The Ohio State University

Center for Frontiers in Nuclear Science

The role of the axial anomaly in polarized deep inelastic scattering

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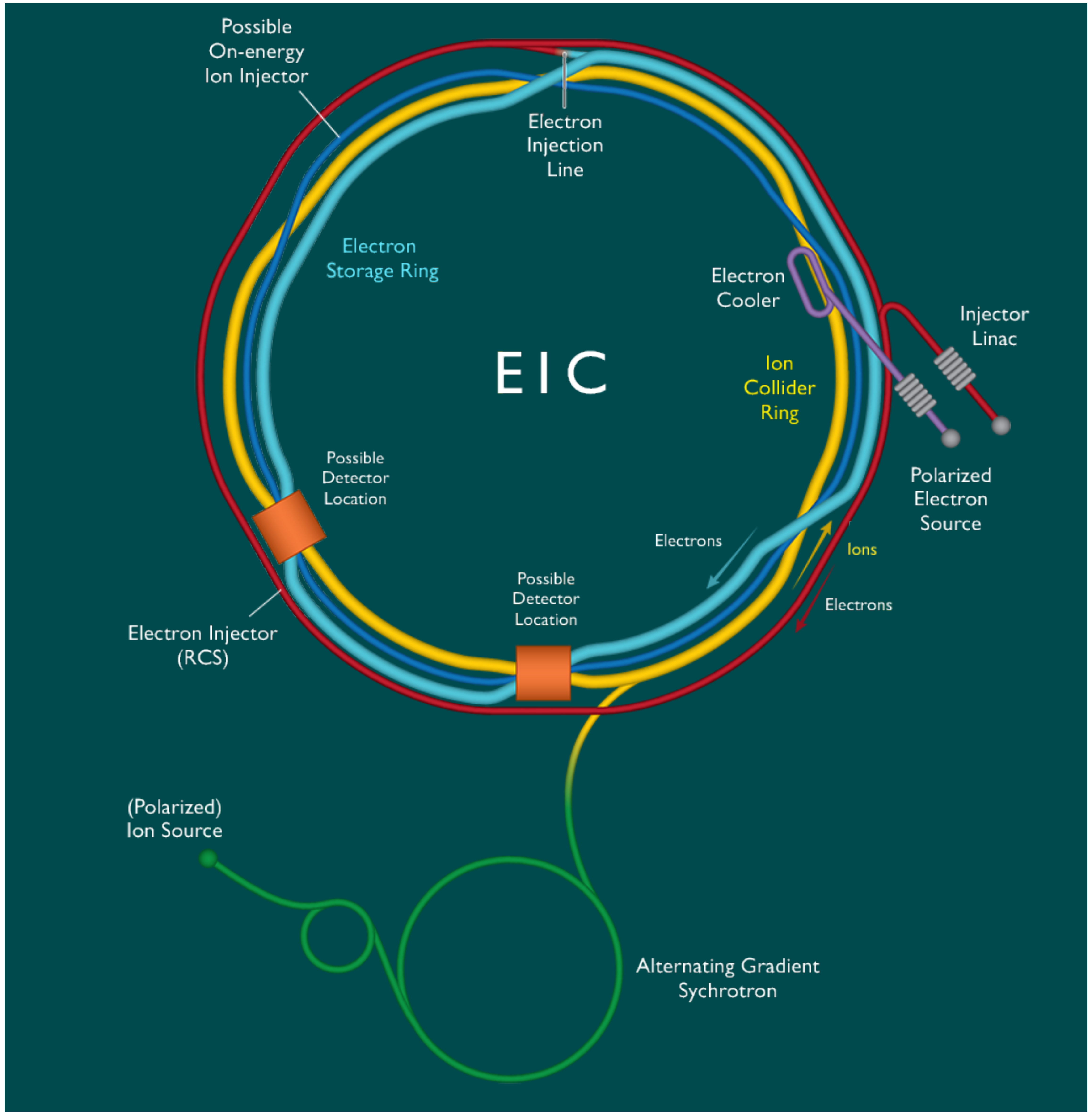
Based on Andrey Tarasov and Raju Venugopalan arXiv:2008.08104

Theory seminar at JLab, October 19th, 2020

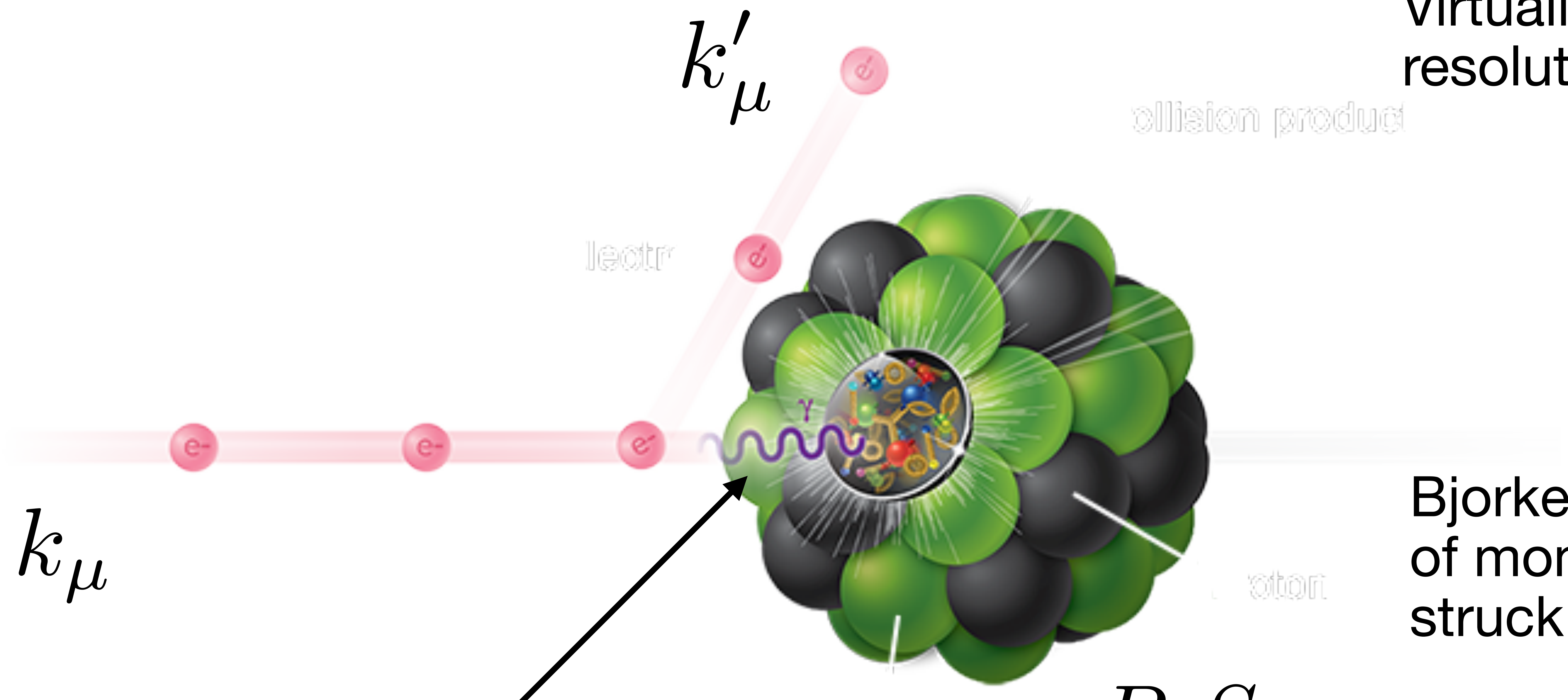
Electron-Ion Collider (EIC)

- Polarized protons up to 275 GeV
- Nuclei up to Z/A*275 GeV
- Electrons up to 18 GeV
- Sqrt(s) = 20-140 GeV
- Luminosity 10^{34} $cm^{-2}sec^{-1}$
- First polarized electron - polarized proton collider
- First electron-nucleus collider



