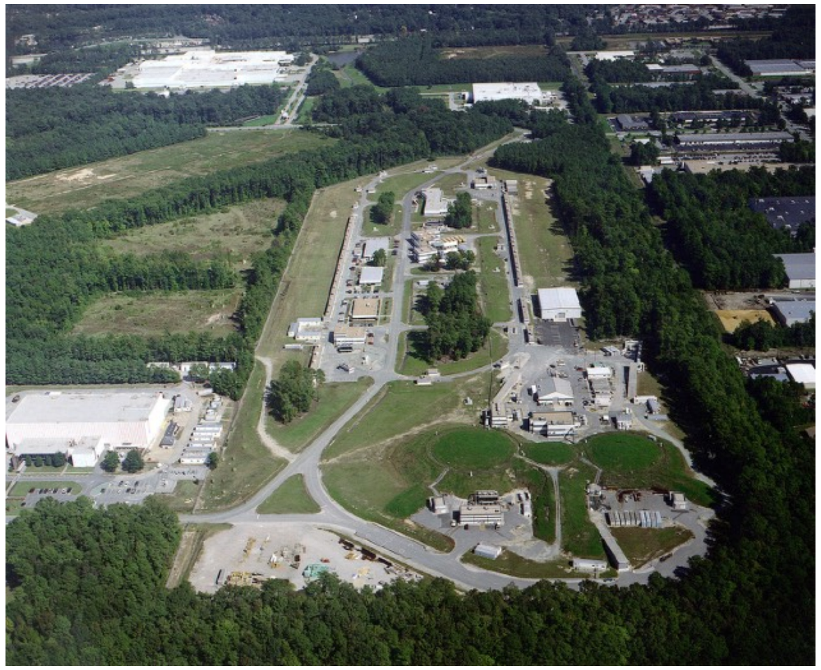
Quantum Monte Carlo Approaches to Lepton-Nucleus Scattering



J. Carlson (LANL)
Jan, 2020

- Motivations
- Nuclear Interactions/Currents
- Inclusive Electron Scattering
- Two-Nucleon Structure/dynamics
- Short-time Evolution
  and two-nucleon dynamics
- Quantum computing approaches
- Summary / Outlook

in collaboration with:
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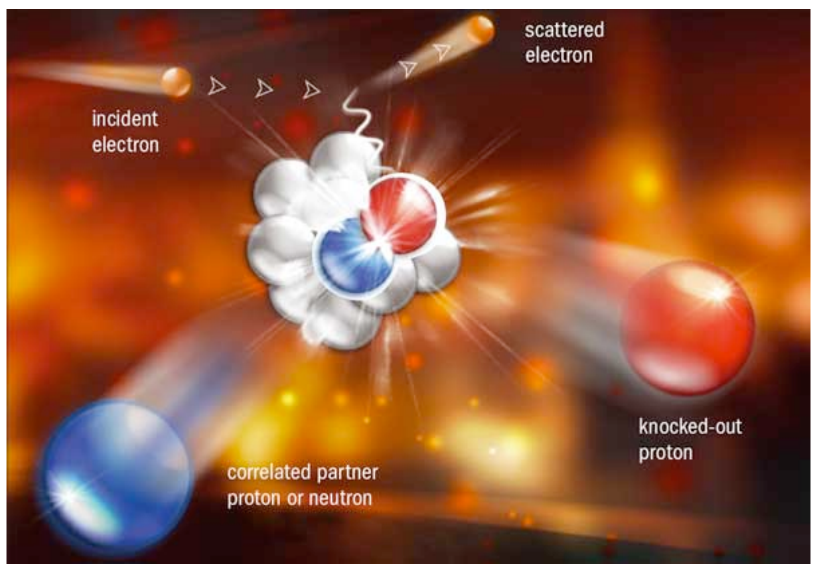
R. B. Wiringa (ANL)

Why study Leptons and Nuclei (nuclear scale and beyond)

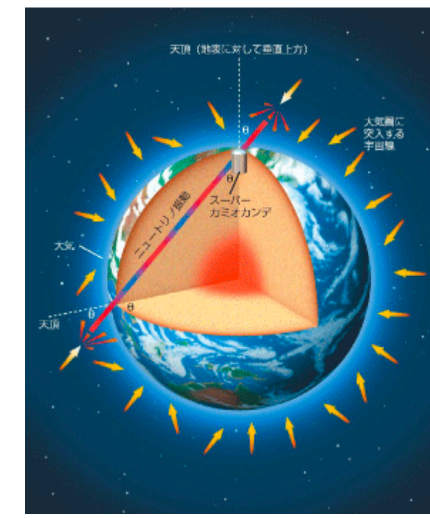
Nuclear structure and dynamics at scale of inter-nucleon spacing
Quasi-elastic scattering : electrons and neutrinos (even 0+ to 0+)
Neutrino Properties: hierarchy, CP violation, double beta decay
Astrophysical Environments: neutron star mergers, supernovae

NP vs PP back-to-back pairs

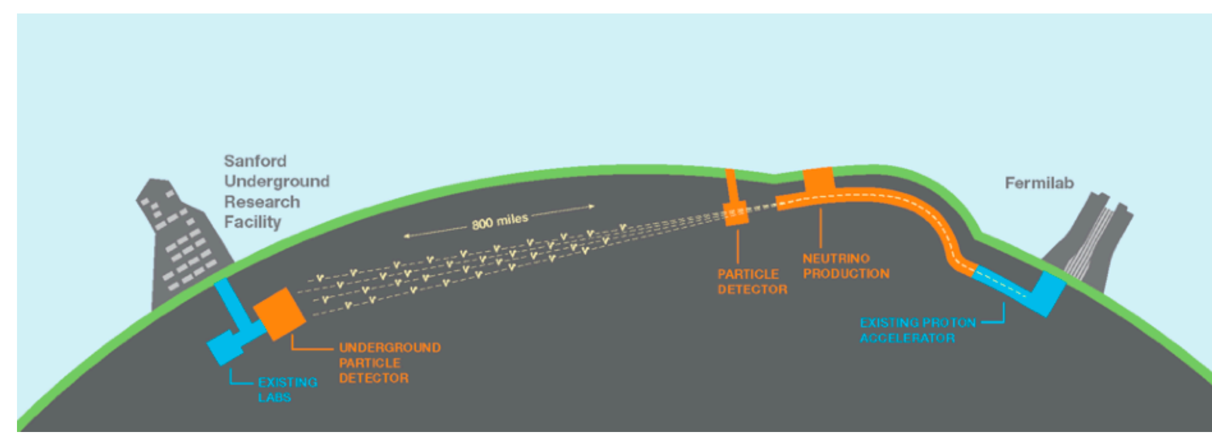


E Piasetzky *et al.* 2006 **Phys. Rev. Lett. 97** 162504.
M Sargsian *et al.* 2005 **Phys. Rev. C 71** 044615.
R Schiavilla *et al.* 2007 **Phys. Rev. Lett. 98** 132501.
R Subedi *et al.* 2008 **Science 320** 1475.

Atmospheric Neutrinos

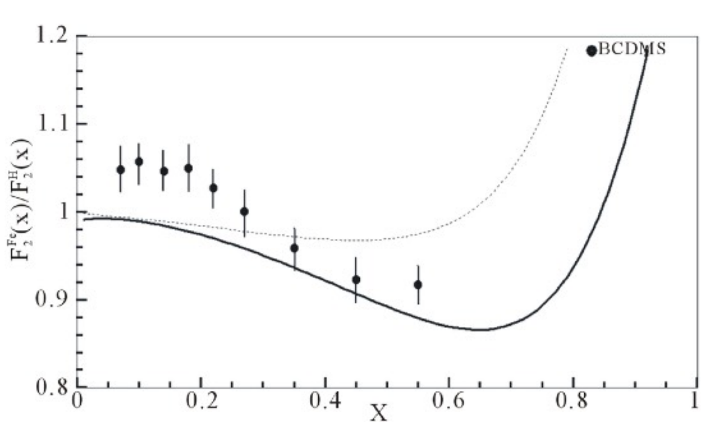


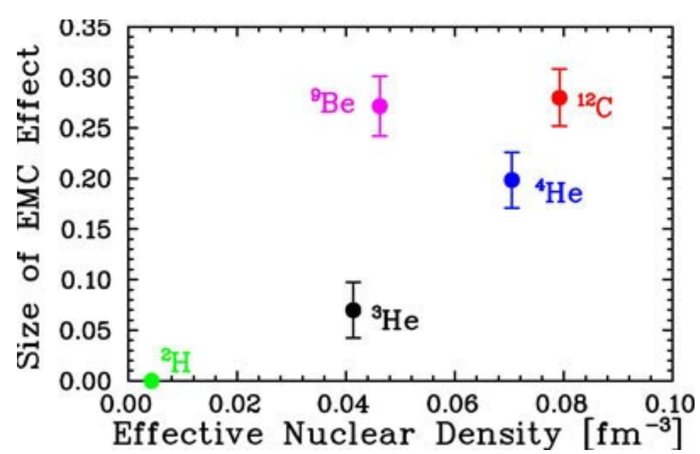
Accelerator Neutrinos



Quasi-Elastic scattering, resonance region and deep inelastic scattering

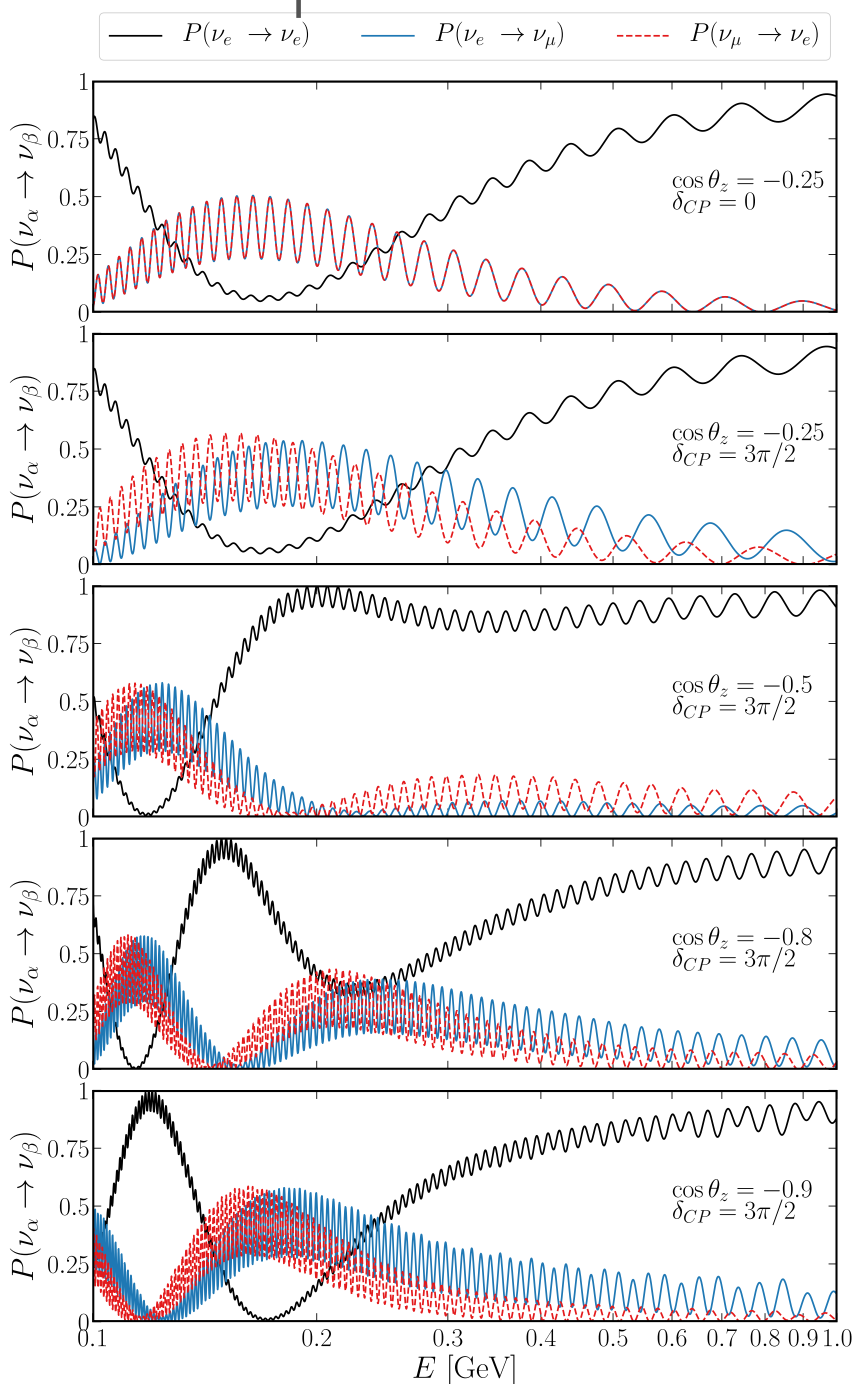
EMC effect (nuclear dependence)





