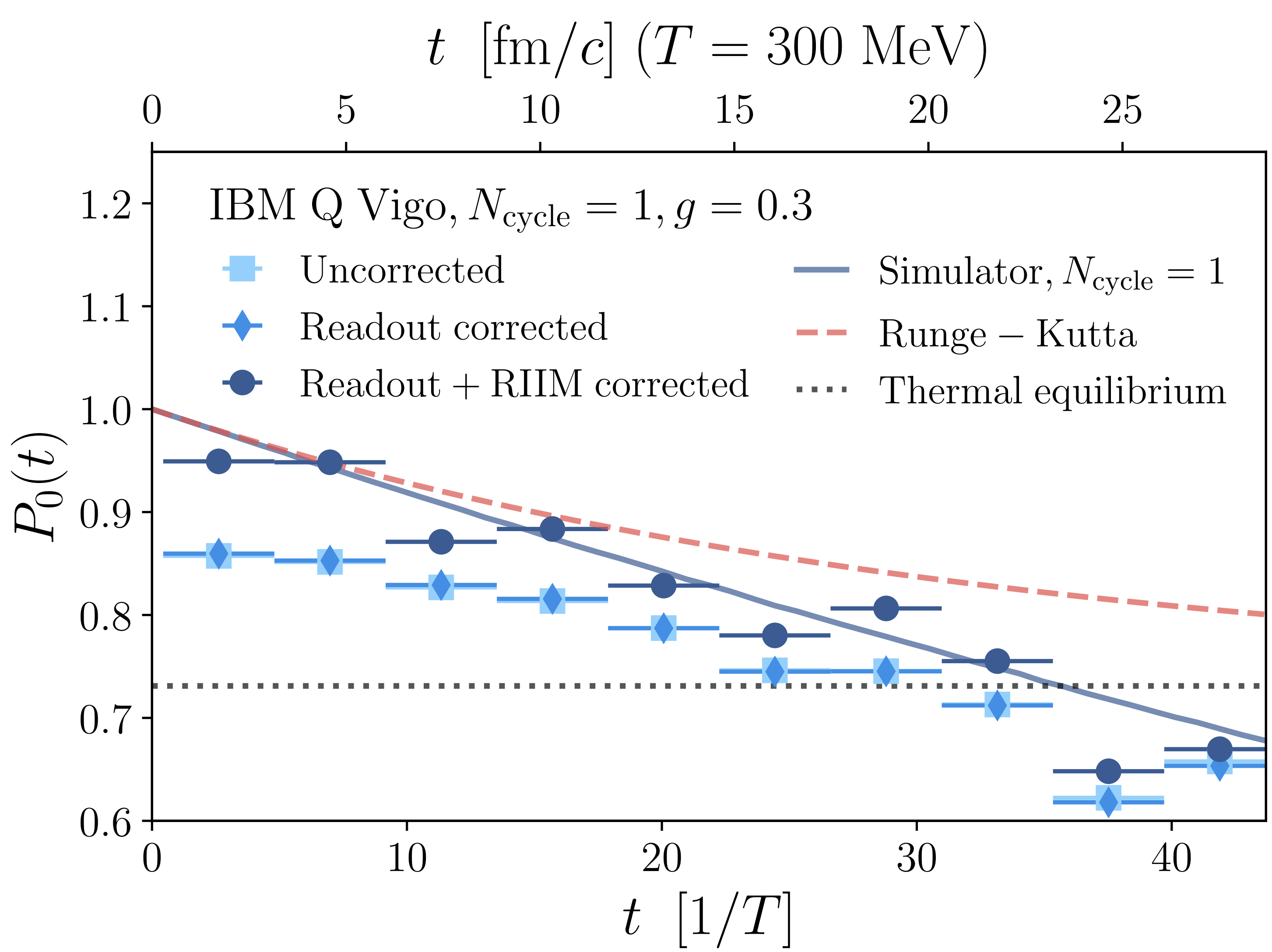

ibmq_vigo device

Readout correction small

CNOT gate error correction gives good agreement

Random Identity Insertion Method (RIIM)

Bauer, He, de Jong, Nachman `20

Proof of concept

Outline

Open quantum systems in heavy-ion collisions

Quantum simulation with IBM Q

Conclusions and outlook

- **Open quantum system formalism describes the real-time evolution of hard probes in heavy-ion collisions**
    - Allows to go beyond semiclassical approximations in current models
- **Proof of concept that these systems can be simulated on current and near-term quantum computers (IBM Q)**
    - NISQ era digital quantum computing
    - Recently developed error mitigation techniques
- **Future steps**
    - More efficient quantum algorithms & error mitigation
    - Extension toward QCD