https://www.bnl.gov/eic/

Deep inelastic scattering



$$q_\mu = k_\mu - k'_\mu$$

Virtuality: measure of resolution power

$$Q^2 = -q^2$$

$$\Delta x \sim \frac{1}{Q}$$

Bjorken variable: measure of momentum fraction of a struck parton

$$x_B = \frac{Q^2}{2P \cdot q}$$

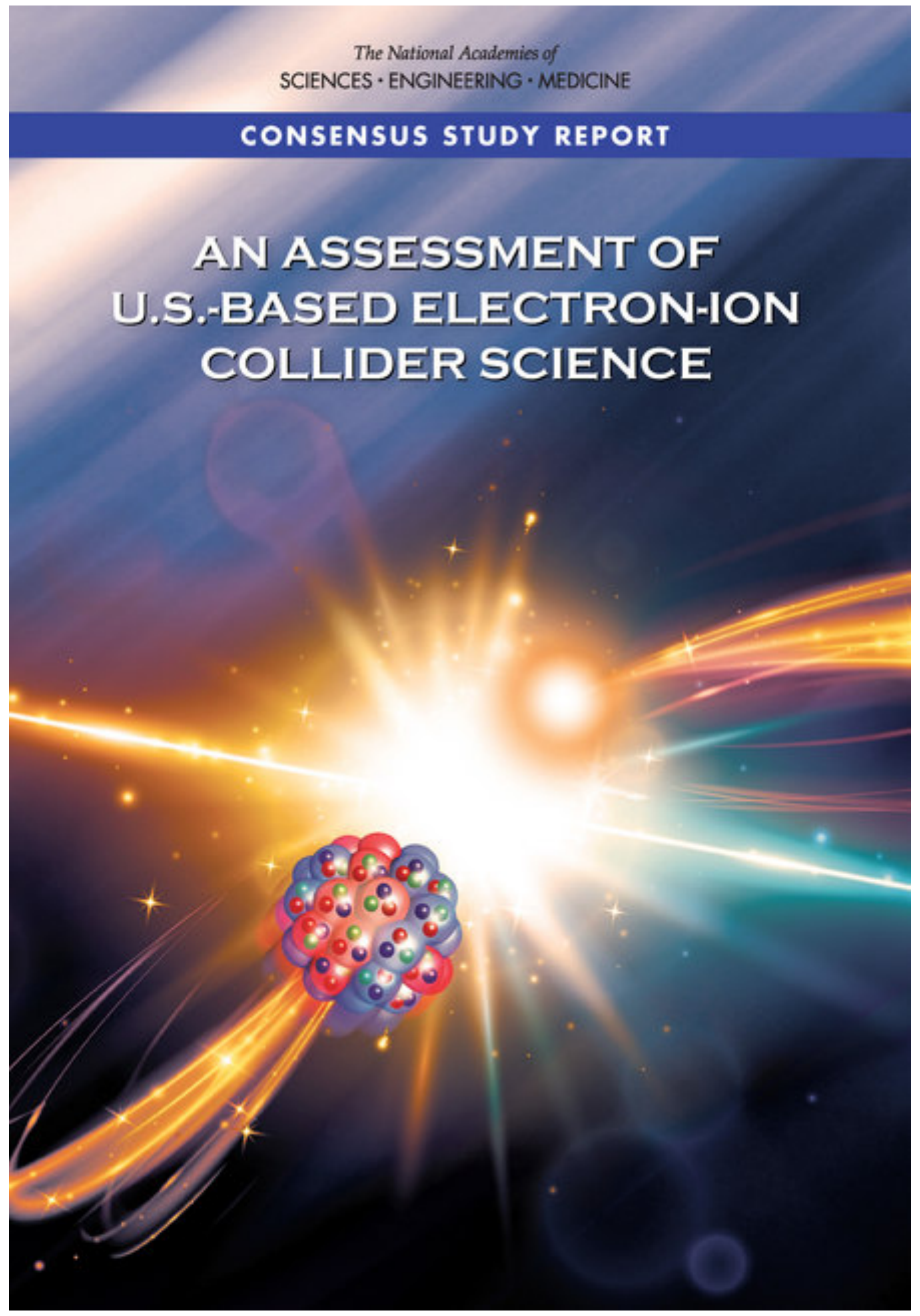
https://www.bnl.gov/eic/

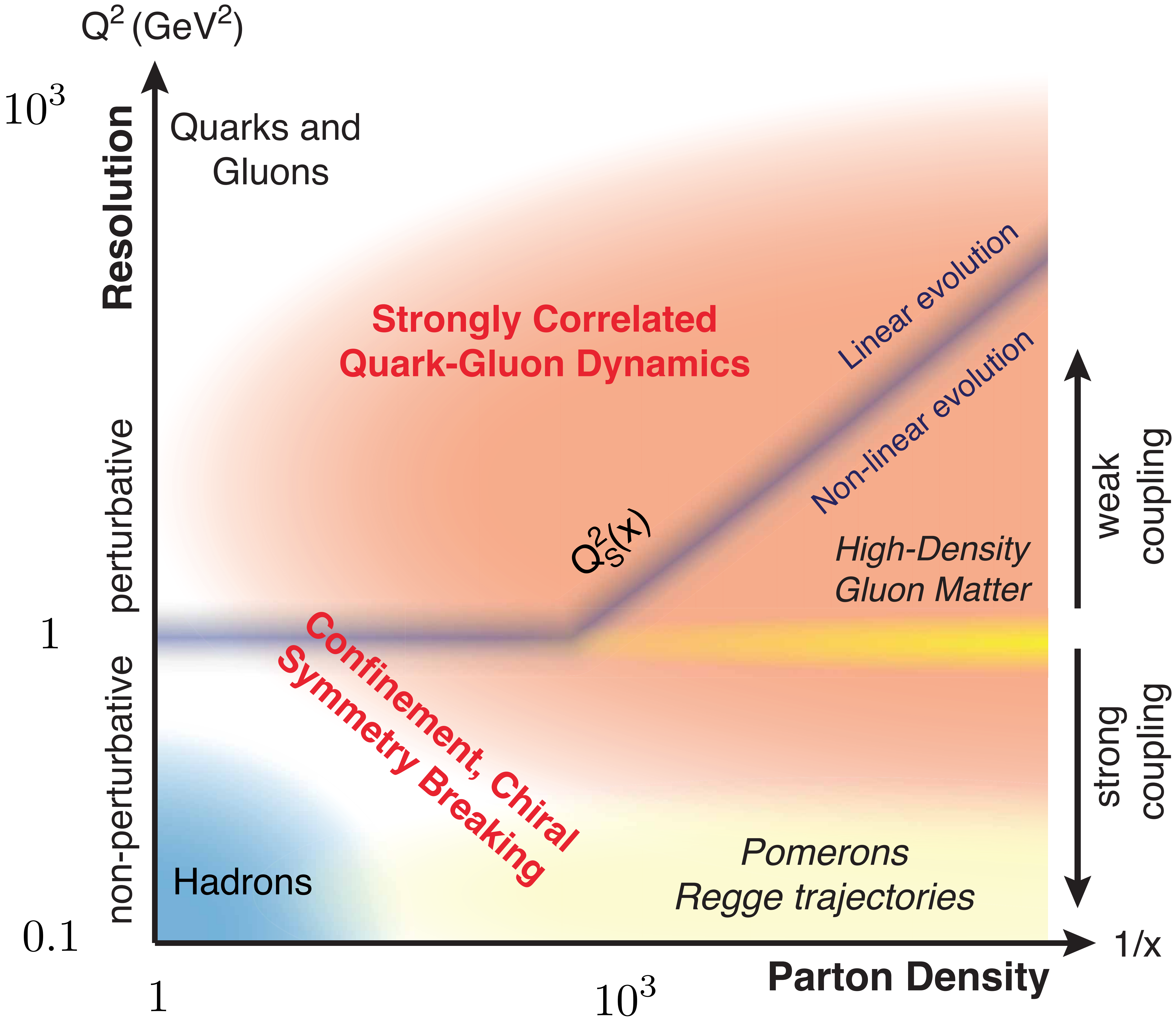
Landscape of QCD

Aschenauer et al., arXiv:1708.01527
Rep. Prog. Phys. 82, 024301 (2019)

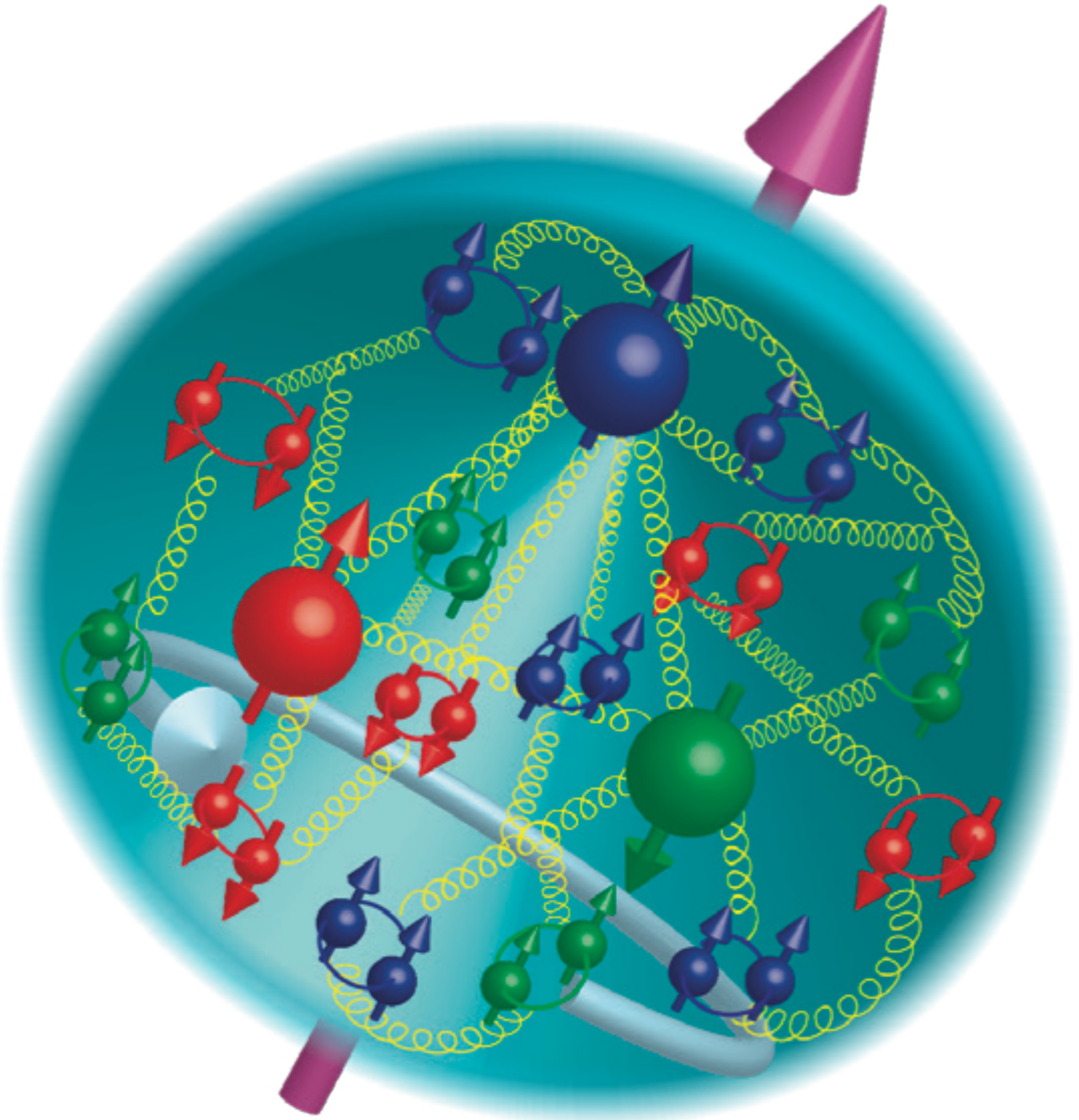
Finding 1: An EIC can uniquely address three profound questions about nucleons—neutrons and protons—and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?





The proton's spin puzzle



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

Spin

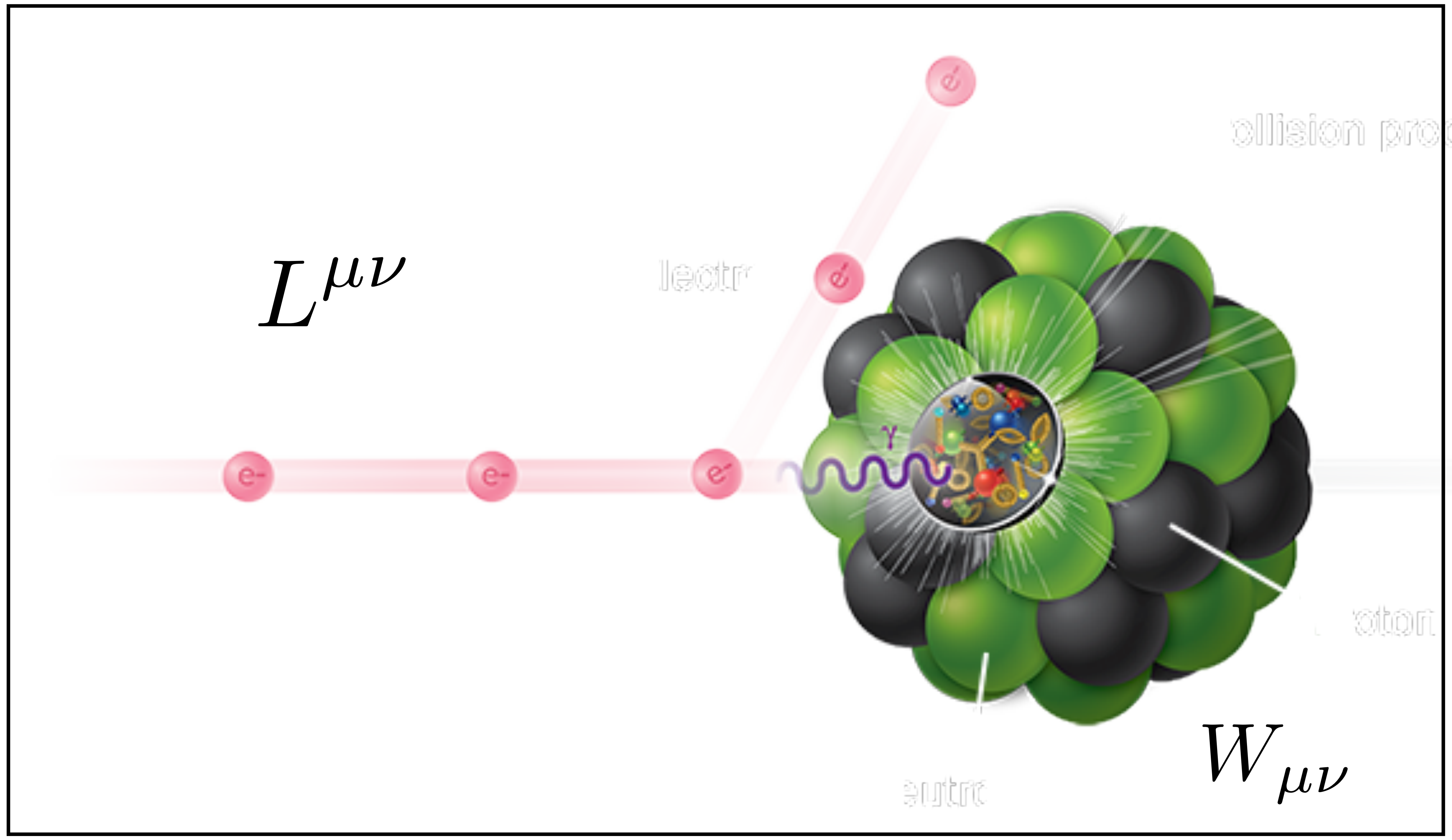
Quark helicity

Gluon helicity

Orbital angular momentum

- DIS experiments showed that quarks carry only about 30% of the proton's spin $\Delta\Sigma = 0.25 \sim 0.3$
- Failure of the constituent quark model to explain spin of the proton - *spin crisis*

DIS structure functions



DIS cross-section:

$$\sigma \sim L^{\mu\nu} W_{\mu\nu}$$

Hadron tensor:

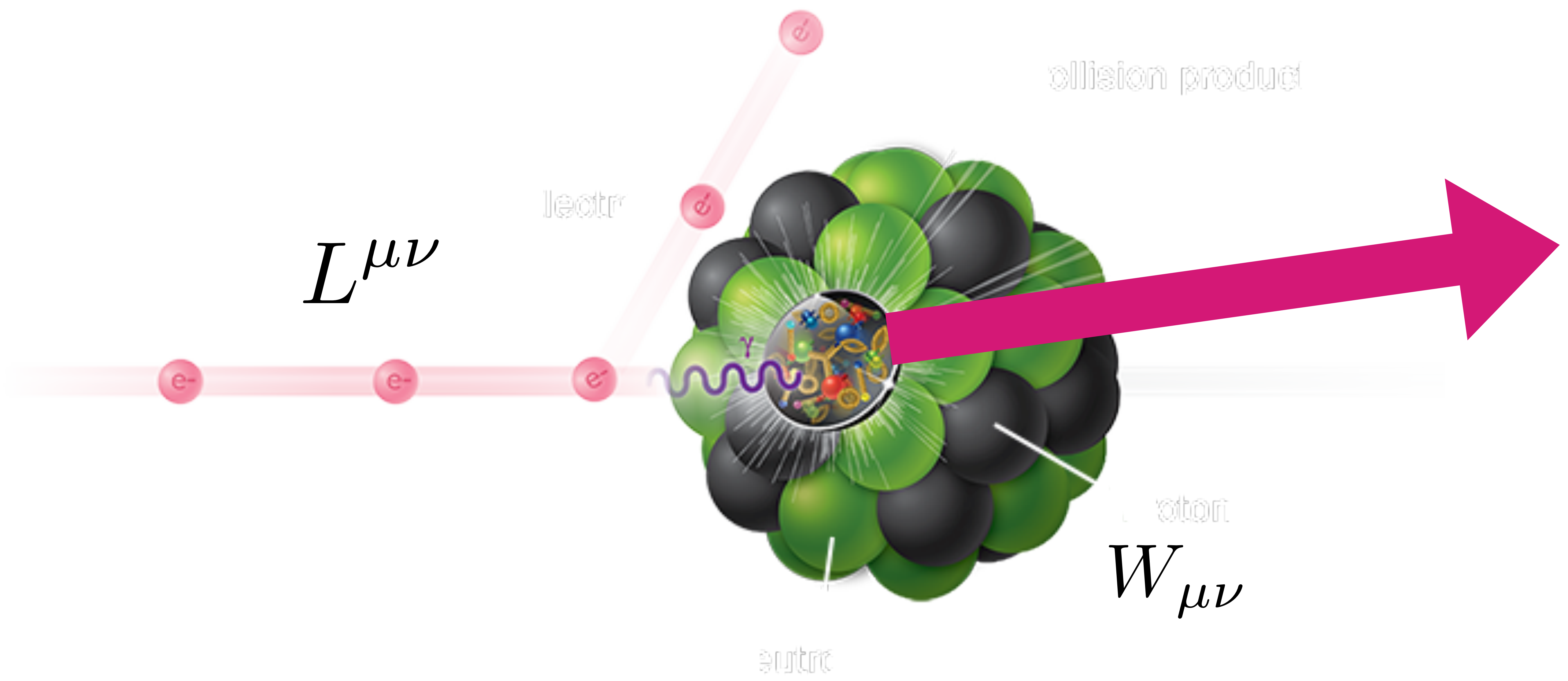
$$W^{\mu\nu}(q, P, S) = \bar{W}^{\mu\nu}_{sym.}(q, P) + i\tilde{W}^{\mu\nu}_{asym.}(q, P, S)$$