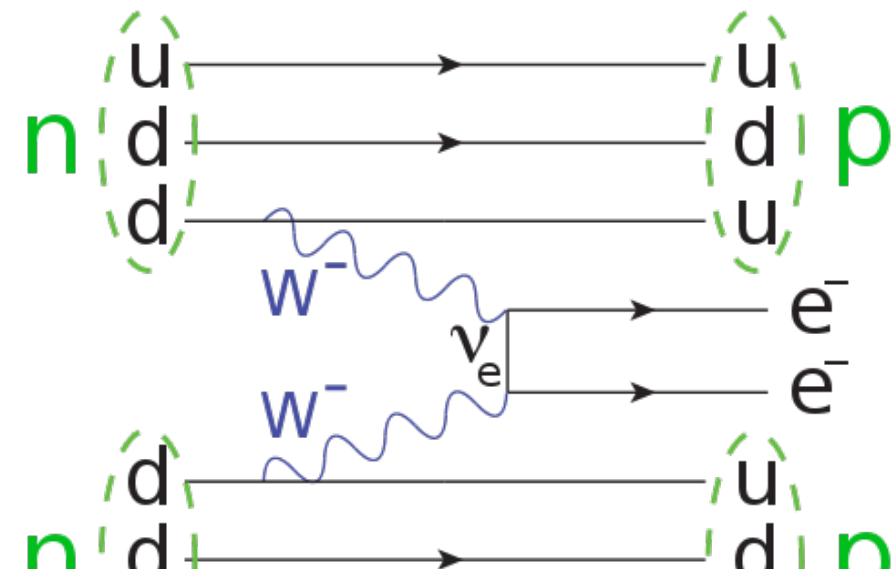
From Nuclear to Hadronic Scales

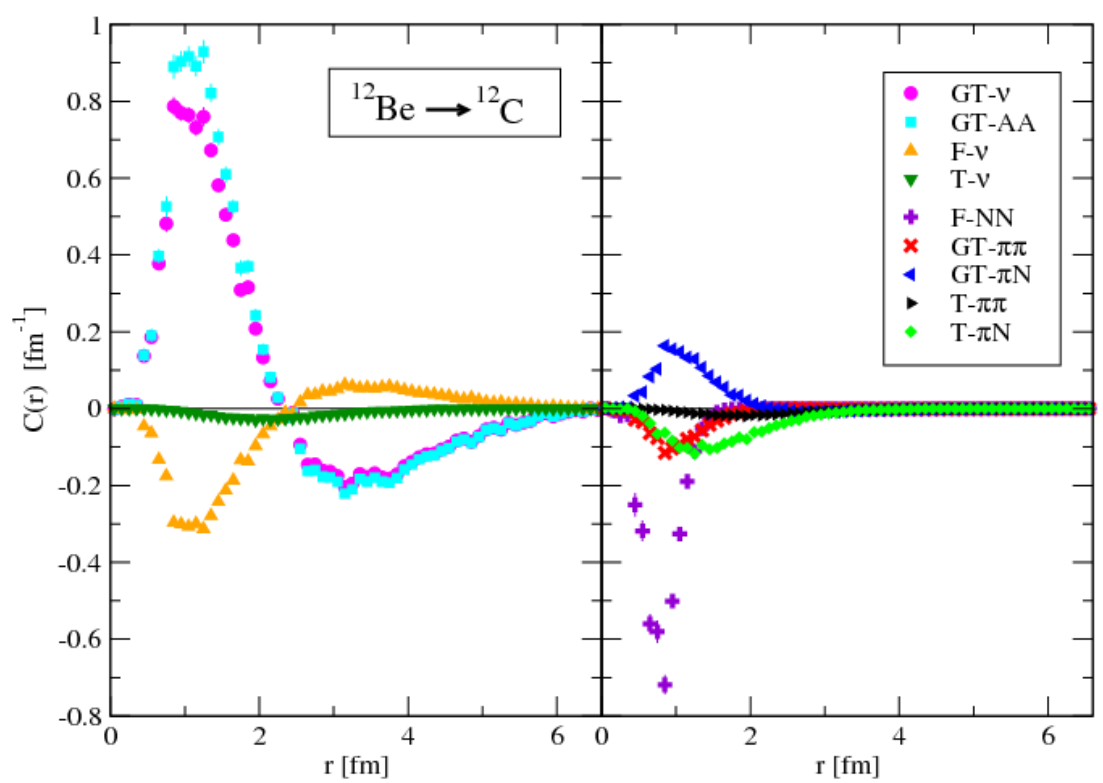
Oscillation Probabilities for Atmospheric Neutrinos



Kelly, et al, [arXiv:1904.02751](https://arxiv.org/abs/1904.02751)