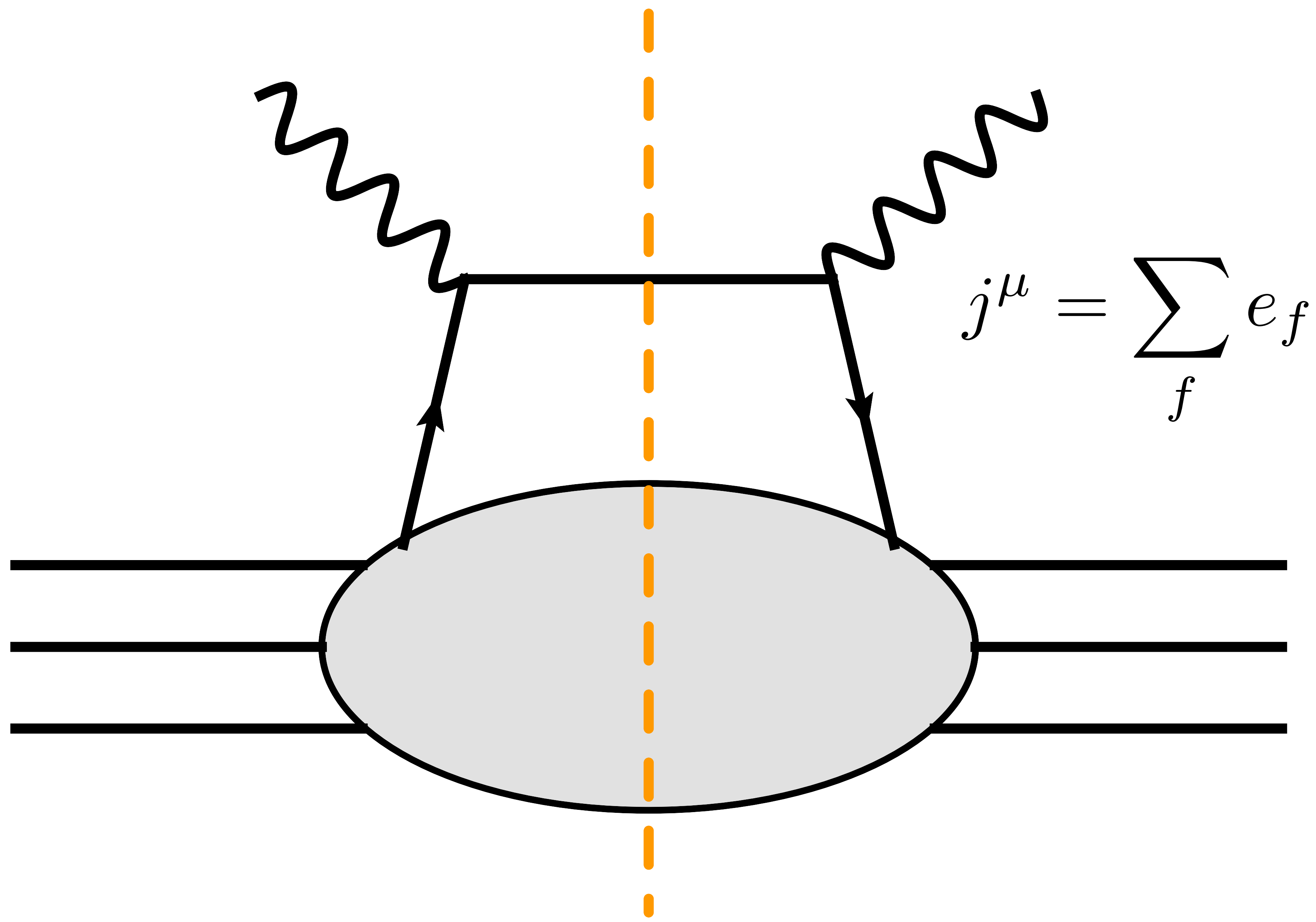
Structure function:

$$\tilde{W}_{\mu\nu}(q, P, S) = \frac{2M_N}{P \cdot q} \epsilon_{\mu\nu\alpha\beta} q^{\alpha} \left\{ S^{\beta} g_1(x_B, Q^2) + \left[S^{\beta} - \frac{(S \cdot q) P^{\beta}}{P \cdot q} \right] g_2(x_B, Q^2) \right\}$$

Structure function g_1

$$W^{\mu\nu}(q,P,S) = \frac{1}{2\pi}\int d^4x\, e^{iqx}\langle P,S|j^\mu(x)j^\nu(0)|P,S\rangle$$





$$j^\mu = \sum_f e_f \bar{\Psi}_f \gamma^\mu \Psi_f$$

$$g_1(x_B,Q^2) = \frac{1}{2}\sum_f e_f^2\left(\Delta q_f(x_B,Q^2) + \Delta\bar{q}_f(x_B,Q^2)\right)$$

polarized parton distribution function (pdf):

$$\Delta q_f(x_B,Q^2) = \frac{1}{4\pi}\int dy^-\, e^{-iy^- x_B P^+}\langle P,S|\bar{\Psi}_f(0,y^-,0_\perp)\gamma^+\gamma_5\Psi_f(0)|P,S\rangle$$

First moment of g_1

$$\int_0^1 dx_B\, g_1(x_B, Q^2) = \frac{1}{18}\left(3F + D + 2\,\Sigma(Q^2)\right)$$

Combination of F and D can be measured in β-decay and hyperon decay experiments

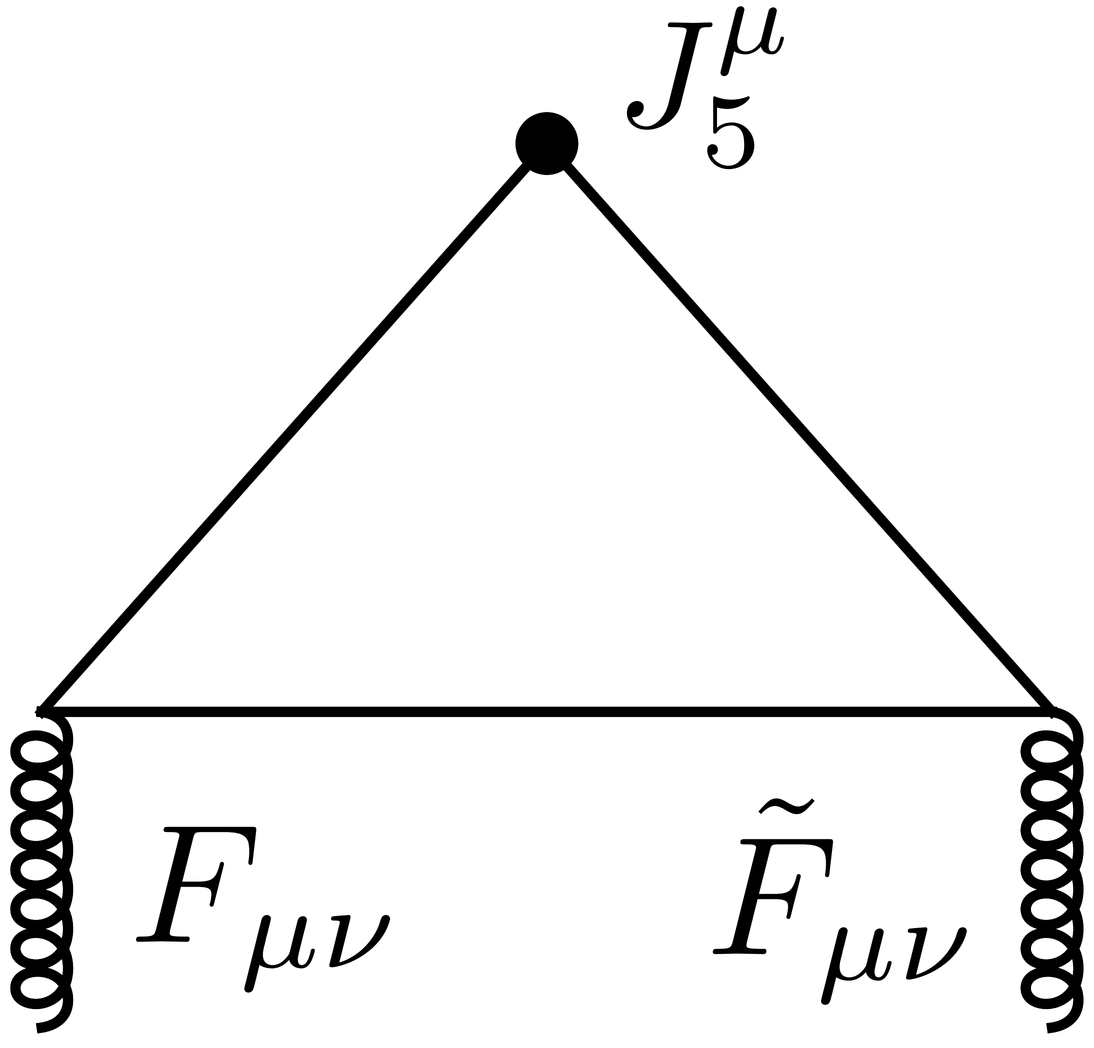
In term of helicity PDFs iso-singlet quark helicity is

$$\Sigma(Q^2) = \sum_f \int_0^1 dx_B\, \left(\Delta q_f(x_B, Q^2) + \Delta \bar{q}_f(x_B, Q^2)\right)$$

Quark spin contribution is defined by iso-singlet axial vector current

$$S^\mu \Sigma(Q^2) = \frac{1}{M_N} \sum_f \langle P, S| \bar{\Psi}_f \gamma^\mu \gamma_5 \Psi_f |P, S\rangle \equiv \frac{1}{M_N} \langle P, S| J_5^\mu(0) |P, S\rangle$$

Iso-singlet axial vector current



$$S^\mu \Sigma(Q^2) = \frac{1}{M_N} \langle P, S | J_5^\mu(0) | P, S \rangle$$

The current is not conserved. The anomaly equation:

$$\partial^\mu J^5_\mu(x) = \frac{n_f \alpha_s}{2\pi} \, \mathrm{Tr}\left(F_{\mu\nu}(x) \tilde{F}^{\mu\nu}(x) \right) = 2\, n_f \, \partial_\mu K^\mu$$

Chern-Simons current:

$$K_\mu = \frac{\alpha_S}{4\pi} \epsilon_{\mu\nu\rho\sigma} \left[A_a^\nu \left(\partial^\rho A_a^\sigma - \frac{1}{3} g f_{abc} A_b^\rho A_c^\sigma \right) \right]$$

Why is Σ small?

Efremov, Teryaev (1988)
Altarelli, Ross (1988)
Carlitz, Collins, Mueller (1988)
Bodwin, Qiu (1990)

Write quark helicity in terms of the conserved current (add and subtract K_μ):

$$S^\mu \Sigma(Q^2) = S^\mu \Sigma(Q^2) - \frac{2n_f}{M_N} \langle P, S|K^\mu|P, S\rangle + \frac{2n_f}{M_N} \langle P, S|K^\mu|P, S\rangle$$

Introduce a conserved curent $\tilde{J}_5^\mu$:

$$S^\mu \Sigma(Q^2) = S^\mu \tilde{\Sigma}(Q^2) + \frac{2n_f}{M_N} \langle P, S|K^\mu|P, S\rangle$$

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$$S^\mu \Sigma(Q^2) = S^\mu \tilde{\Sigma}(Q^2) + \frac{2n_f}{M_N} \langle P, S | K^\mu | P, S \rangle$$

where

$$S^\mu \tilde{\Sigma}(Q^2) = \frac{1}{M_N} \langle P, S | \tilde{J}^5_\mu | P, S \rangle, \quad \tilde{J}^5_\mu = J^5_\mu - 2n_f K_\mu$$

Quark and gluon contribution:

K_μ is not gauge invariant!

$$\tilde{\Sigma}(Q^2) = \Sigma(Q^2) - \frac{n_f \alpha_S}{2\pi} \Delta G$$

Criticized by Jaffe, Manohar in Nucl. Phys., B337:509–546, 1990

If ΔG is large, this would provide a natural explanation of the spin crisis.

Gluon spin contribution in OPE

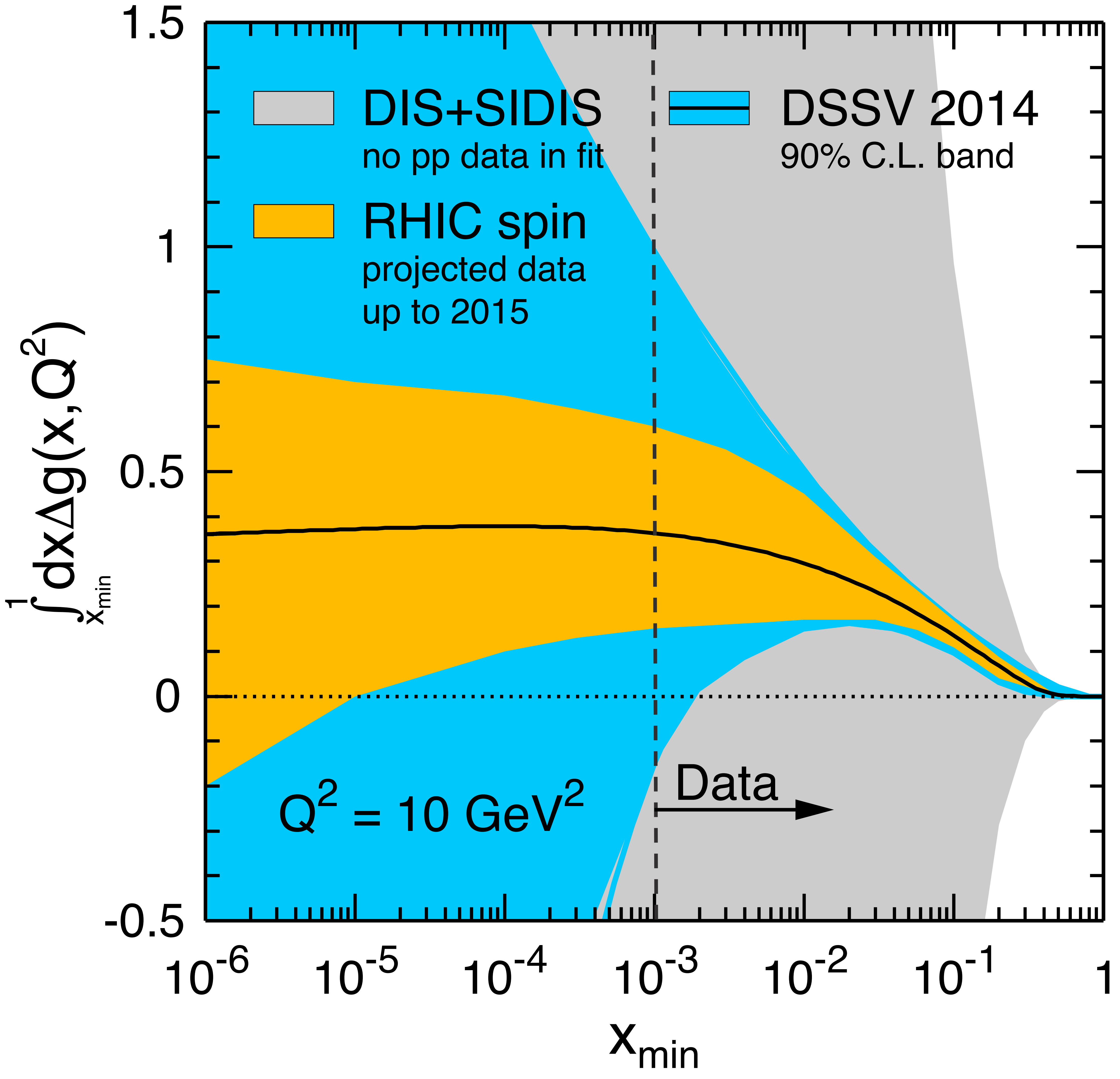
$$\Delta G(x_B, Q^2) = \int_0^1 dx_B \Delta g(x_B, Q^2)$$

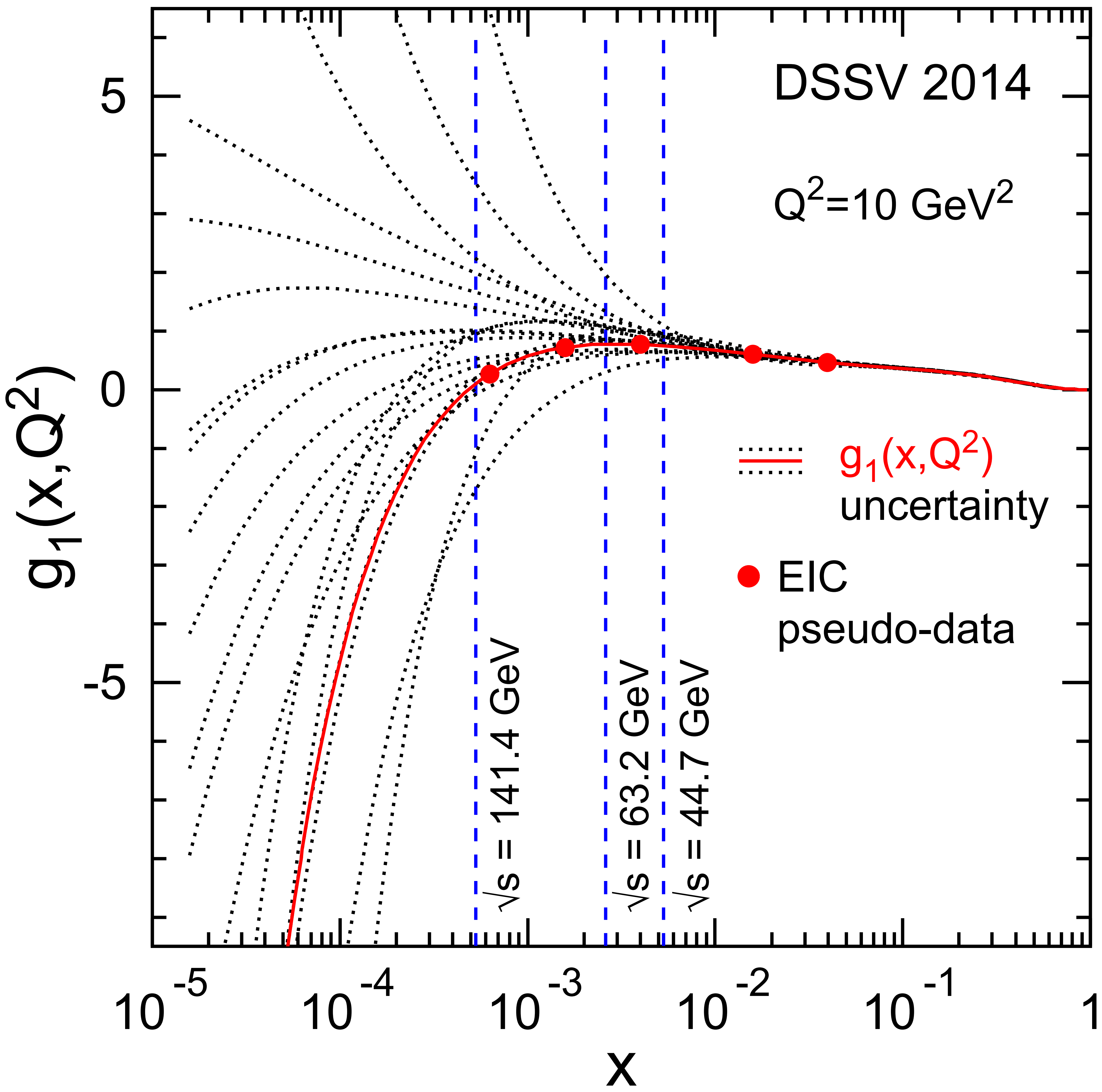
In the Operator Product Expansion (OPE) the gluon spin is defined by
a non-local gauge-invariant operator:

$$\Delta g(x_B, Q^2) = \frac{2i}{x_B} \int \frac{d\xi}{2\pi} e^{-i\xi x_B} \langle P, S | \mathrm{Tr_c} n_\alpha F^{\alpha\mu}(\xi n) n^\beta \tilde{F}_{\beta\mu}(0) | P, S \rangle$$

- ΔG cannot be expressed by a local operator
- It becomes local only in the light-cone gauge and coincides with K^+ (ΔG is no K_μ)

Resolving the proton's spin crisis





D. De Florian, R. Sassot, M. Stratmann, W. Vogelsang, PRL 113 (2014)

Resolving the proton's spin crisis

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\Delta\Sigma \\ \Delta\Gamma\end{pmatrix} = \begin{pmatrix}\Delta P_{\Sigma\Sigma}(a_s) & 0 \\ -\frac{1}{2N_f}\,\Delta P_{\Sigma\Sigma}(a_s) & 0\end{pmatrix}\begin{pmatrix}\Delta\Sigma \\ \Delta\Gamma\end{pmatrix}$$

$$\Delta\Gamma(Q^2) \equiv a_s(Q^2)\Delta G(Q^2)$$

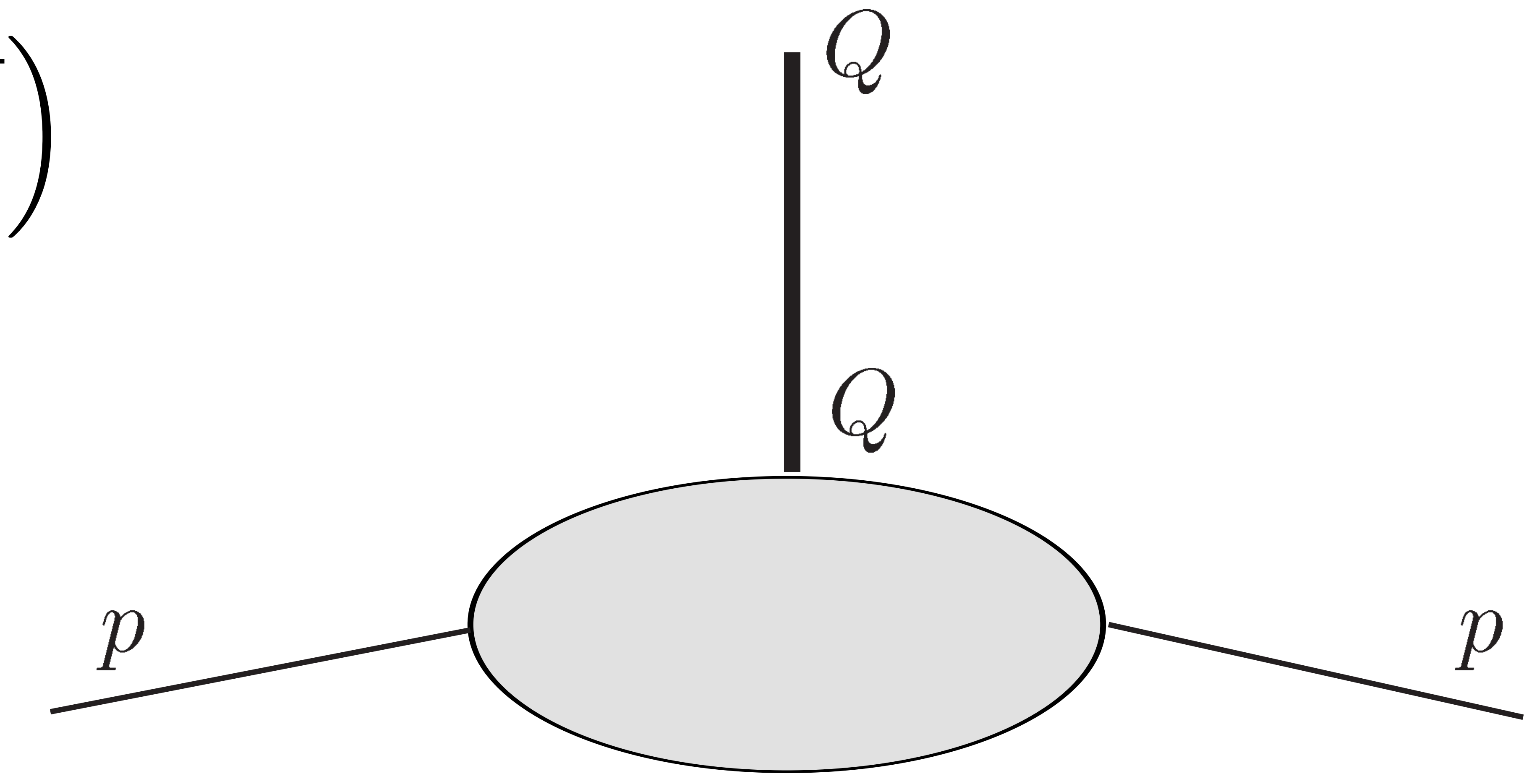
- The evolution is controlled by a single anomalous dimension. This is a consequence of the anomaly equation for the iso-singlet axial current

- This suggests that $\Delta\Sigma$ only mixes with itself

Altarelli and Lampe (1990)
Vogt, Moch, Rogal, Vermaseren (2008)
DeFlorian, Vogelsang (2019)

Alternative picture: Anomalous chiral Ward identities

$$\Sigma(Q^2) = \frac{1}{3m_N}\Delta C_1^S(\alpha_S)\left(g_{QNN}\chi(0) + g_{\eta' NN}\sqrt{\chi'(0)}\right)$$



Topological charge density:

$$Q(x) = \frac{\alpha_S}{8\pi} Tr\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$$

Topological susceptibility: $$\chi_{\text{YM}}(p^2) = i\int dx\, e^{iP\cdot x}\langle 0|T(Q(x)Q(0)|0\rangle$$

In the chiral limit $\chi(0) \to 0$, the slope $\chi'(0)$ is estimated to be small. Topological charge screening of spin.