Double Beta Decay



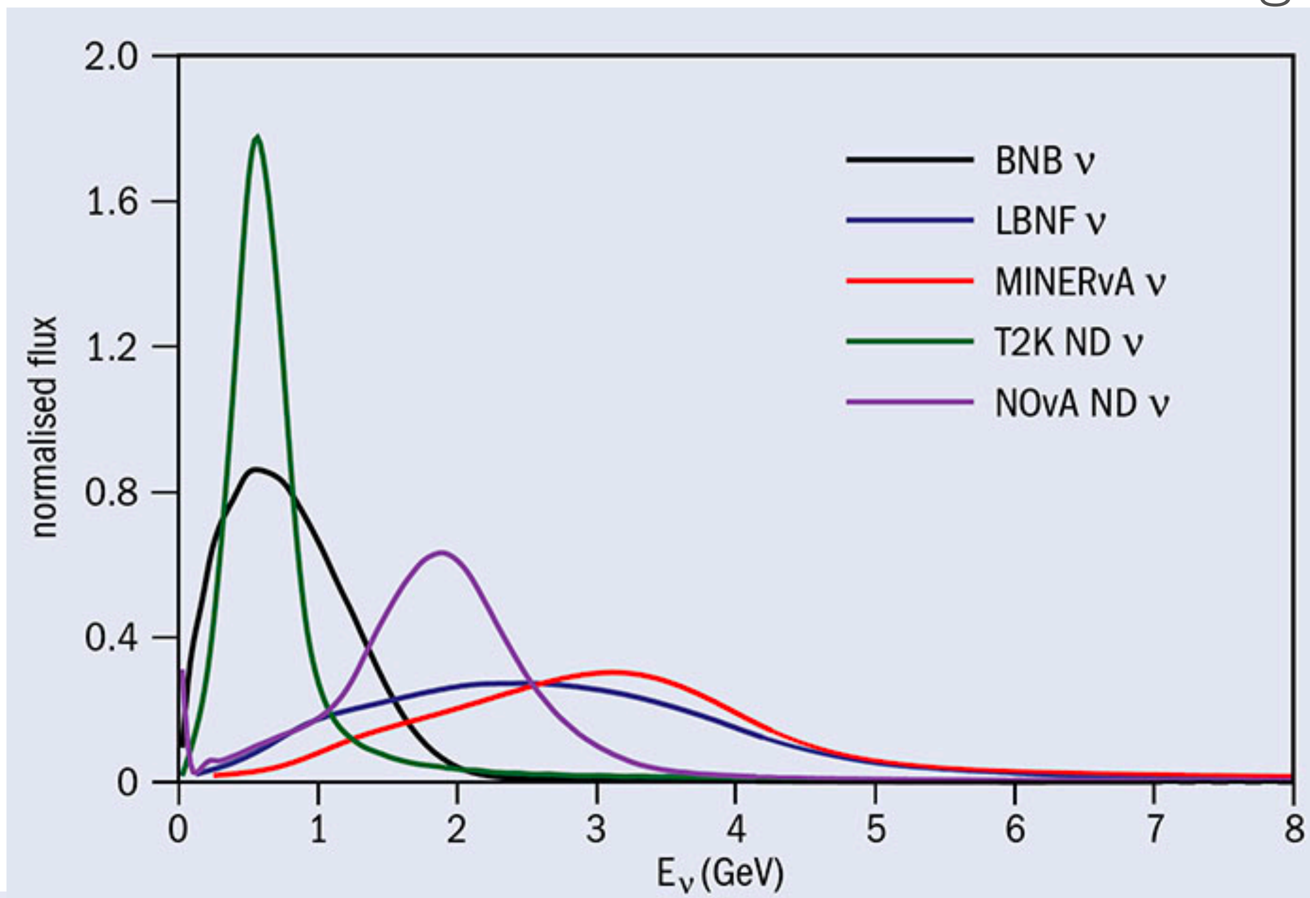


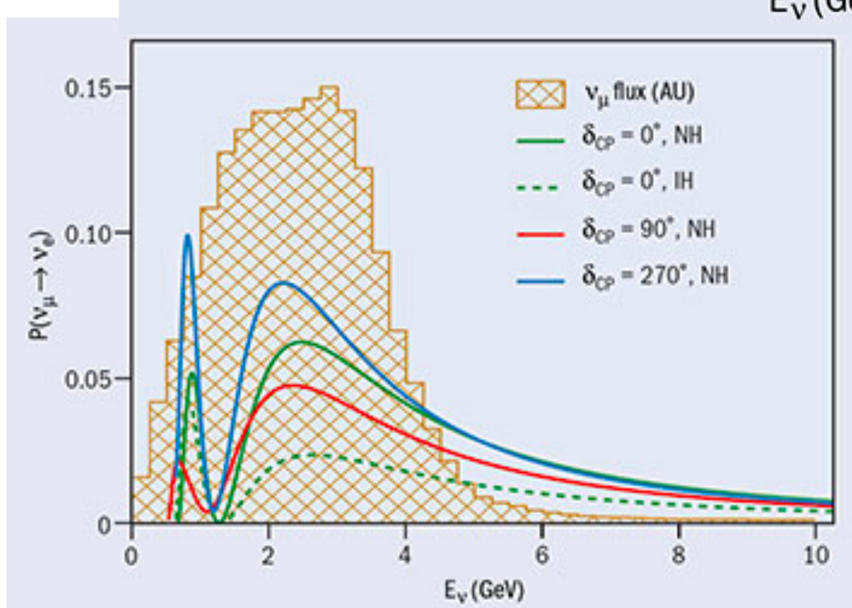
Pastore, et al; PRC 2018

Accelerator Neutrino Experiments

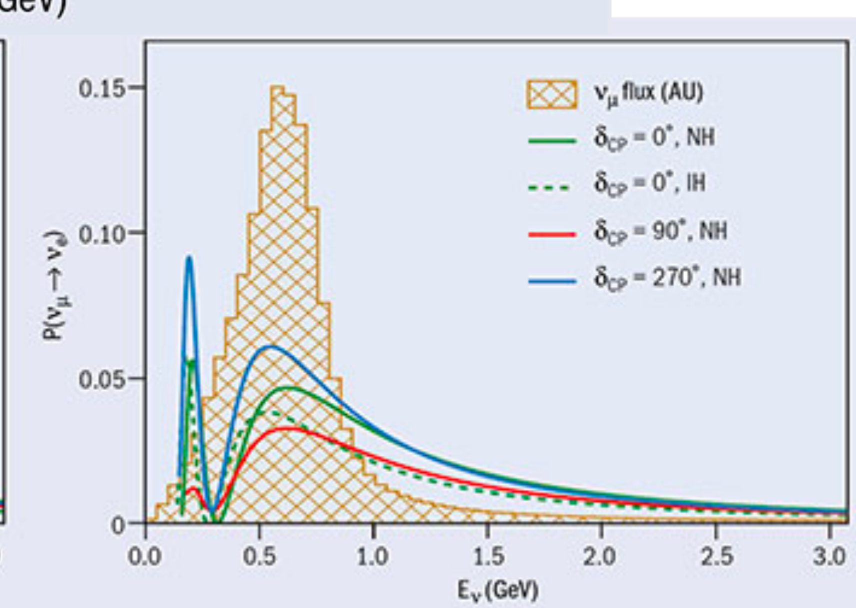
wide range of neutrino energies

importance of oscillations/cross sections for energies ~1-3 GeV



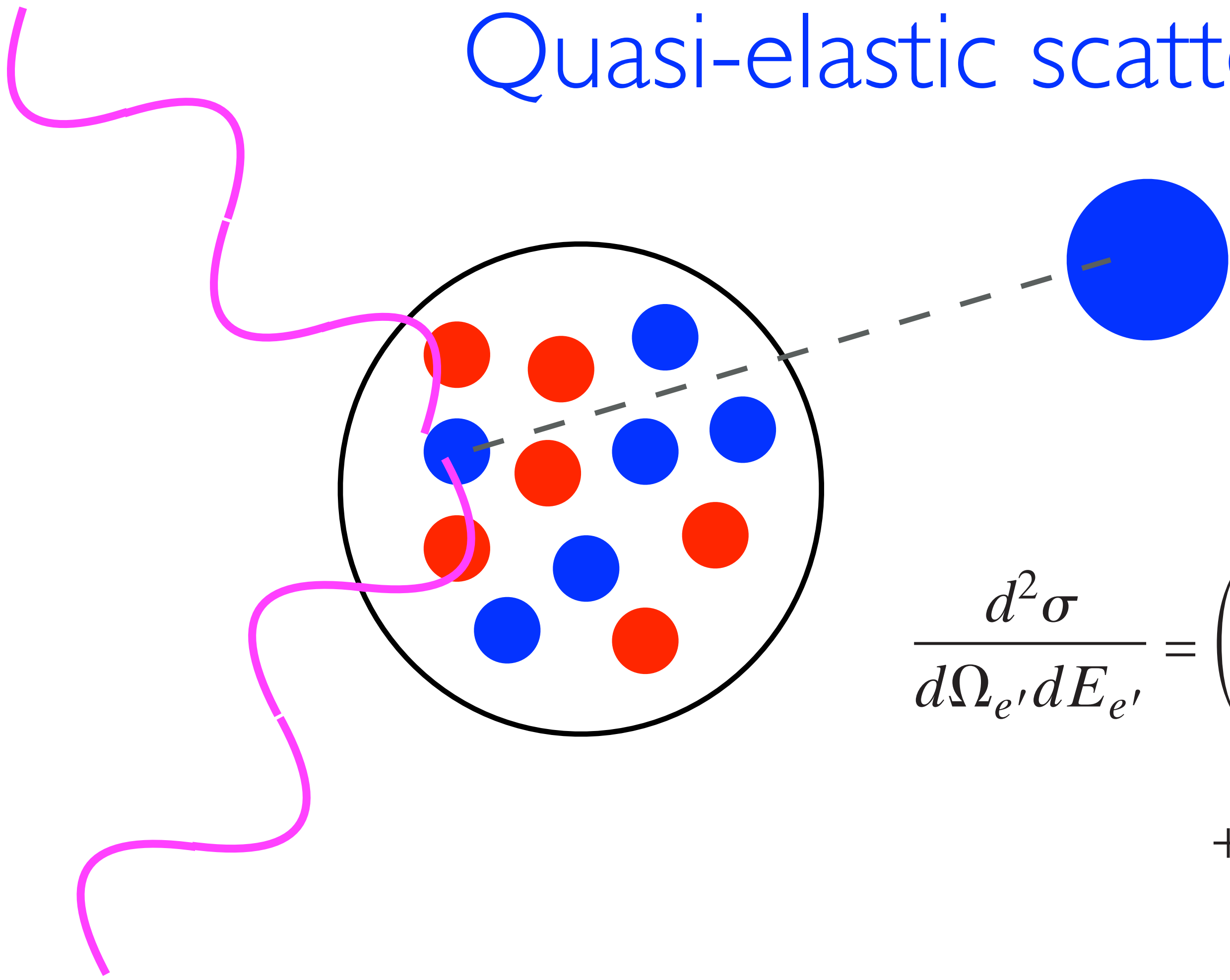


DUNE



T2K

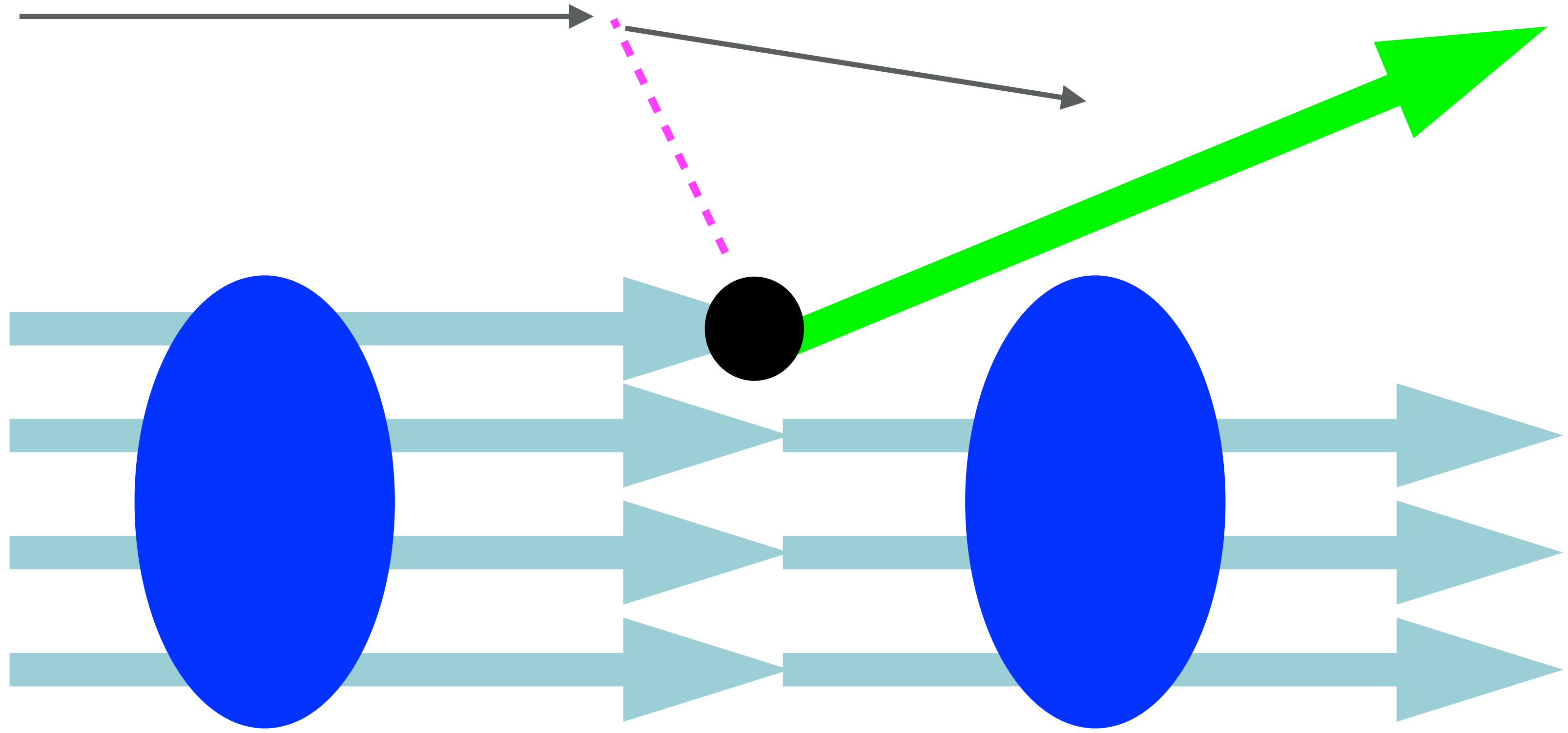
Nuclear Scale:
Quasi-elastic scattering



$$\frac{d^2\sigma}{d\Omega_{e'} dE_{e'}} = \left(\frac{d\sigma}{d\Omega_{e'}}\right)_M \left[\frac{Q^4}{|\mathbf{q}|^4} R_L(|\mathbf{q}|, \omega) + \left(\frac{1}{2}\frac{Q^2}{|\mathbf{q}|^2} + \tan^2\frac{\theta}{2} \right) R_T(|\mathbf{q}|, \omega) \right]$$

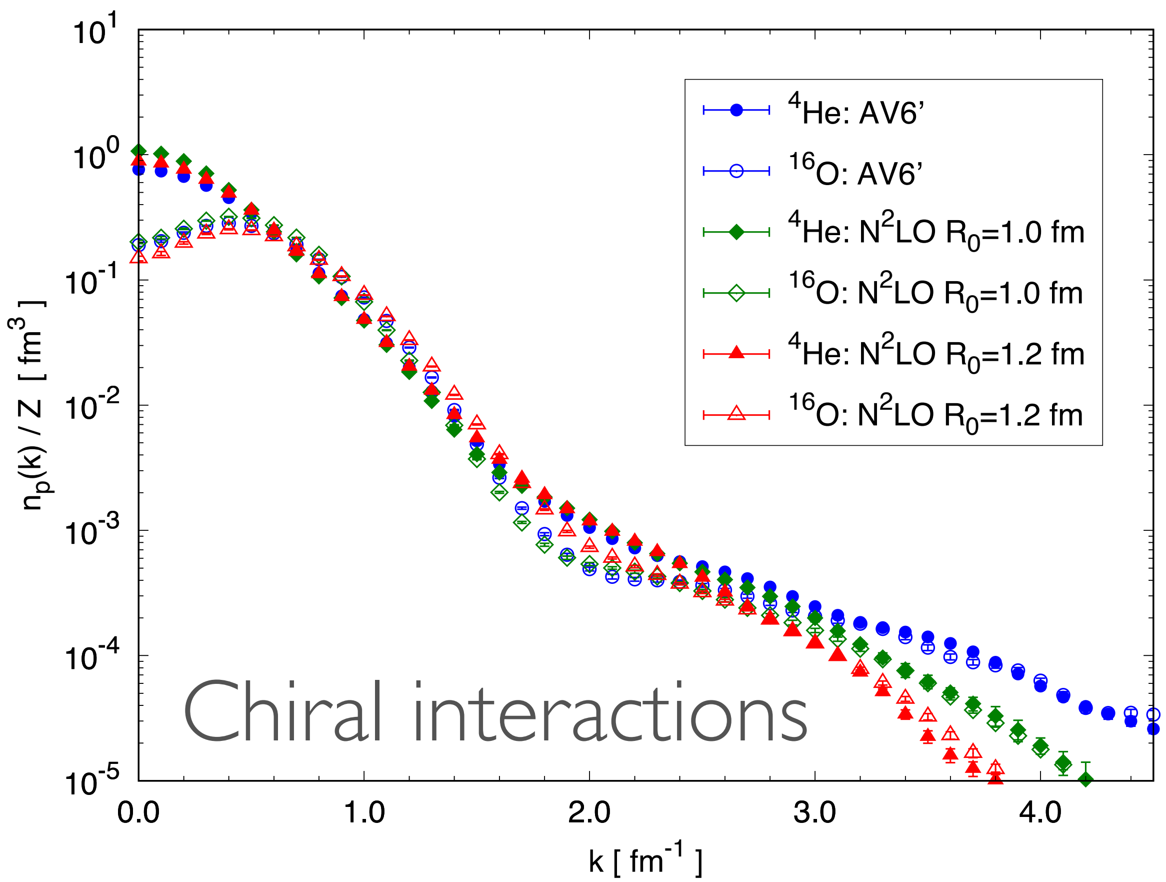
- Scaling with momentum transfer: 'y'-scaling incoherent sum over scattering from single nucleons - scaling of 1st kind
- Target independence: Cross section nearly independent of nuclear target (after counting nucleons) - `superscaling'

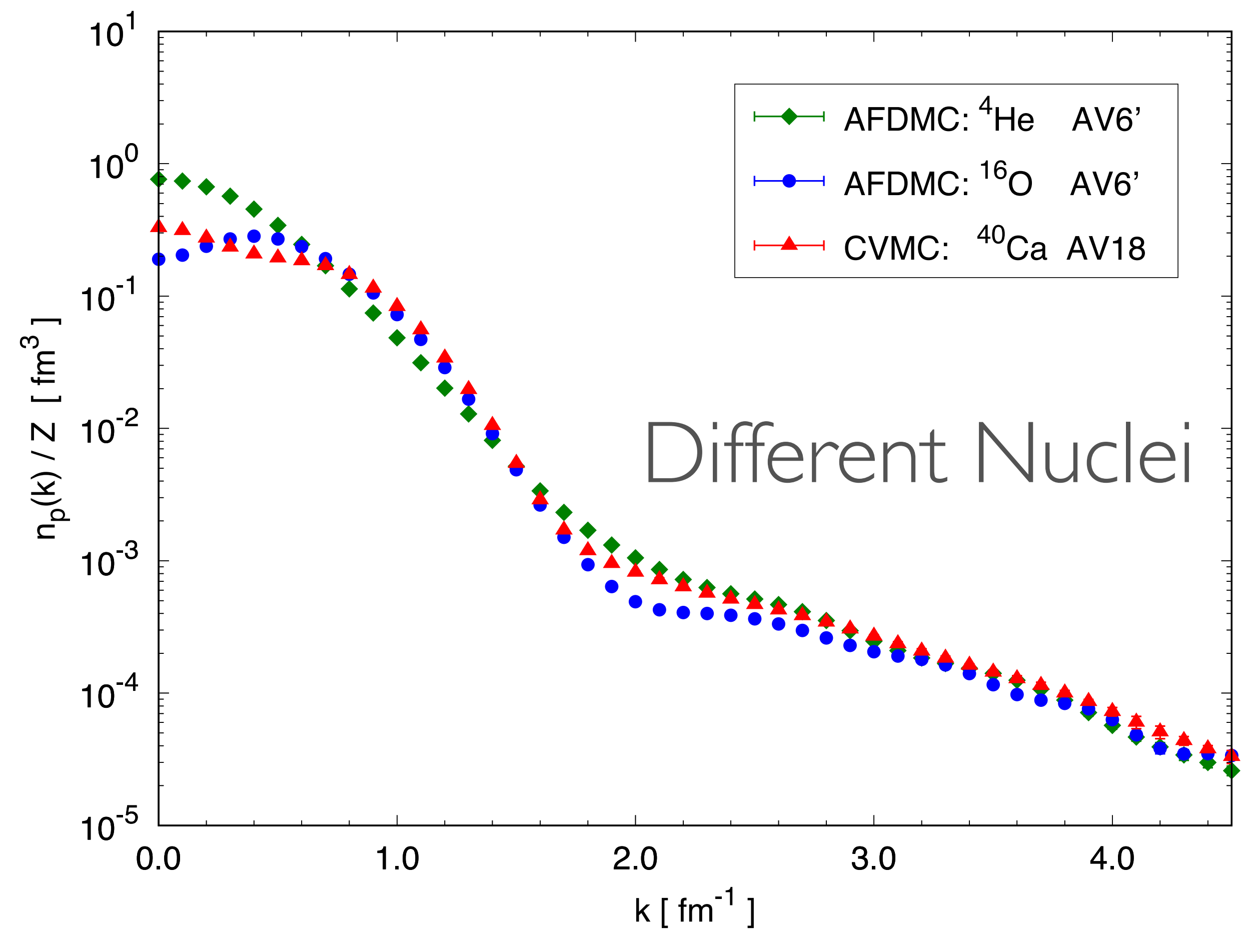
Quasi-Elastic Scattering and Plane Wave Impulse Approximation