Shore, Veneziano (1990)
Narison, Shore, Veneziano, hep-ph/9812333
G. M. Shore, hep-ph/0701171
S. D. Bass, hep-ph/0411005

Perturbative and non-perturbative interplay

The key role of the anomaly is seen from the structure of the triangle graph in the off-forward limit

$$\frac{1}{M_N}\langle P', S|J_5^\mu(0)|P,S\rangle = \Sigma(Q^2,t)\,S^\mu + h(Q^2,t)\,l\cdot S\,l^\mu$$

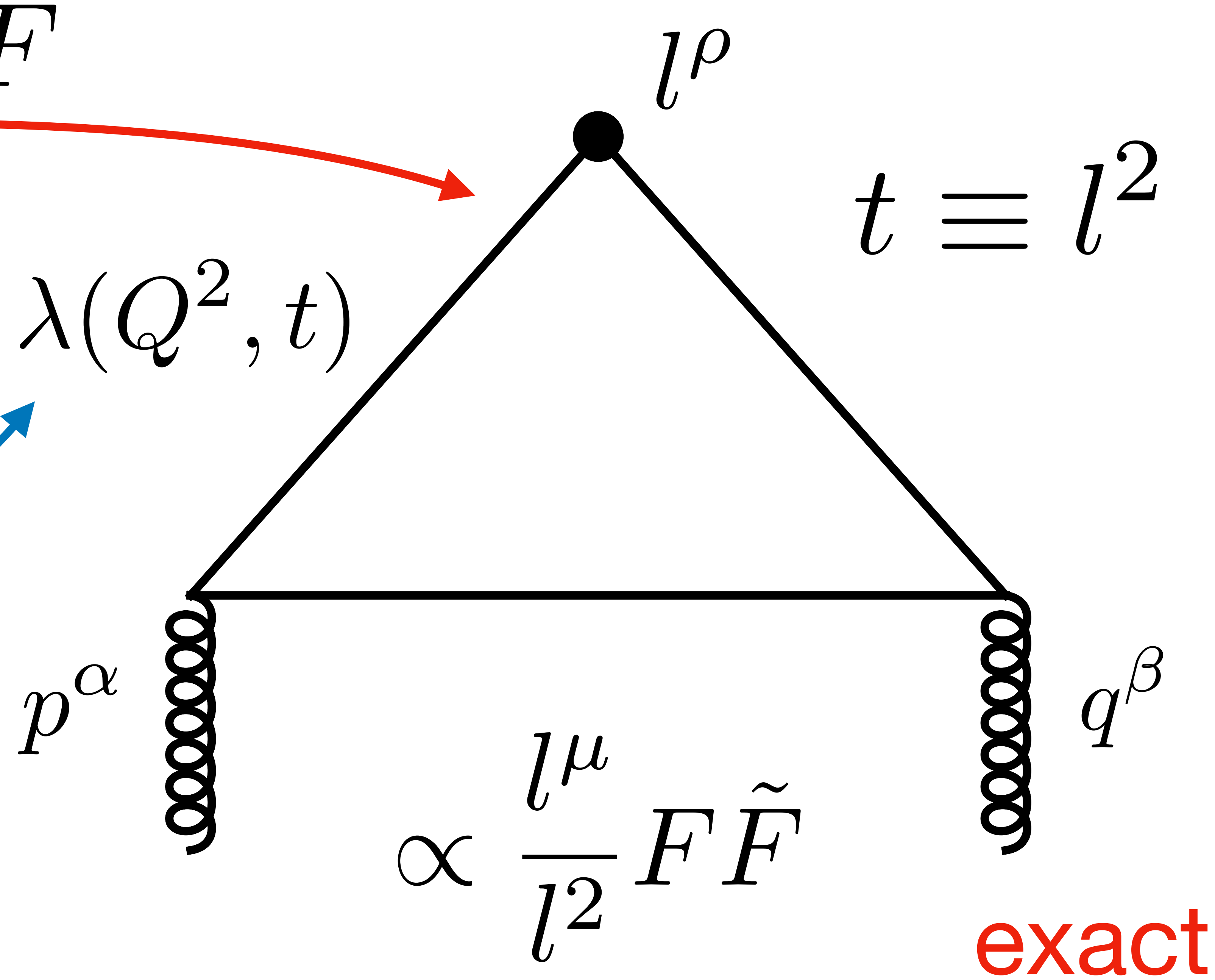
R. L. Jaffe

A. Manohar

In the forward limit only Σ contribute, h doesn't have a pole

The triangle diagram has an infrared pole and $\kappa(t) \propto F\tilde{F}$

$$\frac{1}{M_N}\langle P', S|J_5^\mu(0)|P,S\rangle = \boxed{\frac{l\cdot S\,l^\mu}{l^2}\kappa(Q^2,t)} + \left(S^\mu - \frac{l\cdot S\,l^\mu}{l^2}\right)\lambda(Q^2,t)$$

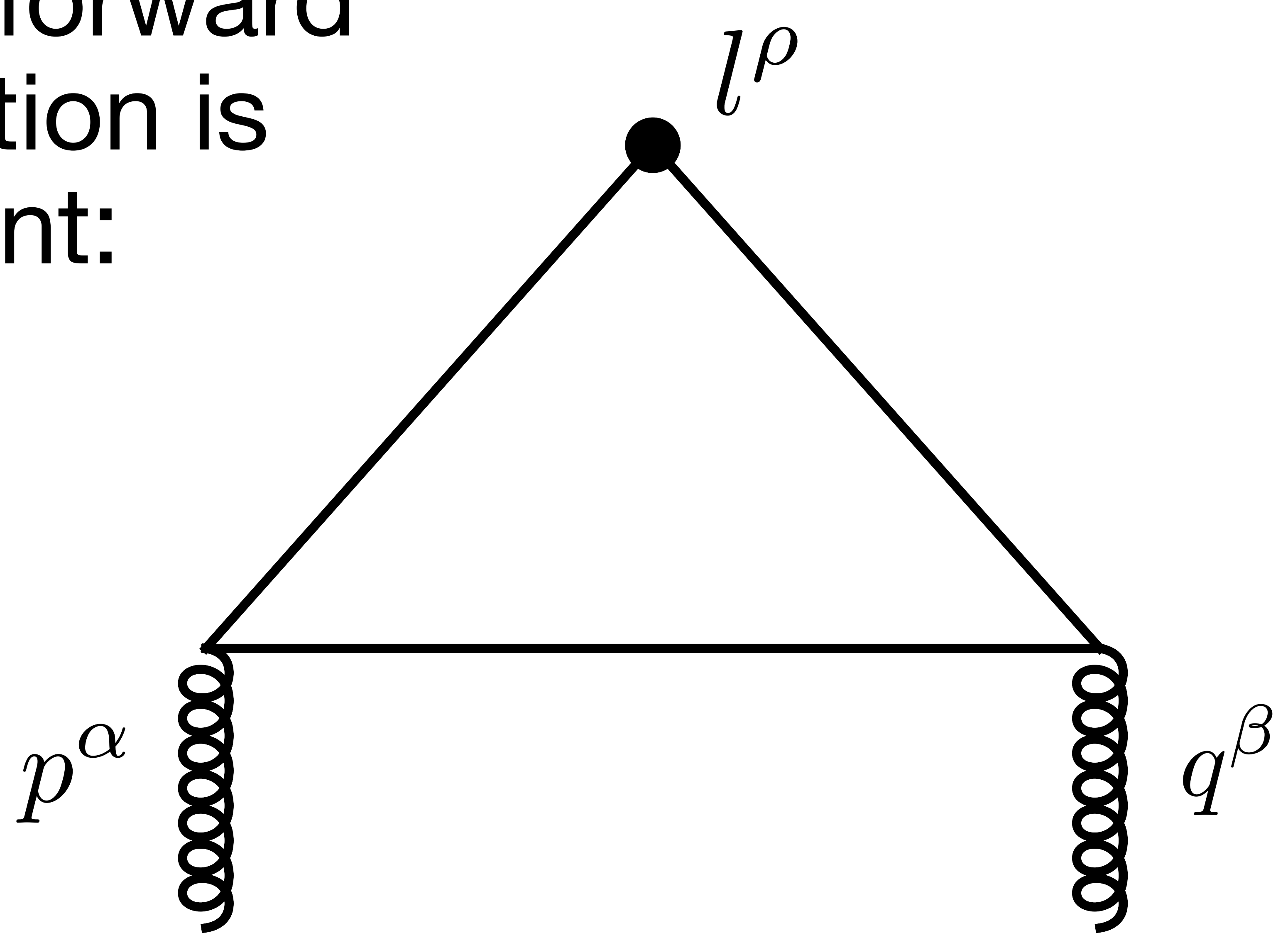


Pseudoscalar contribution

R. L. Jaffe, A. Manohar
Nucl. Phys., B337:509–546, 1990

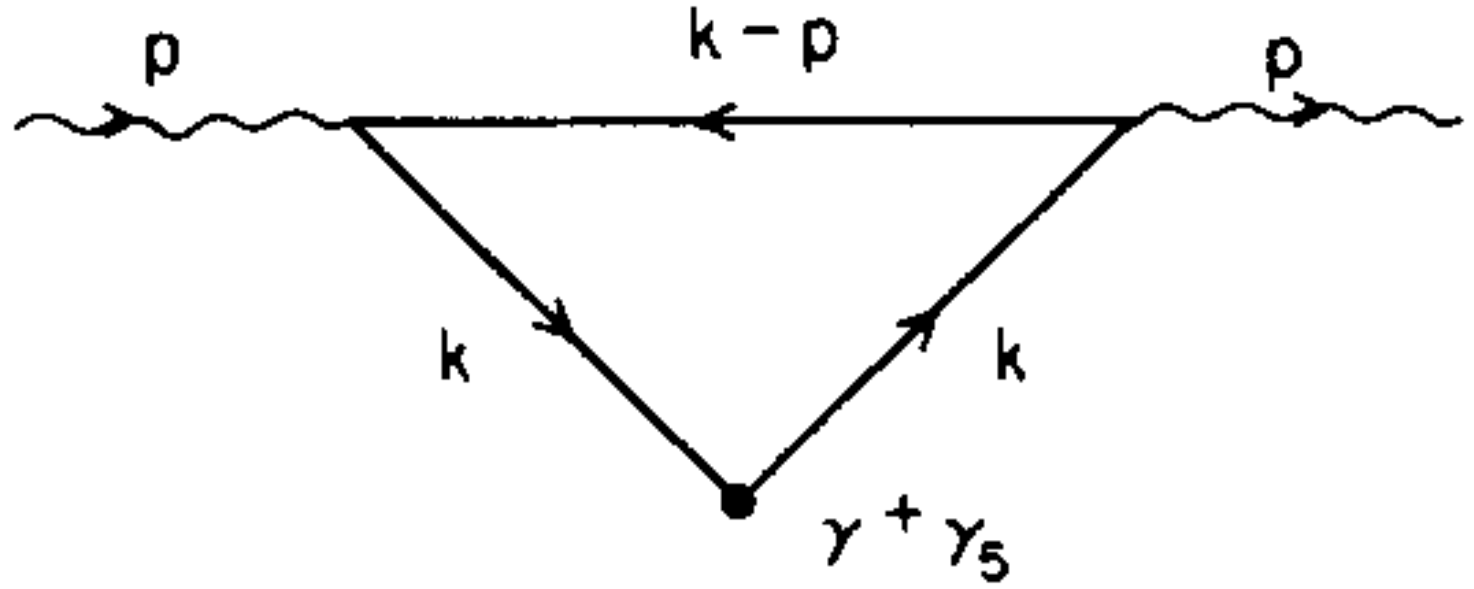
OPE vs. exact off-forward calculation

Exact result of the off-forward calculation is divergent:



$$\propto \frac{l^\mu}{l^2} F\tilde{F}$$

In the forward OPE calculation the result is finite:



$$\lim_{m^2/P^2 \to 0} \Gamma_5^+ = \mp \frac{\alpha_s N_f p^+}{\pi}$$

Carlitz, Collins, Mueller (1988)

Perturbative and non-perturbative interplay

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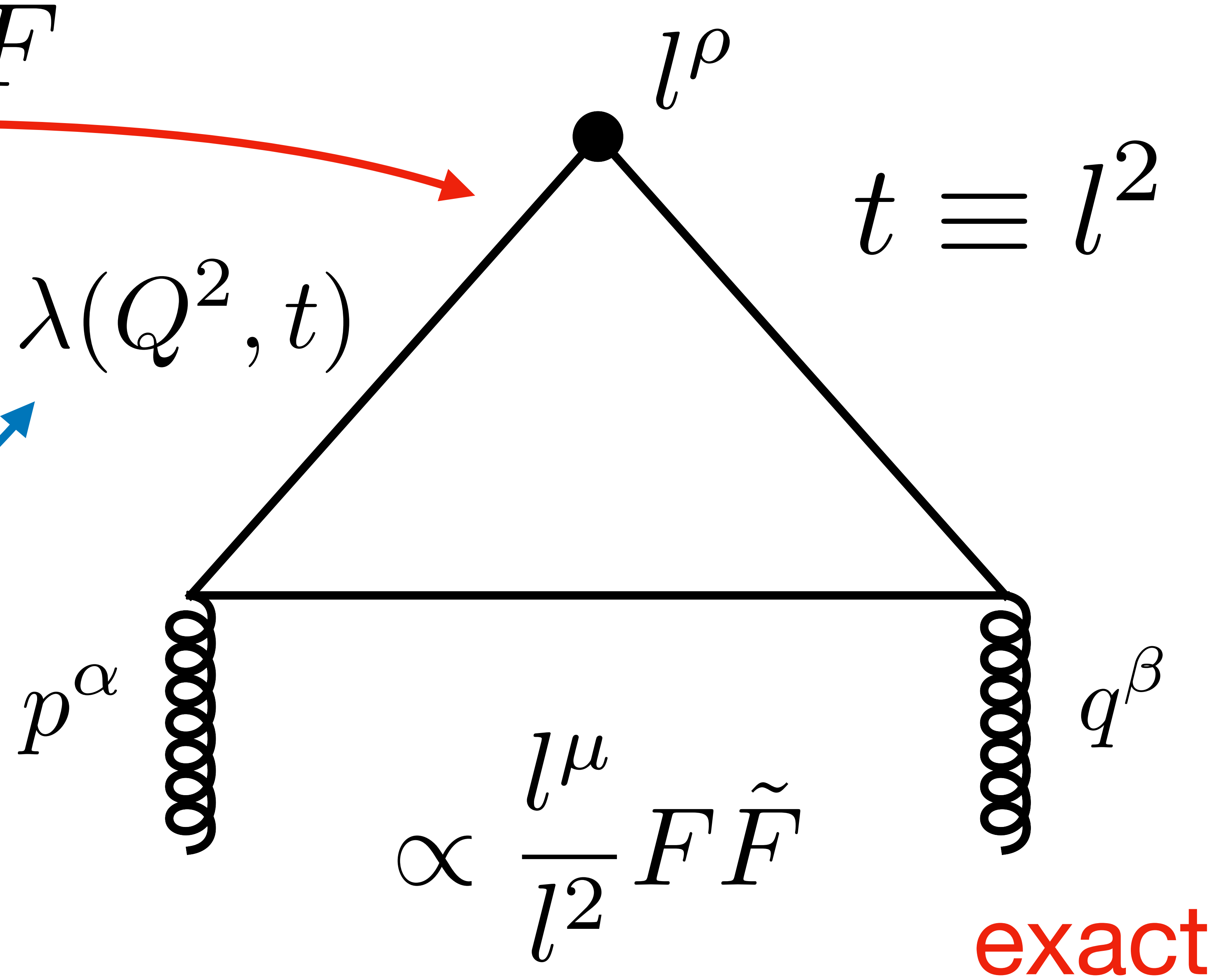