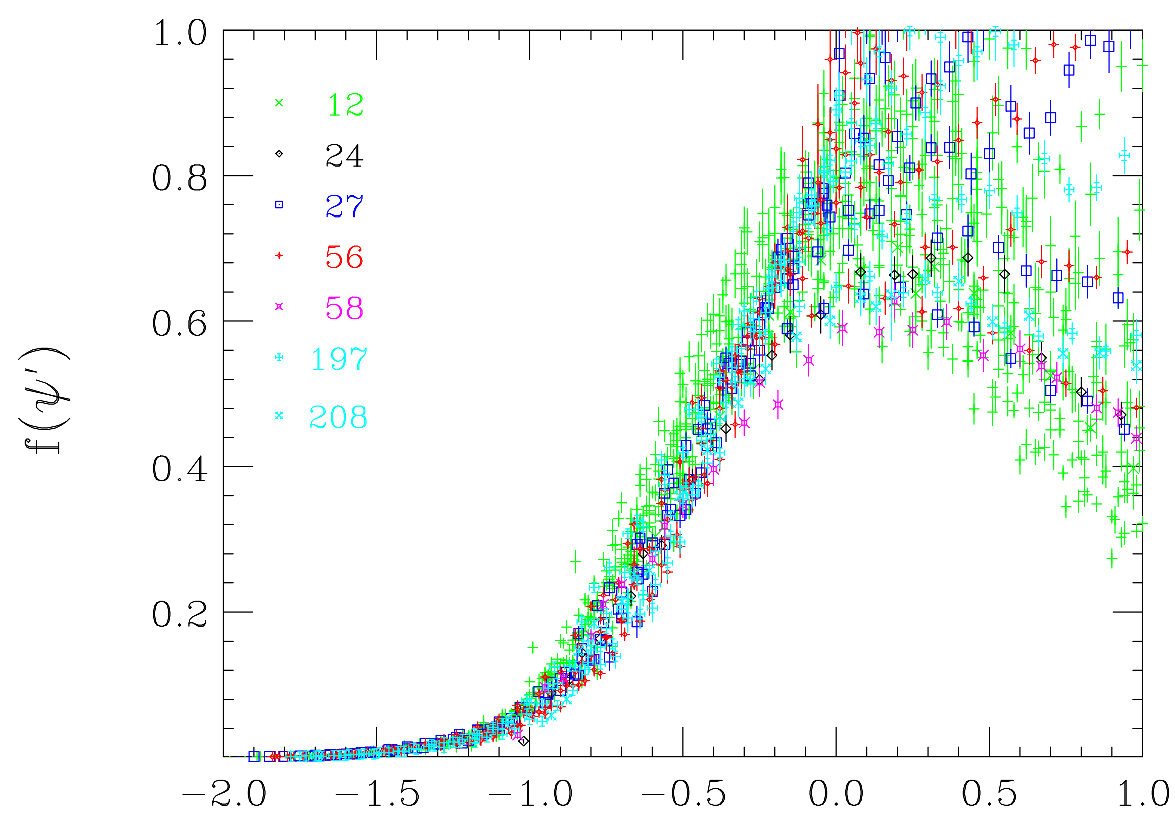
Incorporates incoherent scattering of single nucleons:
n(k) or spectral function S(k,w)
and single-nucleon form factors

Single-Nucleon Momentum Distributions





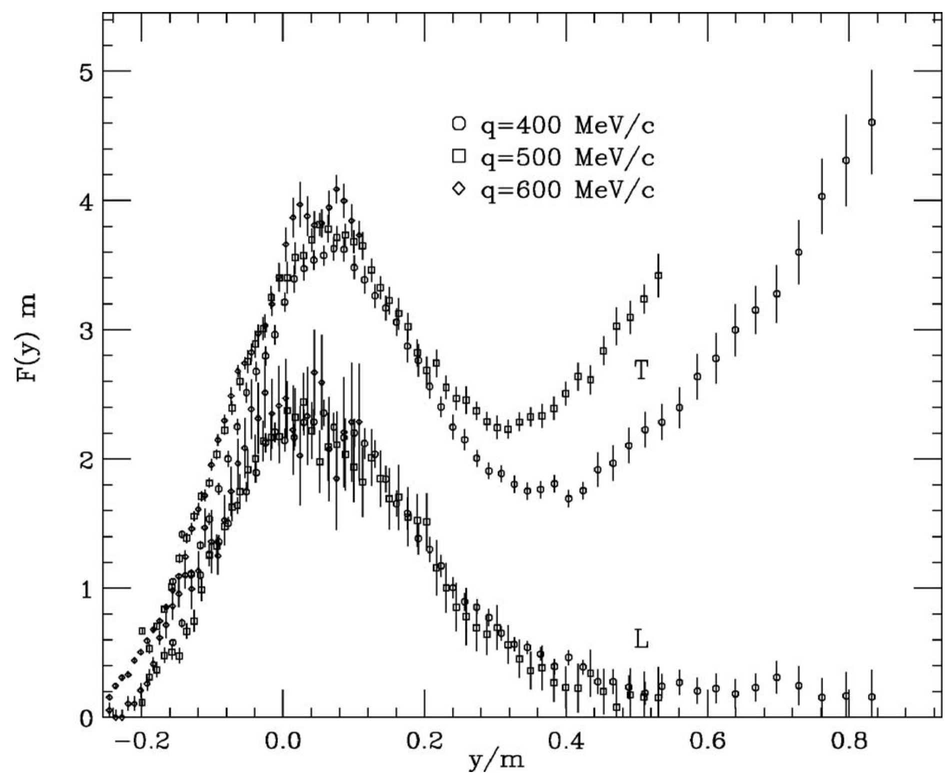
Lonardoni, Gandolfi, Wiringa, Pieper, et al



Integrated Strength: 15-20 % above k_F, Amplitude ~ 0.3-0.4

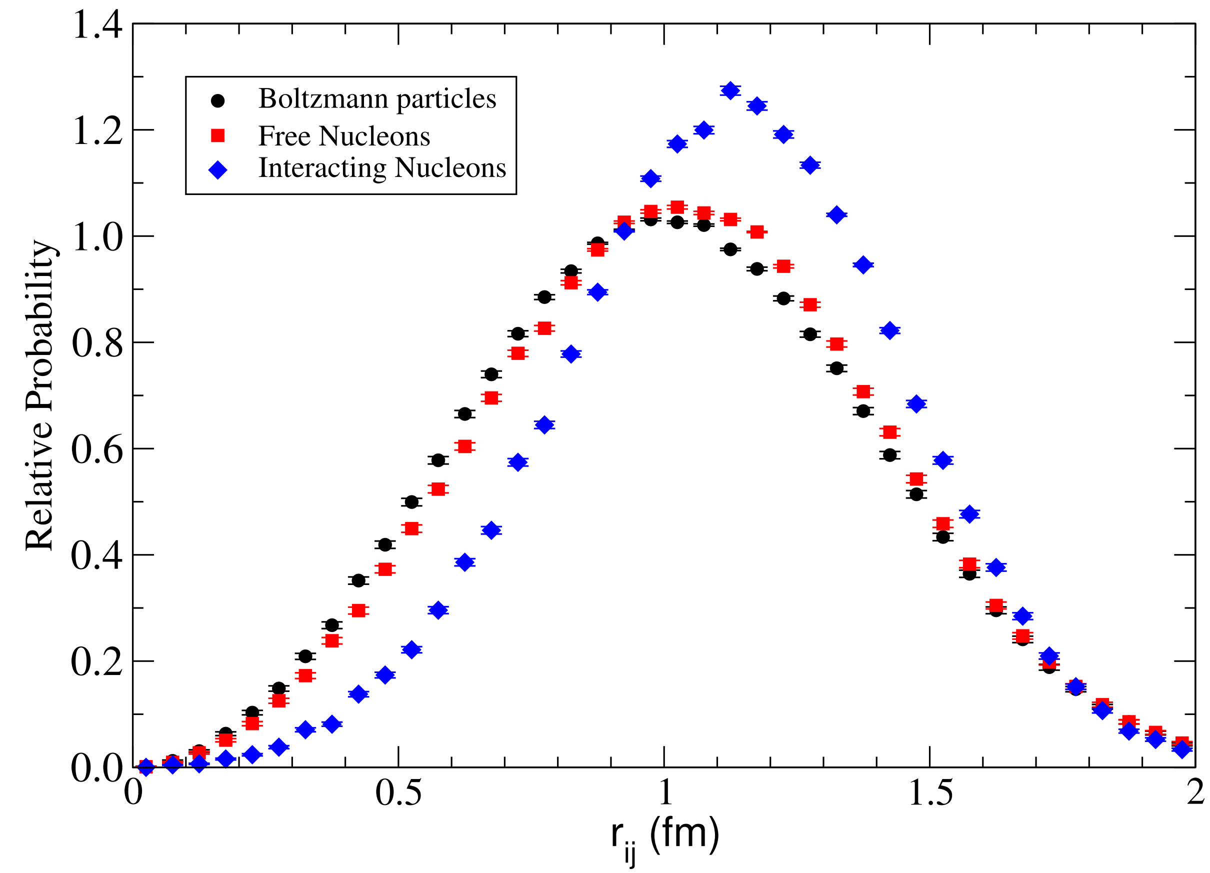
Scaling of the 1st kind (w/ p)
Donnelly & Sick (1999)

But, scattering from a single nucleon not the whole story



Scaled longitudinal vs. transverse scattering from ^{12}C

from Benhar, Day, Sick, RMP 2008
data Finn, et al 1984



Nearest-neighbor distances in nuclear matter

Distances probed at various q

q	$r \sim \pi/q$
0.3 GeV/c	2.1 fm
0.5 GeV/c	1.2 fm
1 GeV/c	0.6 fm

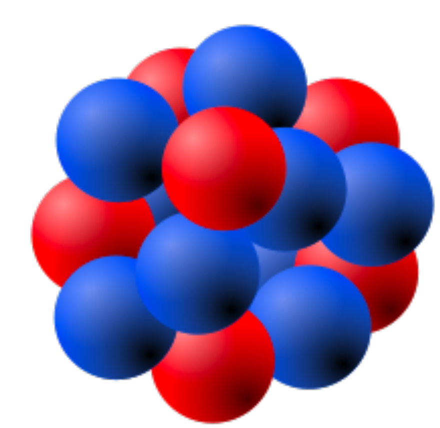
Nearest neighbor nucleons at

$\rho = 0.16\ \mathrm{fm}^{-1} = 1\ /\ (4/3\ \pi\ r^3)$

$r = 1.14$ fm

$d = 2.28$ fm

$1/m_\pi \sim$ **1.5 fm**



Electron Scattering:
Longitudinal and Transverse Response

Transverse (current) response:

$$R_T(q,\omega) \;=\; \sum_f \langle 0|\; \mathbf{j}^\dagger(q)\; |f\rangle\langle f|\; \mathbf{j}(q)\; |0\rangle\; \delta(w-(E_f-E_0))$$

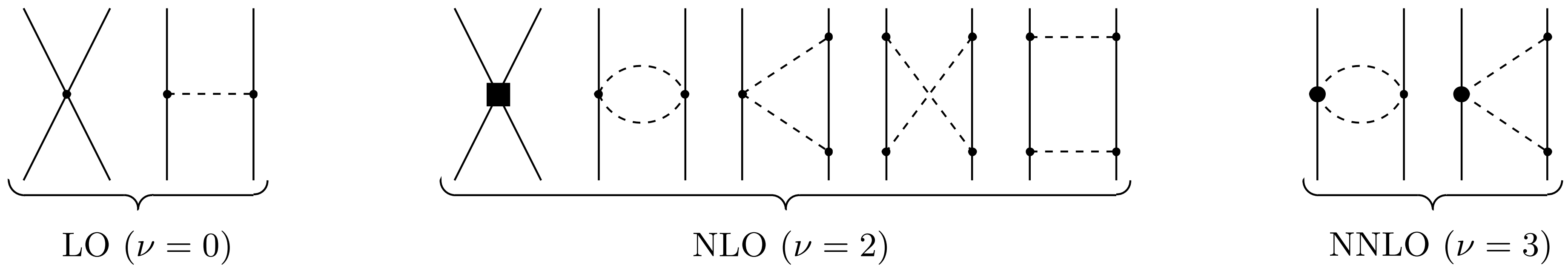
Longitudinal (charge) response:

$$R_L(q,\omega) \;=\; \sum_f \langle 0|\; \rho^\dagger(q)\; |f\rangle\langle f|\; \rho(q)\; |0\rangle\; \delta(w-(E_f-E_0))$$