Pseudoscalar contribution

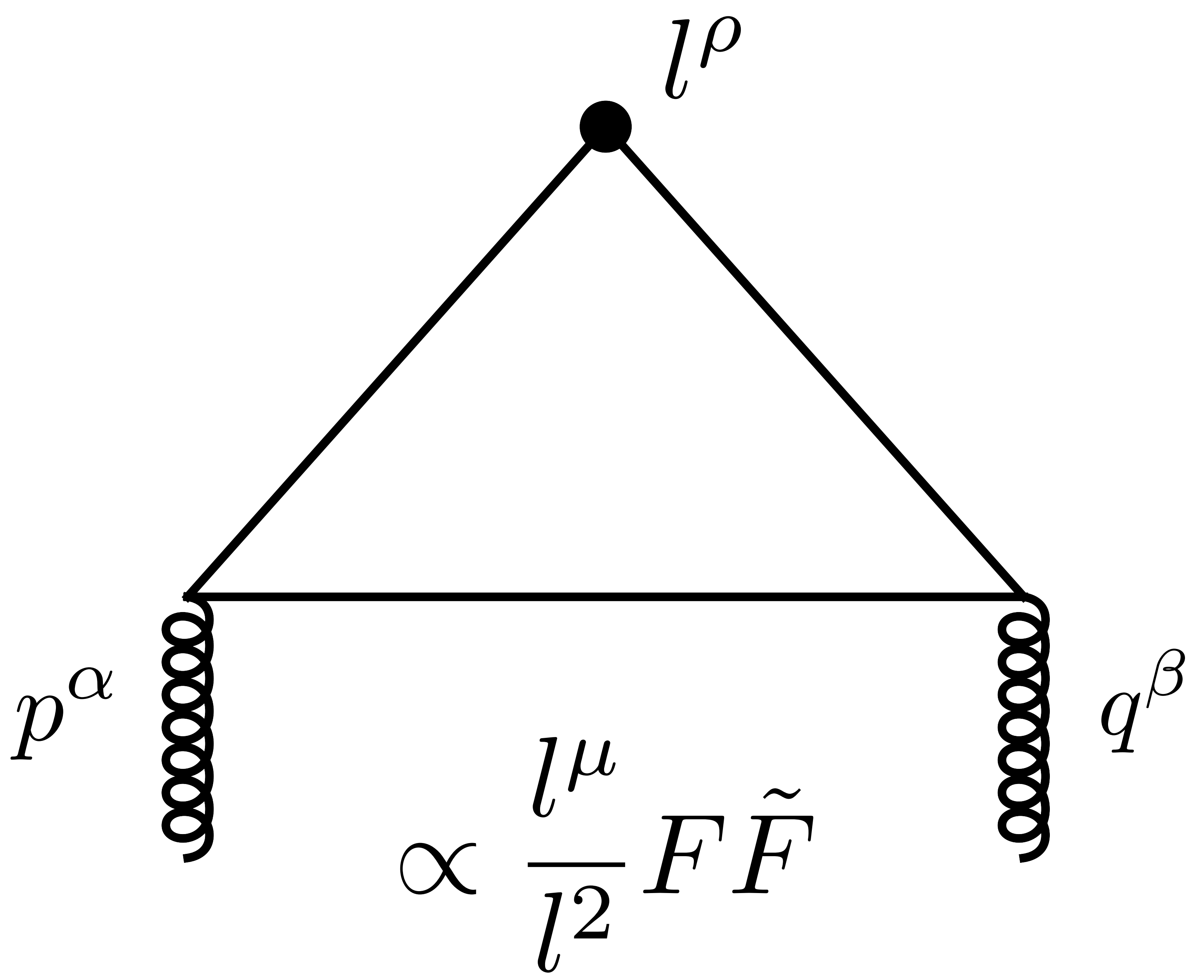
R. L. Jaffe, A. Manohar
Nucl. Phys., B337:509–546, 1990

Perturbative and non-perturbative interplay

The infrared pole of the triangle diagram must be cancelled by a pole in the non-perturbative contribution:

$$\Sigma(Q^2) = \frac{n_f\,\alpha_s}{2\pi\,M_N}\lim_{l_\mu\to 0}\langle P', S|\frac{1}{il\cdot s}\mathrm{Tr}\left(F\tilde{F}\right)(0)|P, S\rangle$$

The result is manifestly gauge invariant!

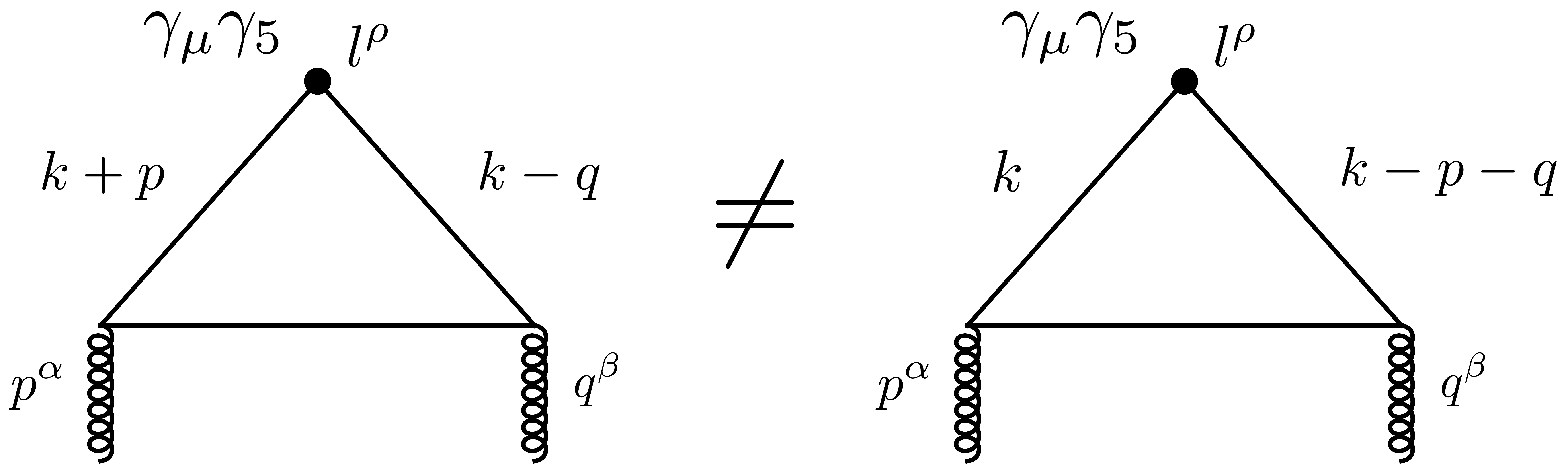


The existence of the infrared pole of the triangle diagram has not been addressed in pQCD calculations. In the OPE in the forward limit the triangle diagram is finite, i.e it doesn't have the infrared pole and $F\tilde{F}$

The triangle diagram

In the Feynman diagram approach the calculation of the triangle anomaly is ambiguous

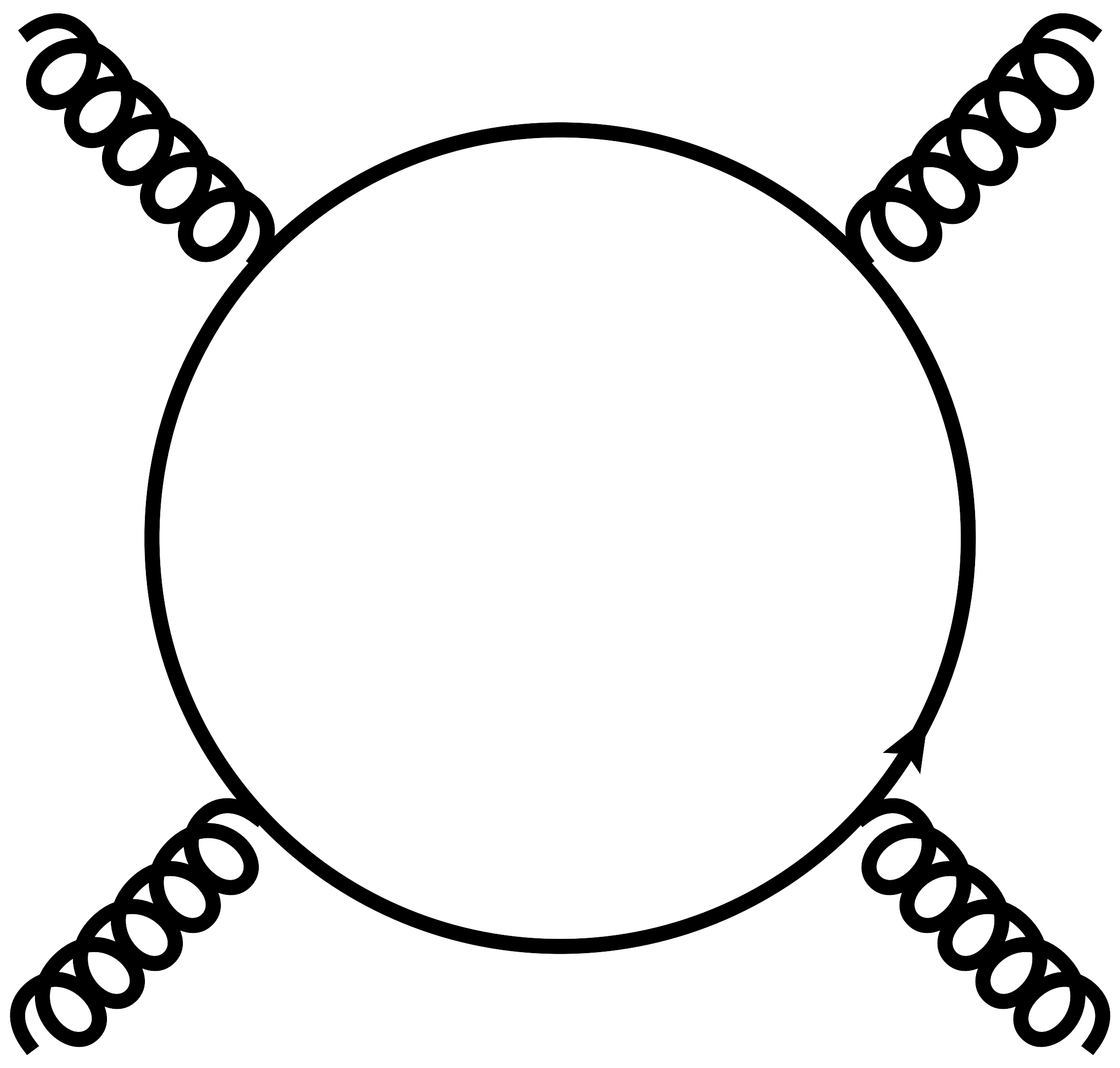
Linearly divergent integrals!



A brute force calculation yields a wrong result. The value of the triangle diagram depends on how the loop momentum is parametrized. The "correct" parametrization can be achieved by imposing the vector Ward identities.

Our approach is to use the worldline formalism which doesn't have these ambiguities

The worldline formalism



One-loop effective action:

$$\Gamma_{QED}[A] = \ln \mathrm{Det}[\not{p} + \not{A}]$$

Heat-kernel regularization:

$$\Gamma_{QED}[A] = -\frac{1}{2}\mathrm{Tr}\int_0^\infty \frac{dT}{T}\exp\left\{-T\left[(p_\mu + eA_\mu)^2 - \frac{e}{2}F_{\mu\nu}\sigma^{\mu\nu}\right]\right\}$$

Bern,Kosower, NPB 379 (1992) 145
Strassler, NPB 385 (1992) 145
Schubert, Phys. Rep. (2001)
D'Hoker, Gagne (1996)
Berezin, Marinov (1976)
Barducci (1977)

One can introduce bosonic and fermionic coherent states and construct a functional integral representation:

Grassmann numbers

$$\int d^4x\,|x\rangle\langle x| = \int d^4p\,|p\rangle\langle p| = 1; \quad \langle x|p\rangle = e^{ipx}$$

$$i\int |\xi\rangle\langle\xi|d^2\xi = -i\int |\bar{\xi}\rangle\langle\bar{\xi}|d^2\bar{\xi} = 1\,; \quad \langle\xi|\bar{\xi}\rangle = e^{\sum_{i=1}^2 \xi_i\bar{\xi}_i}\,; \quad \langle\bar{\xi}|\xi\rangle = e^{\sum_{i=1}^2 \bar{\xi}_i\xi_i}$$

The triangle anomaly in the worldline formalism

Functional integrals over trajectories of a point-like particle

$$\Gamma[A,A_5] = -\frac{1}{2}\mathrm{Tr_c}\int_0^\infty \frac{dT}{T}\int \mathcal{D}x\int_{AP}\mathcal{D}\psi$$

$$\times\exp\Big\{-\int_0^T d\tau\Big(\frac{1}{4}\dot{x}^2+\frac{1}{2}\psi_\mu\dot{\psi}^\mu+ig\dot{x}^\mu A_\mu-ig\psi^\mu\psi^\nu F_{\mu\nu}-2i\psi_5\dot{x}^\mu\psi_\mu\psi_\nu A_5^\nu+i\psi_5\partial_\mu A_5^\mu+(D-2)A_5^2\Big)\Big\}$$

Free "propagation" Vector coupling Axial coupling

D.G.C. McKeon, C. Schubert, Phys. Lett. B 440 (1998) 101

The triangle anomaly:

$$\langle P',S|J_5^\kappa|P,S\rangle = \int d^4y\,\frac{\partial}{\partial A_{5\kappa}(y)}\Gamma[A,A_5]\Big|_{A_5=0}e^{ily}\equiv\Gamma_5^\kappa[l]$$

The triangle anomaly in the worldline formalism

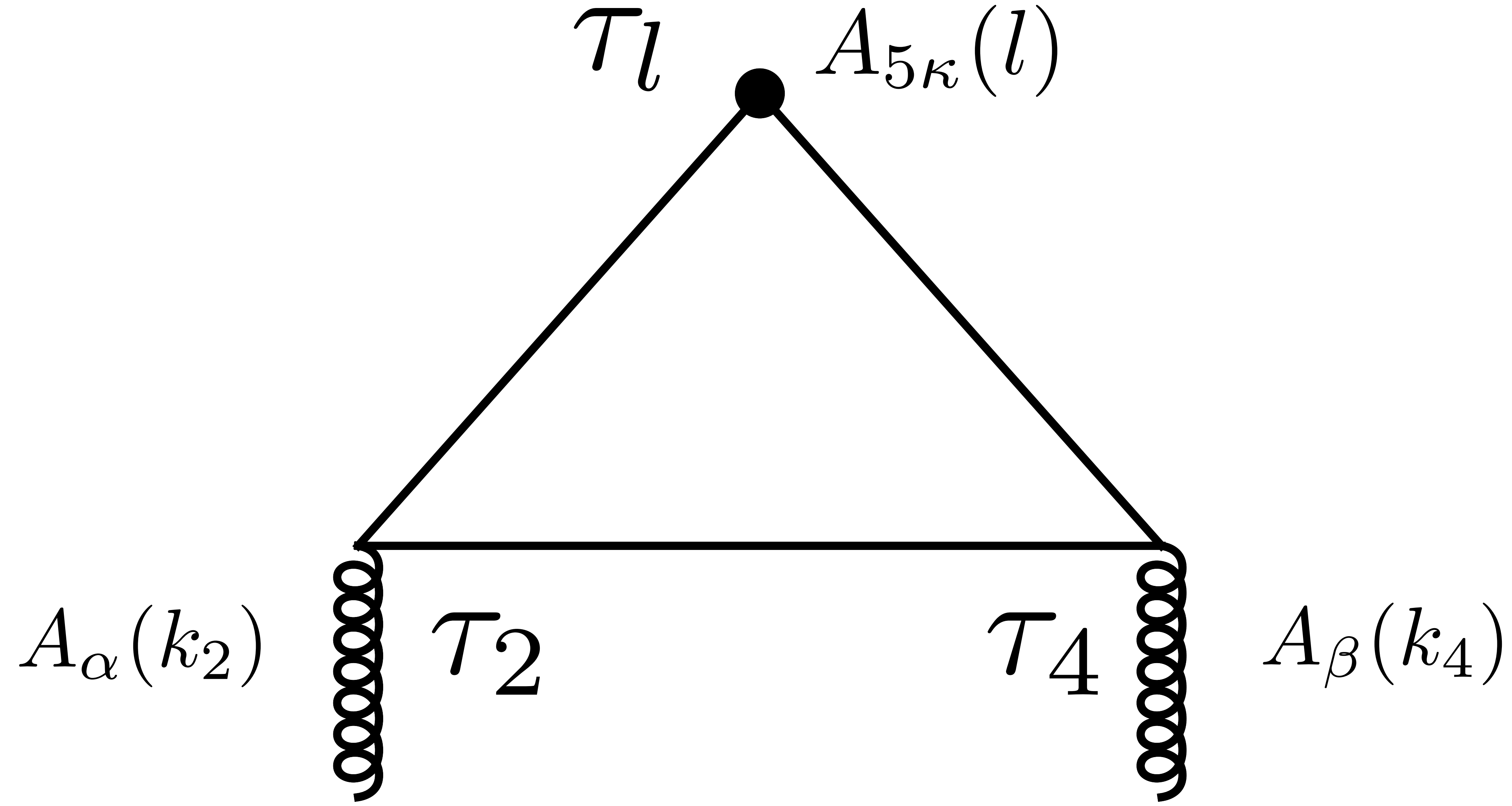
Tarasov, Venugopalan (2020)
c.f. McKeon, Schubert (1998)

Axial current

$$\Gamma_5^{\kappa\alpha\beta}[l,k_2,k_4] \equiv -\frac{ig^2}{2}\int_0^\infty \frac{dT}{T}\int \mathcal{D}x \int_{AP}\mathcal{D}\psi \int_0^T d\tau_l\ \psi_5\Big(il^\kappa + 2\psi_l^\kappa \dot{x}_l\cdot\psi_l\Big)e^{ilx_l}$$

$$\times \int_0^T d\tau_2 \int_0^T d\tau_4 \Big(\dot{x}_2^\alpha + 2i\psi_2^\alpha\psi_2^\lambda k_{2\lambda}\Big)e^{ik_2x_2}\Big(\dot{x}_4^\beta + 2i\psi_4^\beta\psi_4^\eta k_{4\eta}\Big)e^{ik_4x_4}\exp\Big\{-\int_0^T d\tau\Big(\frac{1}{4}\dot{x}^2+\frac{1}{2}\psi_\mu\dot{\psi}^\mu\Big)\Big\}$$

Vector currents



- Calculation of the functional integrals is straightforward and doesn't contain any ambiguity

- ψ_5 changes anti-periodic boundary condition of the Grassmann functional integral to periodic

The triangle anomaly in the worldline formalism

$$\Gamma_5^\kappa[l] = \frac{1}{4\pi^2}\boxed{\frac{l^\kappa}{l^2}}\int \frac{d^4k_2}{(2\pi)^4}\int \frac{d^4k_4}{(2\pi)^4}\ \mathrm{Tr_c} F_{\alpha\beta}(k_2)\tilde{F}^{\alpha\beta}(k_4)\ (2\pi)^4\delta^4(l+k_2+k_4)$$

Steven Adler

John S. Bell

We reproduce famous infrared pole of the anomaly

$$S^\mu \int_0^1 dx_B\, g_1(x_B, Q^2)\Big|_{Q^2\to\infty}$$

$$= \sum_f e_f^2 \frac{\alpha_S}{2i\pi M_N} \lim_{l_\mu \to 0} \frac{l^\mu}{l^2}\langle P', S|\mathrm{Tr_c} F_{\alpha\beta}(0)\tilde{F}^{\alpha\beta}(0)|P, S\rangle$$

Roman Jackiw

The infrared pole must be cancelled by a pseudoscalar contribution

The anomaly contribution in g_1

OPE expansion of the product of two electromagnetic currents (hadronic tensor):

$$i\int d^4x\, e^{iqx}\,\mathrm{T}(J_\mu(x)J_\nu(0)) \simeq [\text{symmetric terms under } \mu\leftrightarrow\nu]$$

$$-i\epsilon_{\mu\nu\lambda\sigma}q^\lambda\sum_n \tfrac{1}{2}[1-(-)^n]\left(\frac{2}{-q^2}\right)^n q_{\mu_1}\ \dots\ q_{\mu_{n-1}}\sum_i E^n_{1,i}(-q^2,g)$$

$$\times R^{\sigma\mu_1\cdots\mu_{n-1}}_{1,i} - i(\epsilon_{\mu\rho\lambda\sigma}q_\nu q^\rho - \epsilon_{\nu\rho\lambda\sigma}q_\mu q^\rho - q^2\epsilon_{\mu\nu\lambda\sigma})$$

$$\times\sum_n \tfrac{1}{2}[1-(-)^n]\frac{n-1}{n}\left(\frac{2}{-q^2}\right)^n q_{\mu_1}\dots q_{\mu_{n-2}}\sum_i E^n_{2,i}(-q^2,g)R^{\lambda\sigma\mu_1\dots\mu_{n-2}}_{2,i}$$

In the Bjorken limit when $Q^2\to\infty$ the leading contribution corresponds to $n=1$ and is defined by the axial current

$$R^n_{1,\psi} = (i)^{n-1}S\bar{\psi}\gamma_5\gamma^\sigma D^{\mu_1}\dots D^{\mu_{n-1}}\psi$$

Kodaira (1980)

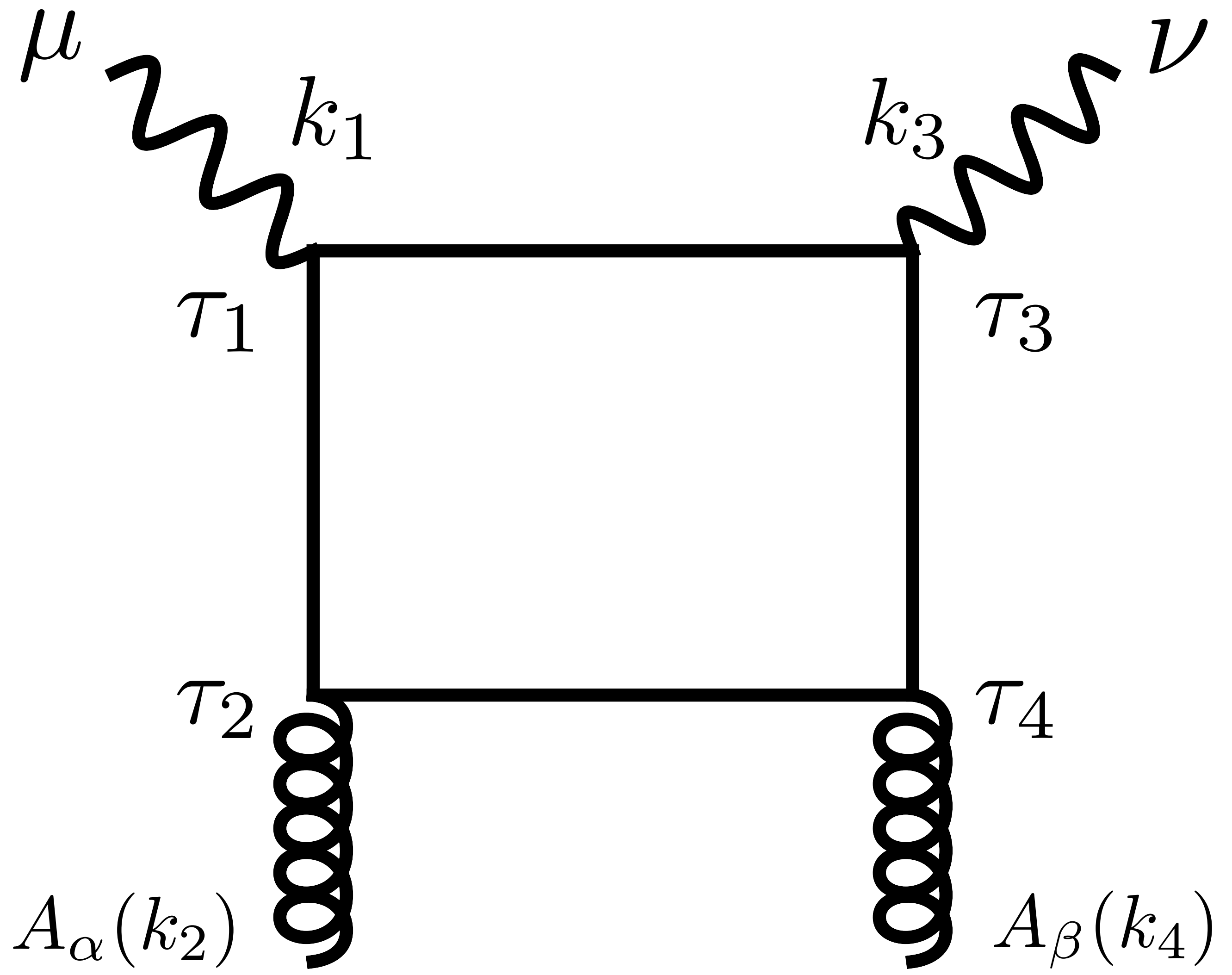
The box diagram in the worldline approach

Tarasov, Venugopalan (2019)

$$\Gamma_A^{\mu\nu}[k_1,k_3] = \int \frac{d^4k_2}{(2\pi)^4}\int \frac{d^4k_4}{(2\pi)^4}\ \Gamma_A^{\mu\nu\alpha\beta}[k_1,k_3,k_2,k_4]\ \mathrm{Tr_c}(\tilde{A}_\alpha(k_2)\tilde{A}_\beta(k_4))$$