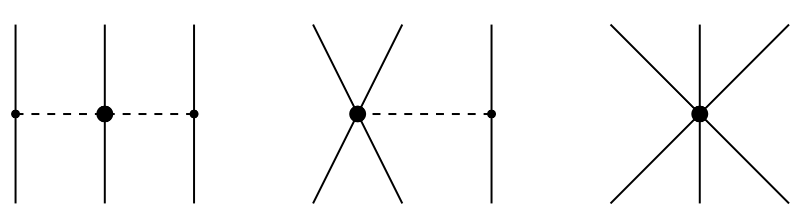
Requires models of nuclear interactions and currents

Basic building blocks: Nuclear interactions and currents

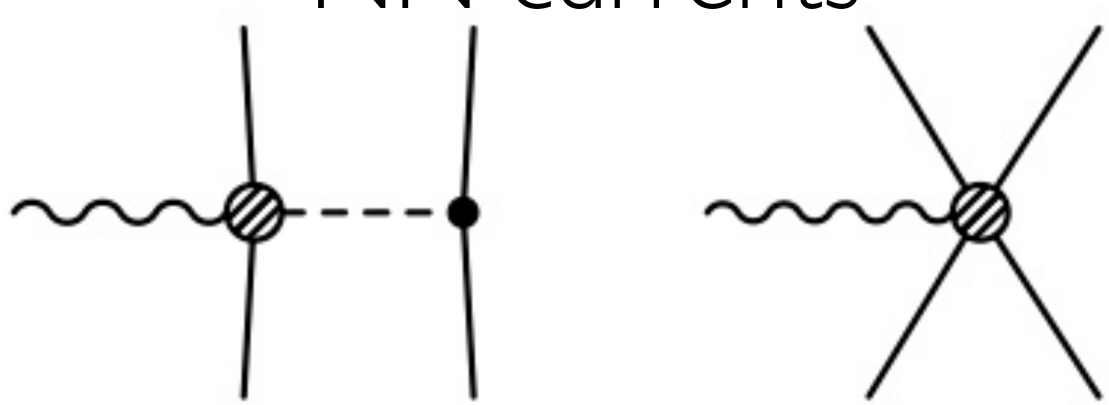
NN interactions



3N interactions

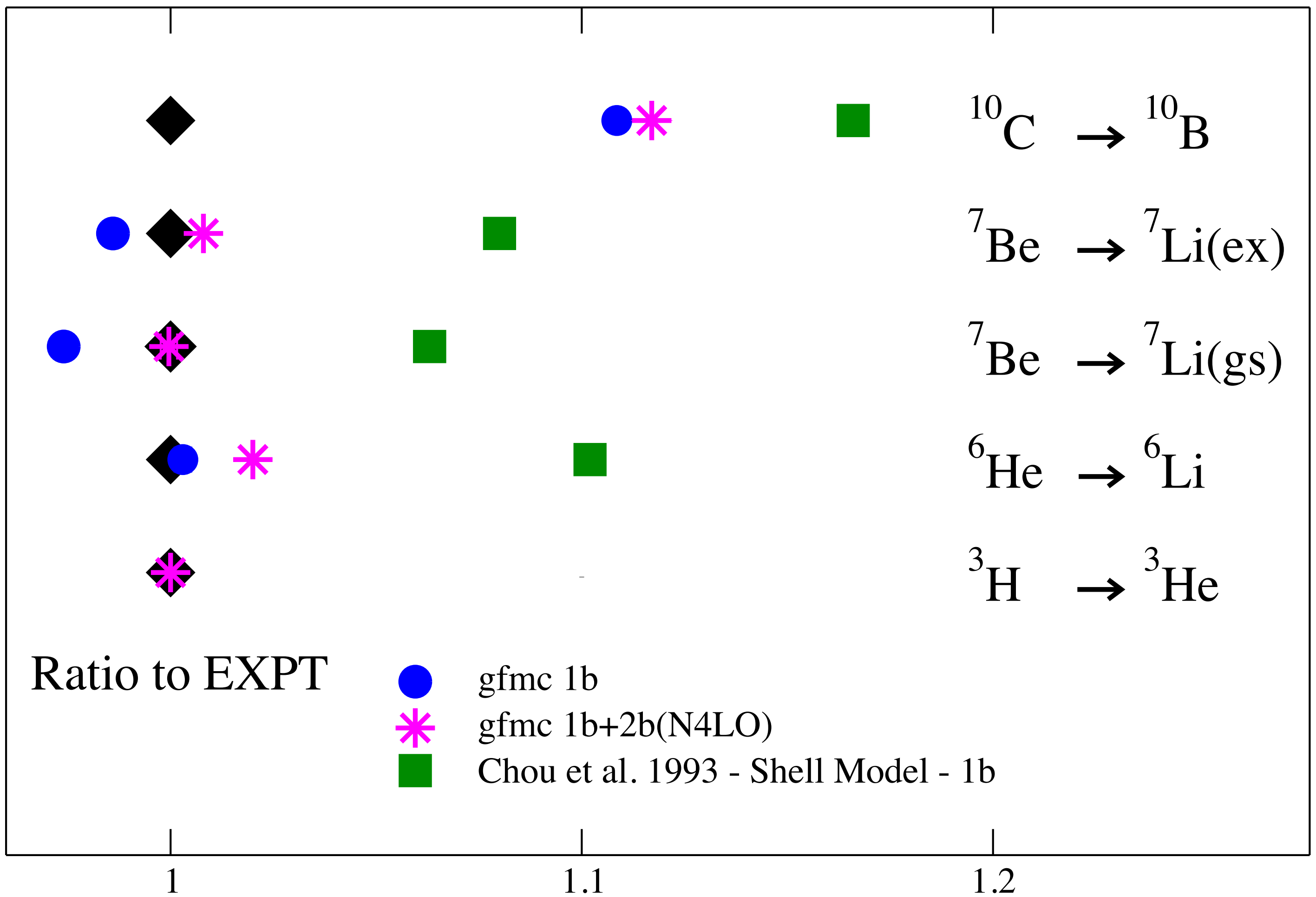


NN currents



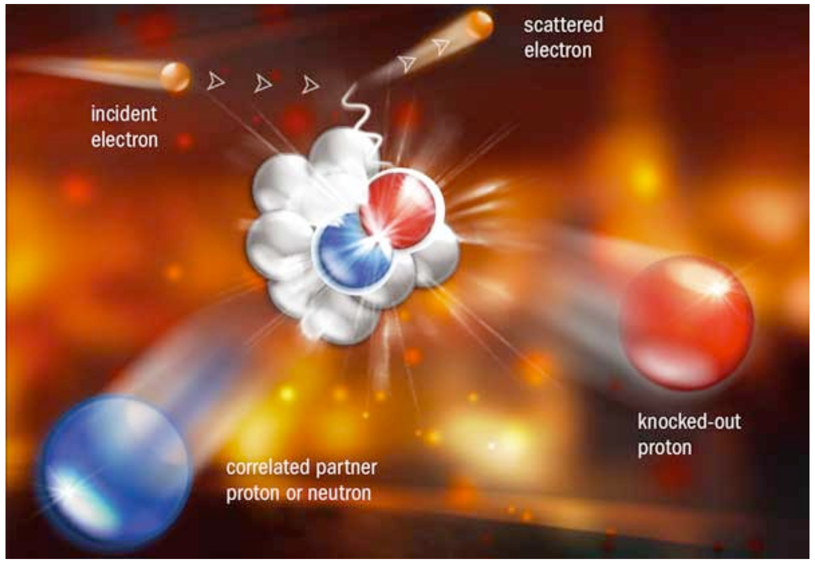
Low Momenta - Beta Decay in Light Nuclei

Pastore, et al, 2017

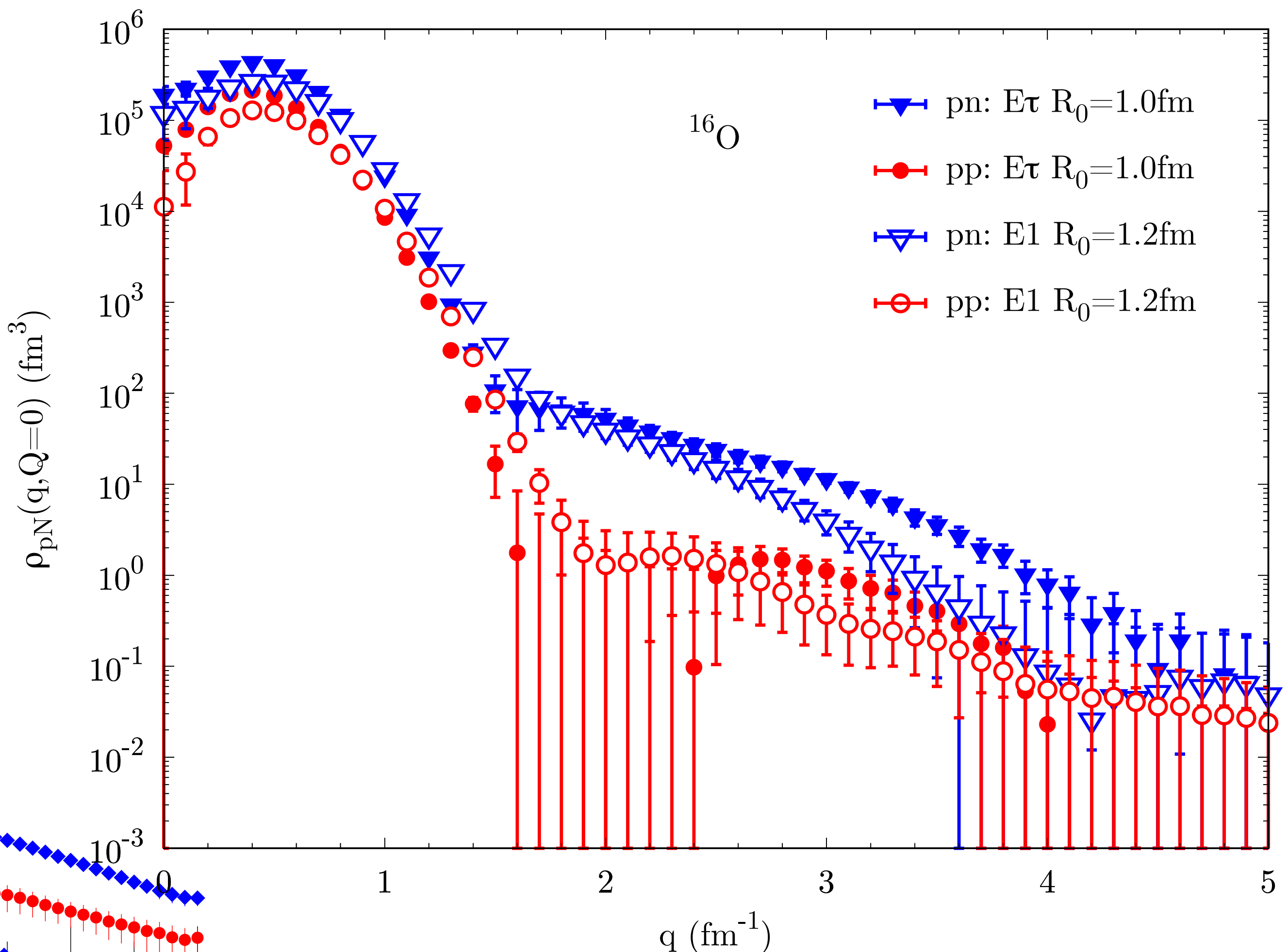


- Contact fit to Tritium beta decay
- Substantial reduction due to two-body correlations
- Modest 2N current contribution
- Good description of experimental data, explains 'quenching'
- Many calculations with larger nuclei underway

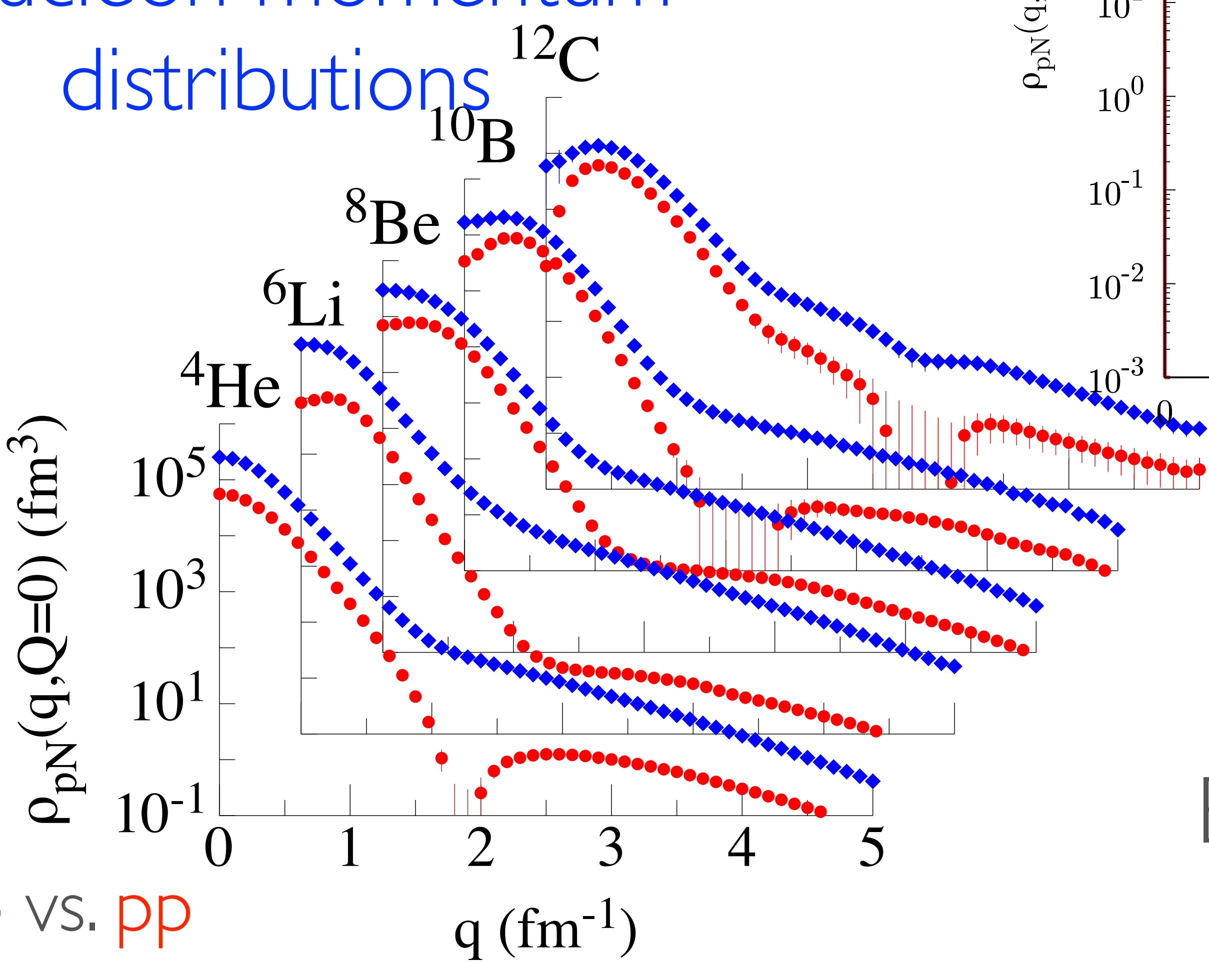
Back to Back Nucleons (total Q~0)
np pairs dominate over nn and pp



E Piasetzky *et al.* 2006 **Phys. Rev. Lett. 97** 162504.
M Sargsian *et al.* 2005 **Phys. Rev. C 71** 044615.
R Schiavilla *et al.* 2007 **Phys. Rev. Lett. 98** 132501.
R Subedi *et al.* 2008 **Science 320** 1475.



2-nucleon momentum distributions

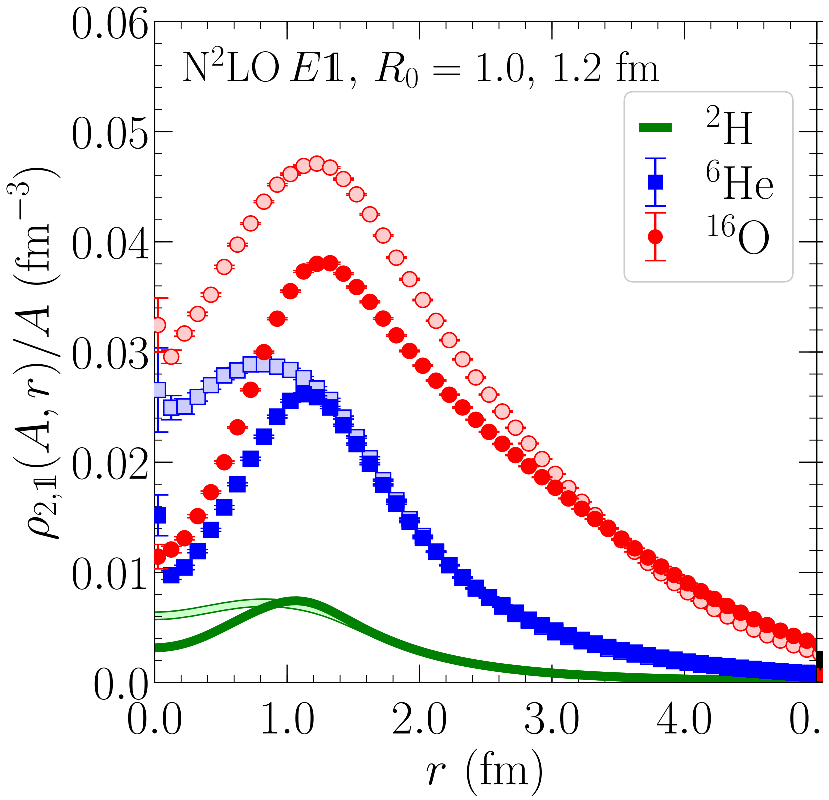


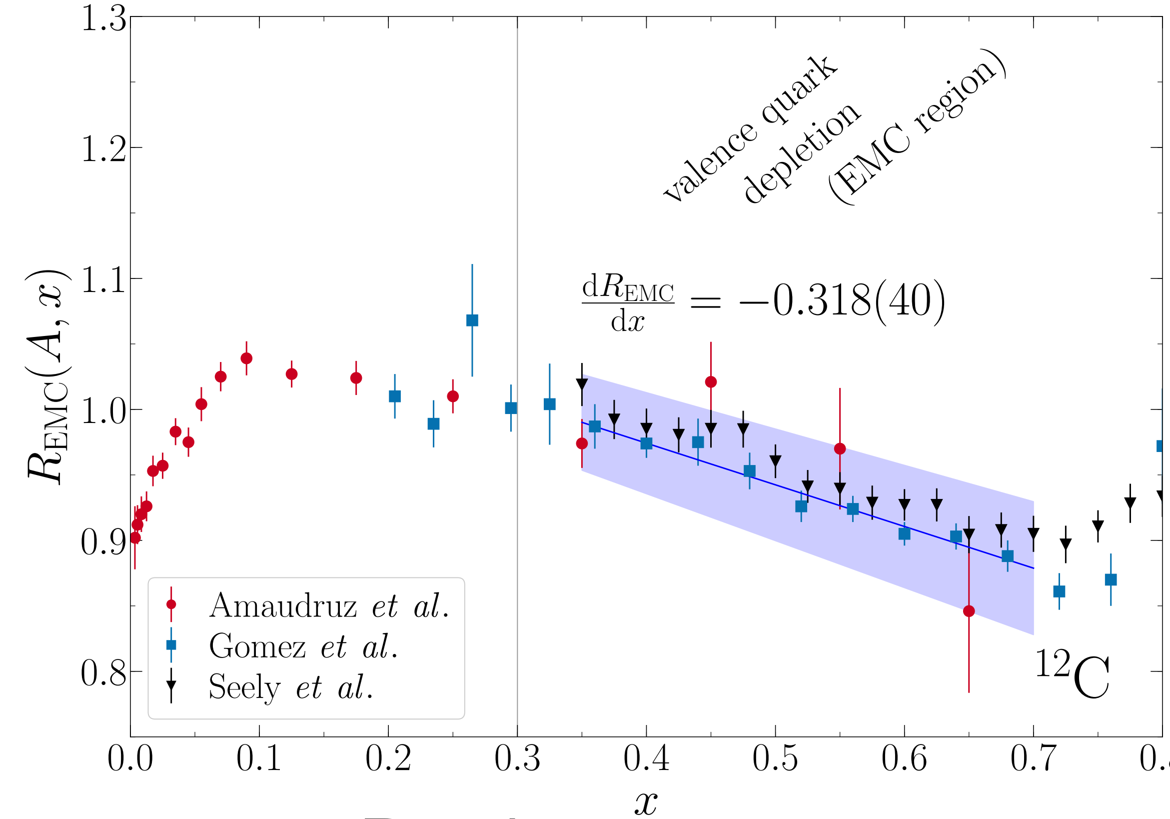
np vs. pp

Wiringa et al.; Carlson, et al, RMP 2015

Bob Wiringa, Diego Lonardoni

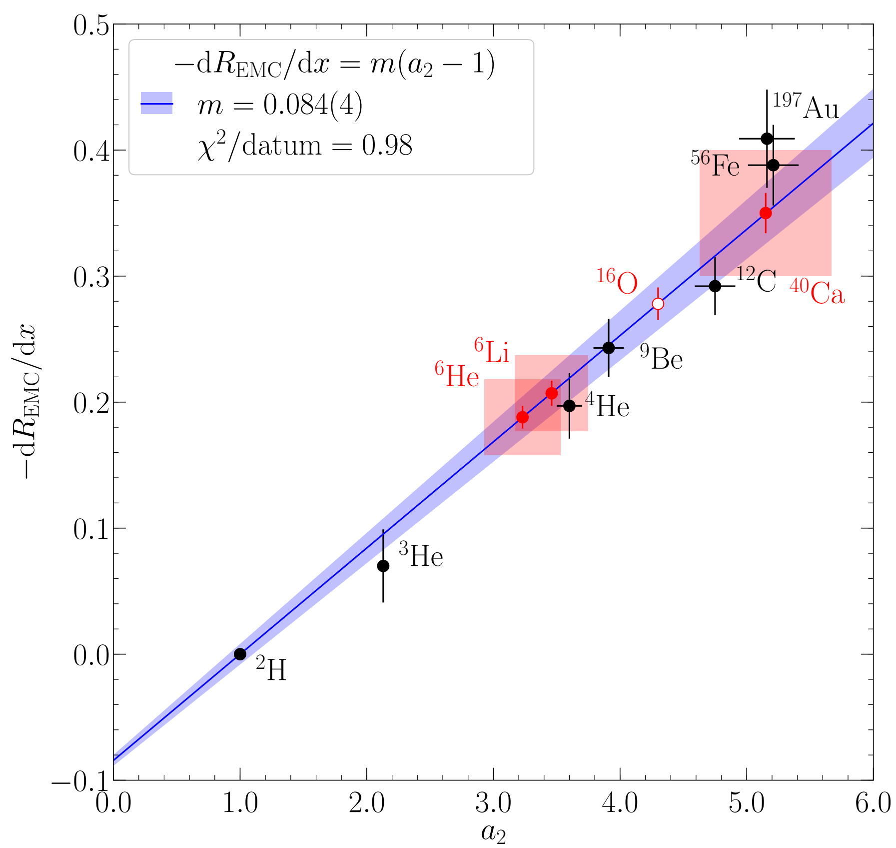
Nucleon pairs at short distance and nuclear EMC



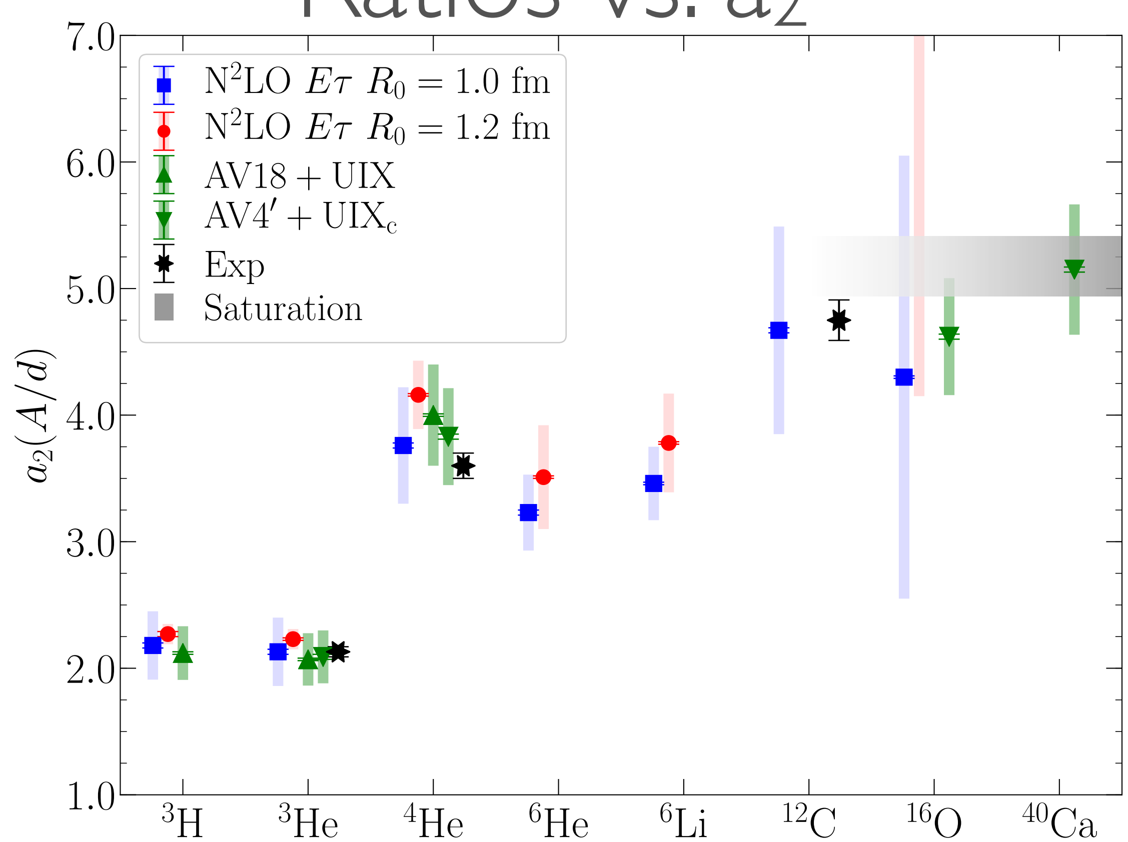


EMC

EMC slope vs A



Ratios vs. a_2



Lynn, et al, 1903.12587

Electron Scattering:
Longitudinal and Transverse Response

Transverse (current) response:

$$R_T(q,\omega) \; = \; \sum_f \langle 0| \; \mathbf{j}^\dagger(q) \; |f\rangle\langle f| \; \mathbf{j}(q) \; |0\rangle \; \delta(w - (E_f - E_0))$$