Hadronic tensor

Box diagram



Worldline representation of the box diagram:

$$\Gamma_A^{\mu\nu\alpha\beta}[k_1,k_3,k_2,k_4] \equiv -\frac{g^2e^2e_f^2}{2}\int_0^\infty \frac{dT}{T}\ \int \mathcal{D}x \int \mathcal{D}\psi\ \exp\Big\{-\int_0^T d\tau\Big(\frac{1}{4}\dot{x}^2+\frac{1}{2}\psi\cdot\dot{\psi}\Big)\Big\}$$
$$\times\Big[V_1^\mu(k_1)V_3^\nu(k_3)V_2^\alpha(k_2)V_4^\beta(k_4) - (\mu\leftrightarrow\nu)\Big]$$

where the vector current $\quad V_i^\mu(k_i) \equiv \int_0^T d\tau_i(\dot{x}_i^\mu + 2i\psi_i^\mu k_j\cdot\psi_j)e^{ik_i\cdot x_i}$

The box diagram in the Bjorken limit

In the Bjorken limit $Q^2 \to \infty$ and x_B is fixed

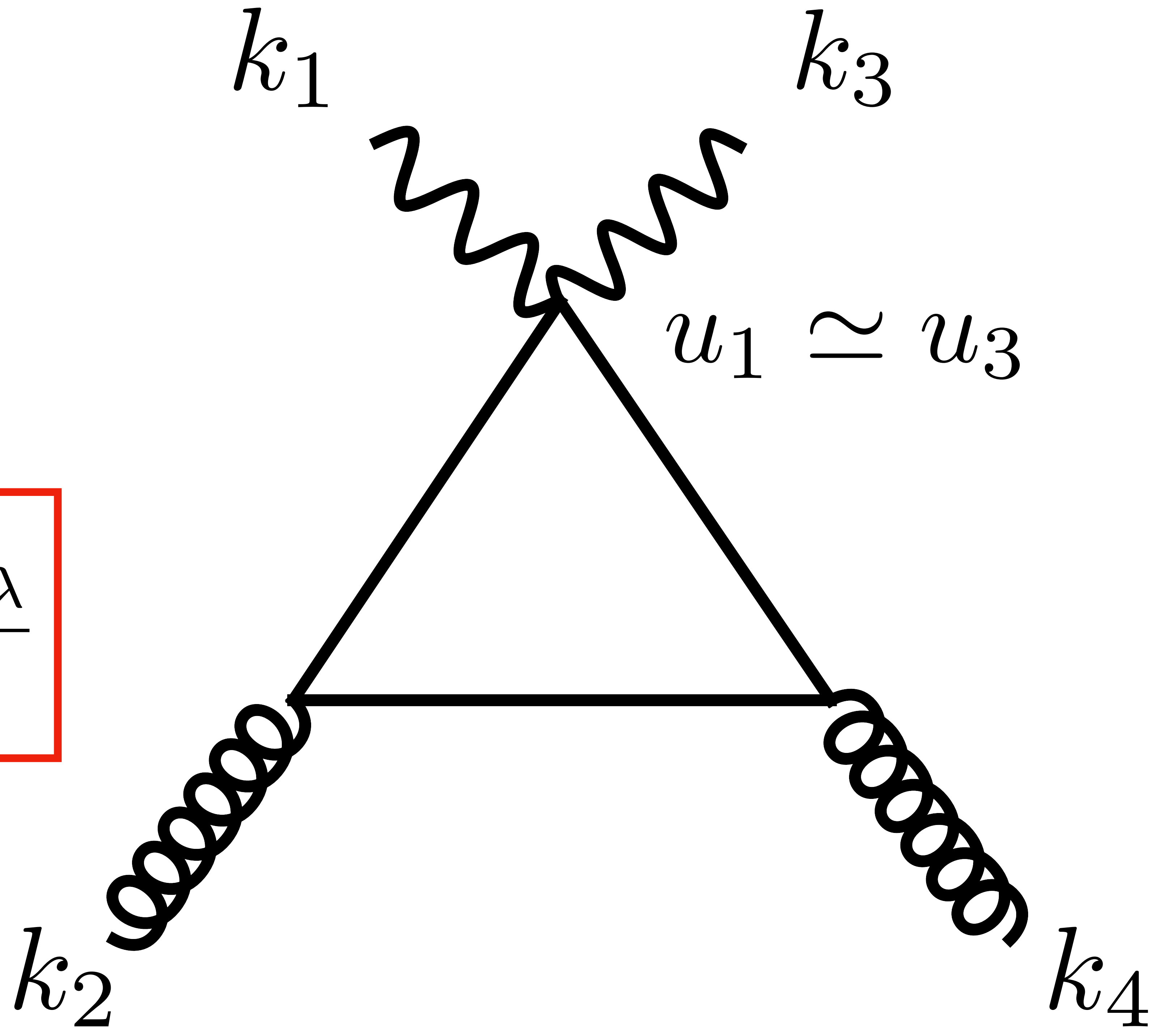
$$\Gamma_A^{\mu\nu\alpha\beta}[k_1,k_3,k_2,k_4]\Big|_{Q^2\to\infty}$$

$$= -2\frac{g^2e^2e_f^2}{\pi^2}\epsilon^{\mu\nu\eta}{}_{\kappa}(k_{1\eta}-k_{3\eta})(2\pi)^4\delta^{(4)}(\sum_{i=1}^{4}k_i)\boxed{\frac{(k_2^\kappa+k_4^\kappa)\epsilon^{\alpha\beta\sigma\lambda}k_{2\sigma}k_{4\lambda}}{(k_2+k_4)^2}}$$

$$\times\prod_{k=2}^{4}\int_0^1 d\beta_k\ \delta(1-\sum_{k=2,3,4}\beta_k)\frac{1}{(k_1+k_2)^2\beta_3+k_1^2\beta_4+k_3^2\beta_2}$$

triangle diagram

Kinematic factor, dependence on x_B and Q^2



$u_1 - u_3 \sim \Lambda_{\mathrm{QCD}}^2/Q^2$

The structure function g_1 in the Bjorken limit

infrared pole

non-local operator

non-pole contribution

$$S^\mu g_1(x_B, Q^2)\Big|_{Q^2\to\infty}$$
$$= \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N} \int_{x_B}^1 \frac{dx}{x} \left(1 - \frac{x_B}{x}\right) \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_\mu\to 0} \frac{l^\mu}{l^2} \langle P', S|\mathrm{Tr_c} F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)|P, S\rangle + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^2}\right)$$

The structure function g_1 is dominated by the triangle anomaly, hence g_1 is topological.

First moment of g_1:

$$S^\mu \int_0^1 dx_B\, g_1(x_B, Q^2)\Big|_{Q^2\to\infty} = \sum_f e_f^2 \frac{\alpha_S}{2i\pi M_N} \lim_{l_\mu\to 0} \frac{l^\mu}{l^2} \langle P', S|\mathrm{Tr_c} F_{\alpha\beta}(0)\tilde{F}^{\alpha\beta}(0)|P, S\rangle$$

matches calculation of the triangle diagram

The pole must be removed by contribution of the pseudo scalar sector

The structure function g_1 in the Regge limit

In the Regge limit $x_B \to 0$ and Q^2 is fixed

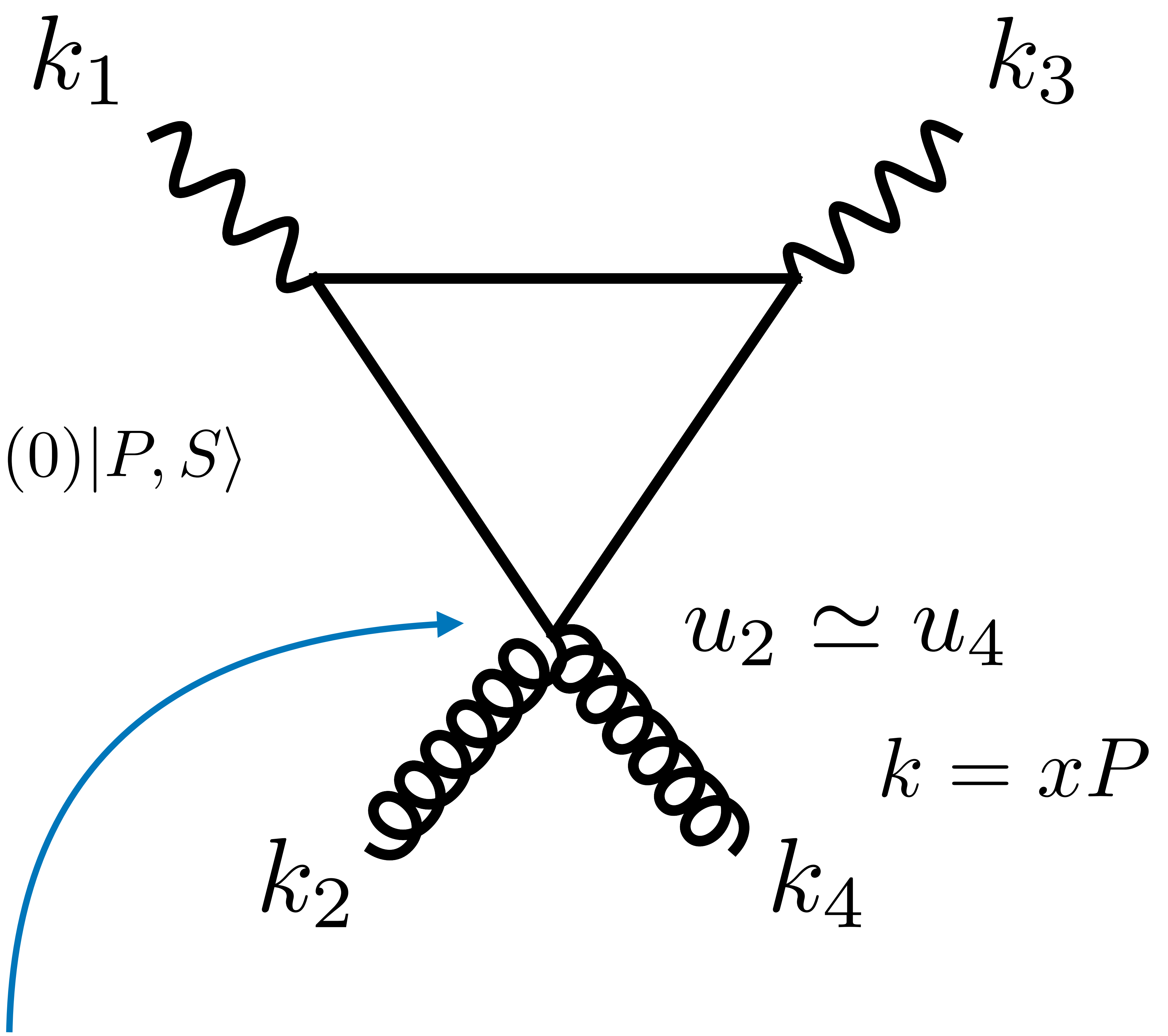
$$S^\mu g_1(x_B, Q^2)\Big|_{Q^2\to\infty}$$

$$= \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N} \int_{x_B}^{1} \frac{dx}{x} \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_\mu \to 0} \frac{l^\mu}{l^2} \langle P', S | \mathrm{Tr}_c F_{\alpha\beta}(\xi n) \tilde{F}^{\alpha\beta}(0) | P, S \rangle$$

In Regge asymptotics: $x > x_B$ and $x_B \to 0$

The result for $g_1(x_B, Q^2)$ is formally identical in Bjorken and Regge limits

$g_1(x_B, Q^2)$ is topological in both asymptotic limits of QCD



Shock-wave approximation

$$u_2 - u_4 \sim \frac{Q_s^2}{2xP \cdot q} \sim x_B$$

Pseudoscalar contribution

To take into account contribution of the pseudoscalar sector we use the most general form of the imaginary part of the effective action

$$W_{\Im}[\Phi, \Pi, A] = \frac{1}{8}\int_{-1}^{1} d\alpha \int_{0}^{\infty} dT\,\mathcal{N} \int_{\mathrm{PBC}} \mathcal{D}x\,\mathcal{D}\psi\,\mathrm{tr}_c\,\mathcal{J}(0)\,\mathcal{P}\,e^{-\int_0^T d\tau \mathcal{L}_\alpha}$$

D'Hoker, Gagne,hep-th/9508131

where the worldline Lagrangian

scalar field

pseudoscalar field

$$\mathcal{L}_\alpha = \frac{\dot{x}^2}{2\mathcal{E}} + \frac{1}{2}\psi_A\dot{\psi}_A - i\dot{x}\cdot A + \frac{i}{2}\mathcal{E}\psi_\mu F_{\mu\nu}\psi_\nu + \frac{1}{2}\mathcal{E}\alpha^2\Phi^2 + \frac{1}{2}\mathcal{E}\Pi^2$$
$$+ i\alpha\mathcal{E}\psi_\mu\psi_6 D_\mu\Phi + i\mathcal{E}\psi_\mu\psi_5 D_\mu\Pi + \alpha\mathcal{E}\psi_5\psi_6[\Pi,\Phi] + \text{ axial vector coupling}$$

We want to perform a perturbative analysis to better understand how the anomalous Ward identities are satisfied

Summary

- We revisit the role of the chiral “triangle” anomaly in deeply inelastic scattering (DIS) of electrons off polarized protons employing a powerful worldline formalism
- We demonstrate how the triangle anomaly appears at high energies in the DIS box diagram
- We show that the anomaly appears in both the Bjorken limit of large Q^2 and in the Regge limit of small x_B
- We find that the infrared pole in the anomaly arises in both limits
- The leading contribution to g_1, in both asymptotics, is given by the expectation value of the topological charge density (it’s relation to ΔG is unclear), which is a generalization of a result previously argued by Jaffe and Manohar for the first moment of g_1
- The cancellation of this pole involves a subtle interplay of perturbative and nonperturbative physics that is deeply related to the $U_A(1)$ problem in QCD