Longitudinal (charge) response:

$$R_L(q,\omega) \; = \; \sum_f \langle 0| \; \rho^\dagger(q) \; |f\rangle\langle f| \; \rho(q) \; |0\rangle \; \delta(w - (E_f - E_0))$$

Requires models of nuclear interactions and currents

Connections to Lattice QCD: one- and two-N matrix elements

- Elastic Nucleon form factors (particularly axial)
- Inelastic form factors:
  - Inclusive (sum over all all hadronic final states): constrains hadronic input
  - Exclusive (e.g. specific pi-N final state)
- Two-Nucleon matrix elements w/ current insertions (particularly for NN final state)

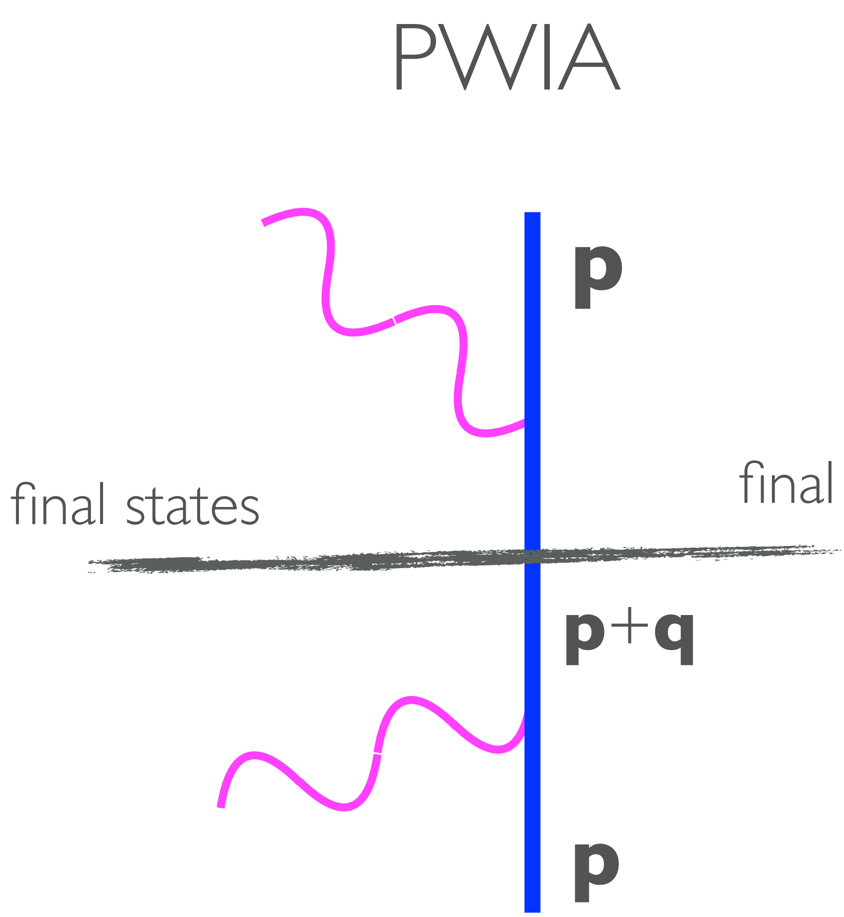
Solutions or advances on dealing with sign problem imaginary to real time response and dynamics

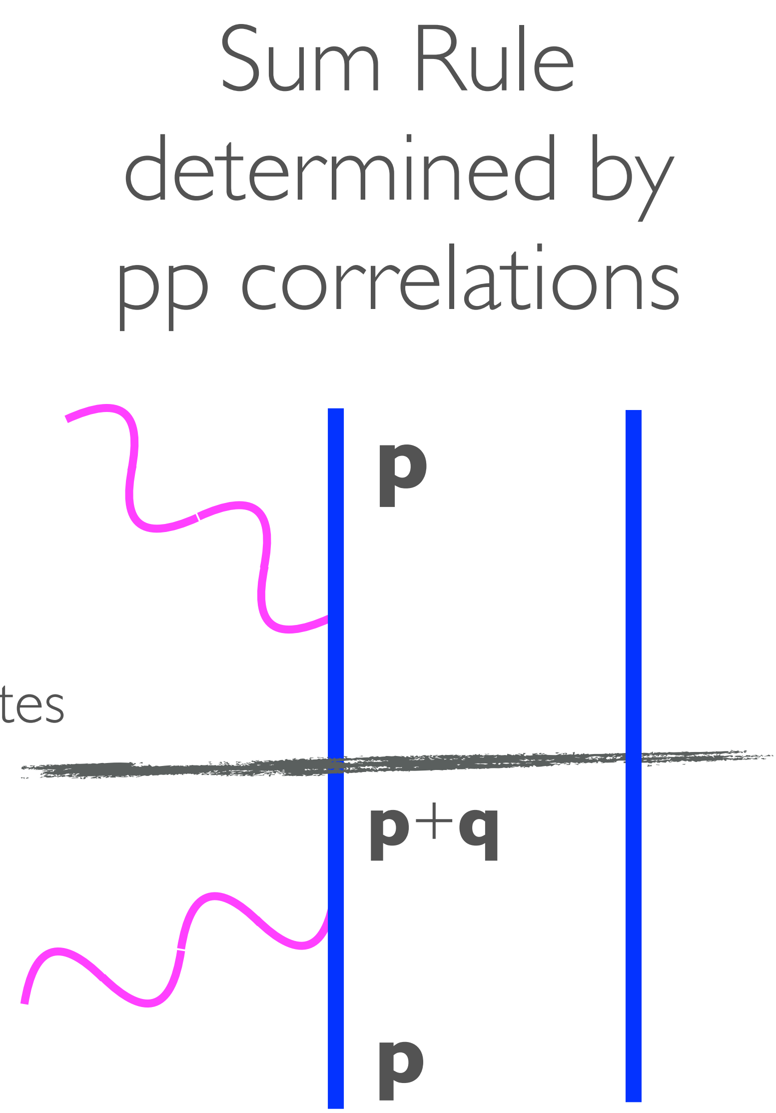
....

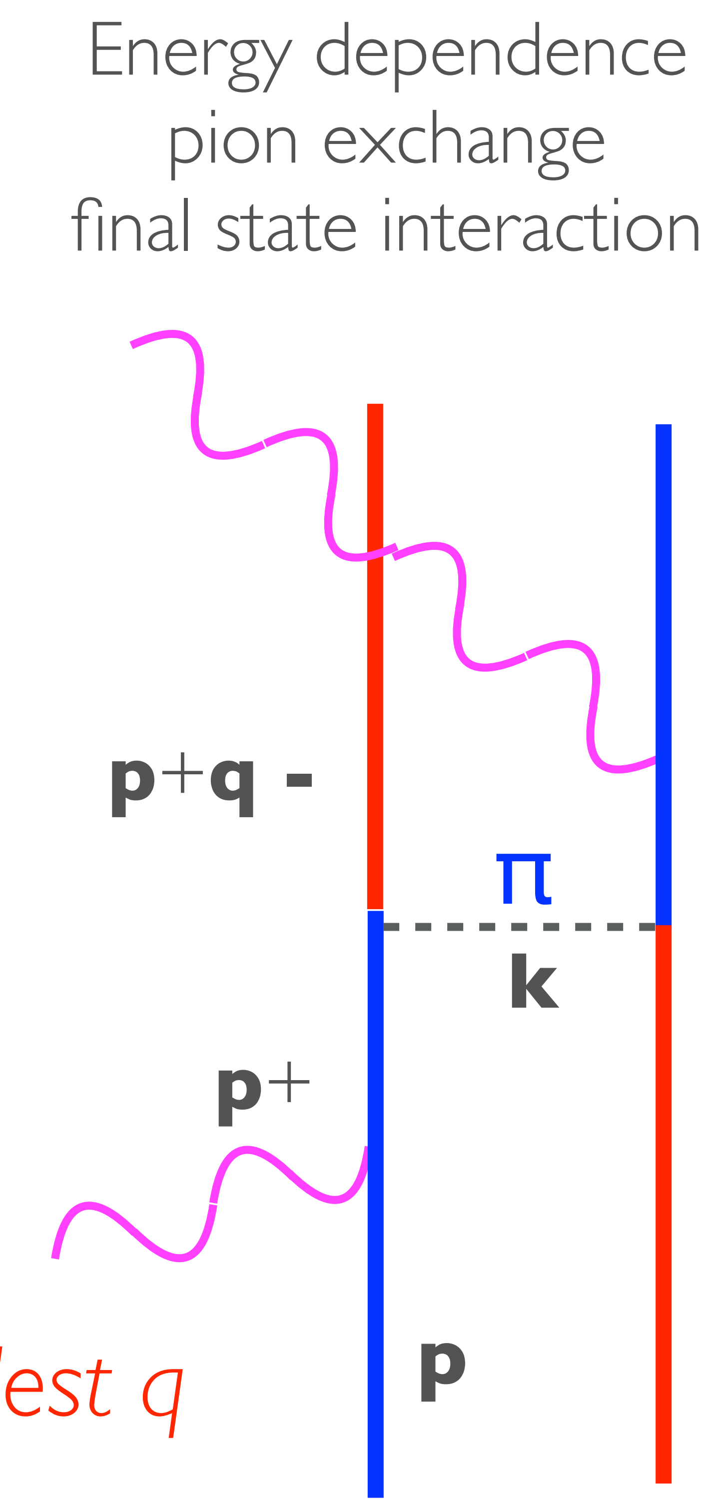
Nearly Static Poperty: Sum Rules (Longitudinal Response)

$$S\ (q)\ =\ \langle\ 0\ |\ \mathbf{j}^{\dagger}(q)\ \mathbf{j}(q)\ 0\ \rangle$$

Gives an indication of total strength,
but not energy dependence







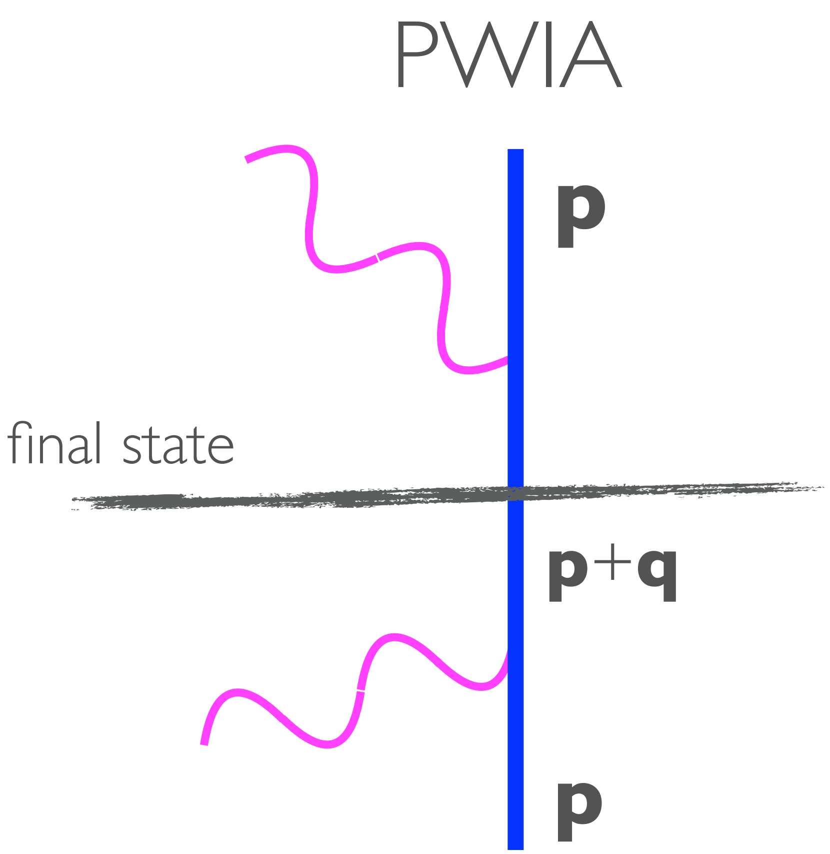
Two-body dynamics broadens peak for modest q

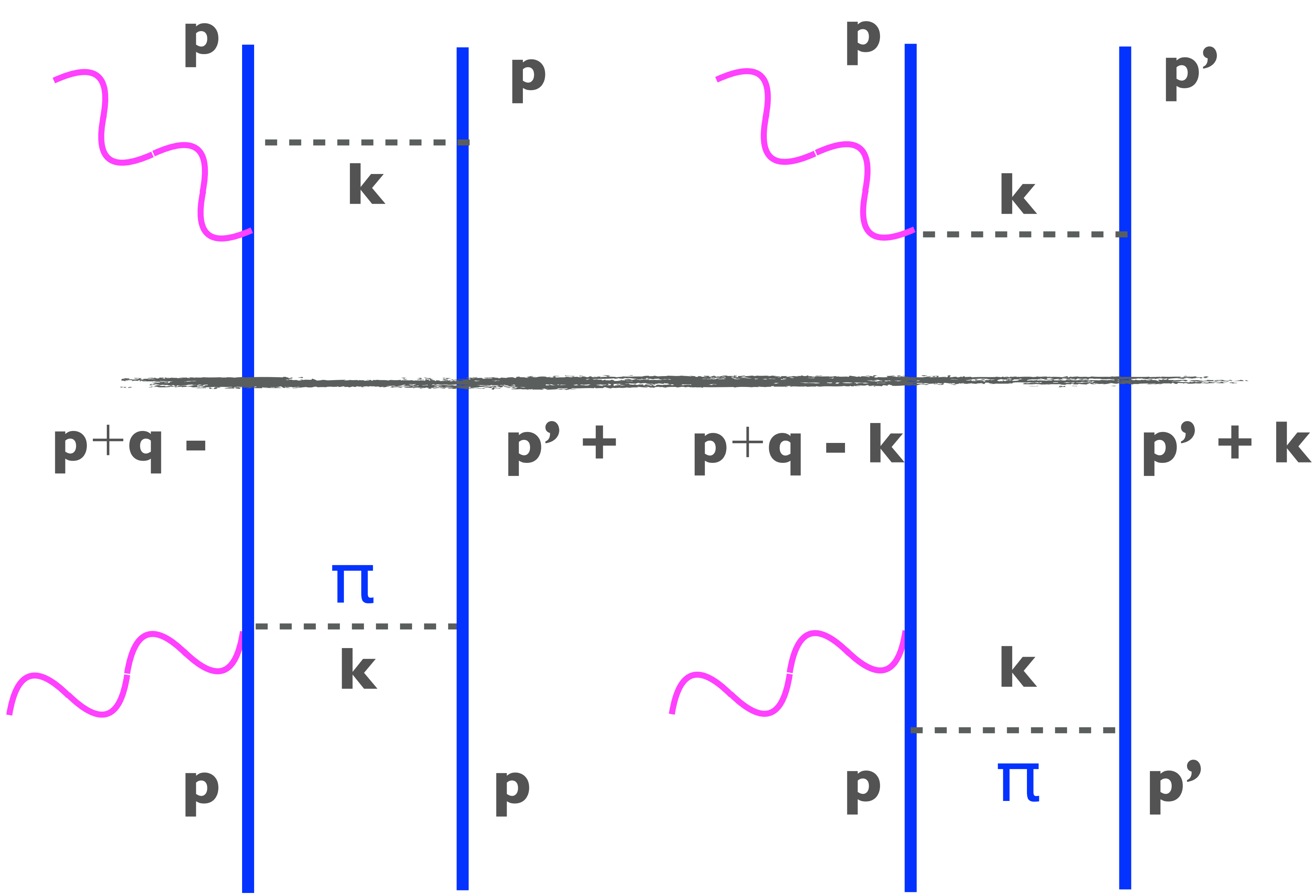
S. Pastore, et al., 2019

2 Nucleon Currents also important: Vector Current Sum Rule

Sum Rule: Constructive Interference
between 1- and 2-body currents
w/ tensor correlations

S. Pastore, et al., 2019



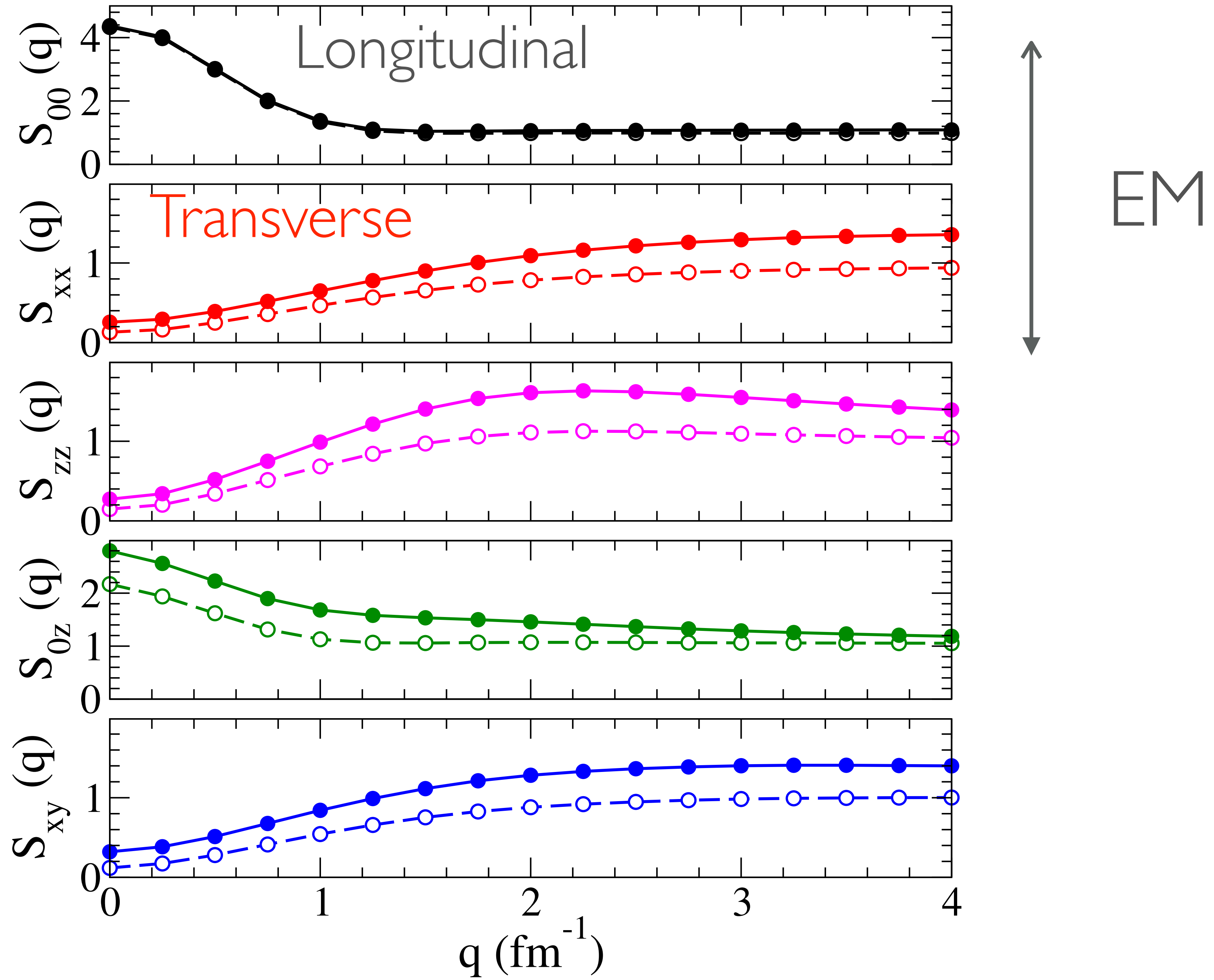


Large enhancement from
initial state correlations
and two-nucleon currents
similar in axial response

Note enhancement from
final states have larger momenta

$$\propto \; \sigma_i \cdot \mathbf{k} \; \sigma_i \cdot \mathbf{q} \; (\sigma_j \cdot \mathbf{k})^2 \; (\tau_i \cdot \tau_j)^2 \; v_\pi^2(k)$$

Sum rules in ^{12}C: neutral current scattering



Lovato, et. al PRL 2014

Single Nucleon currents (open symbols) versus
Full currents (filled symbols)

Full treatment of (inclusive) dynanics: Euclidean Response

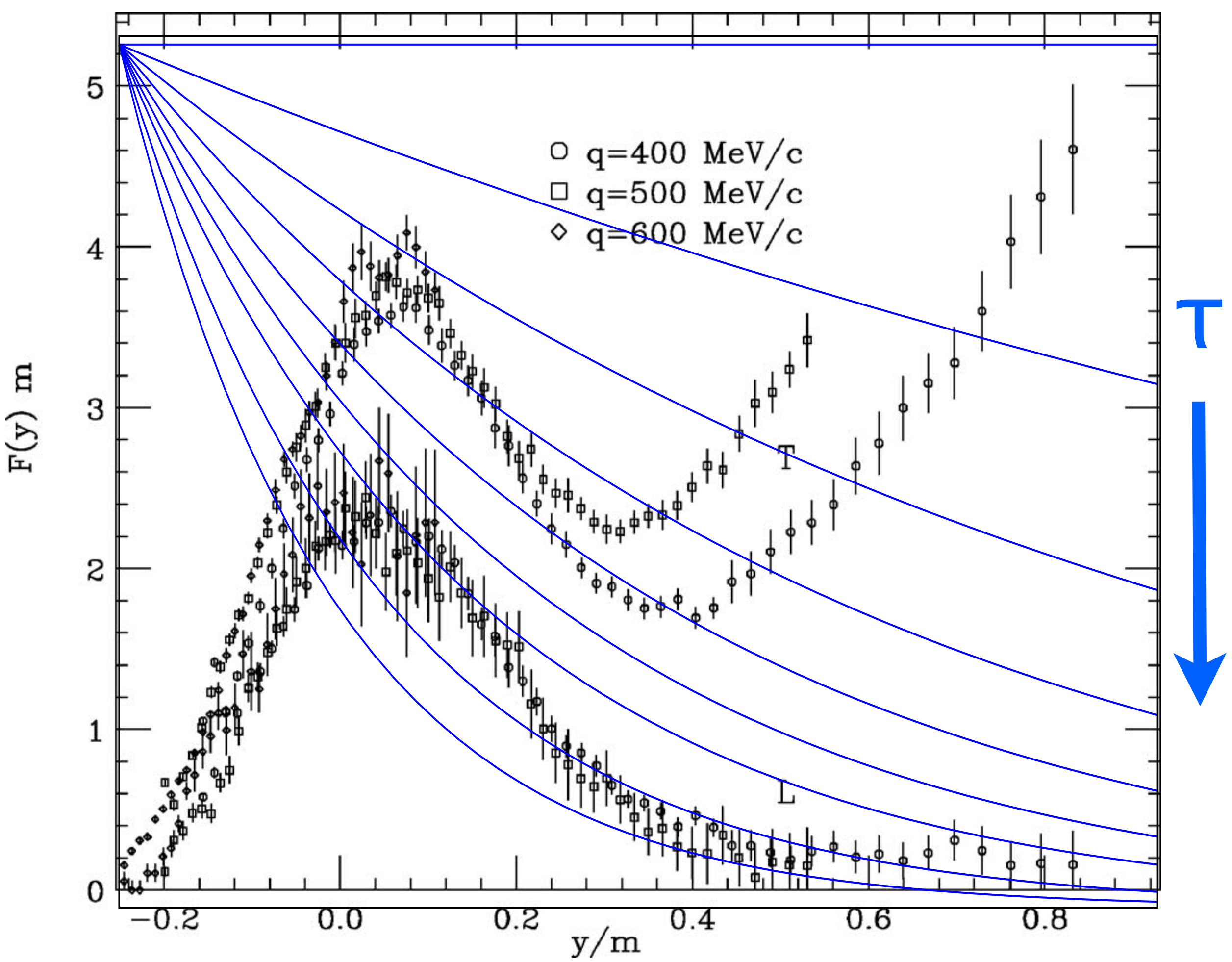
Want to calculate

$$R(q,\omega) \;=\; \int dt\; \langle 0|\; \mathbf{j}^\dagger \exp[i(H-\omega)t]\; \mathbf{j}\; |0\rangle$$

Can calculate

$$\tilde{R}(q,\tau) \;=\; \langle 0|\; \mathbf{j}^\dagger\;\; \exp[-(\mathbf{H}-\mathbf{E_0}-\mathbf{q^2}/(\mathbf{2m}))\tau]\;\; \mathbf{j}\;|\mathbf{0}\rangle >$$

- Exact given a model of interactions, currents
- `Thermal' statistical average
- *Full final-state interactions*
- *All contributions included - elastic, low-lying states, quasi elastic, …*

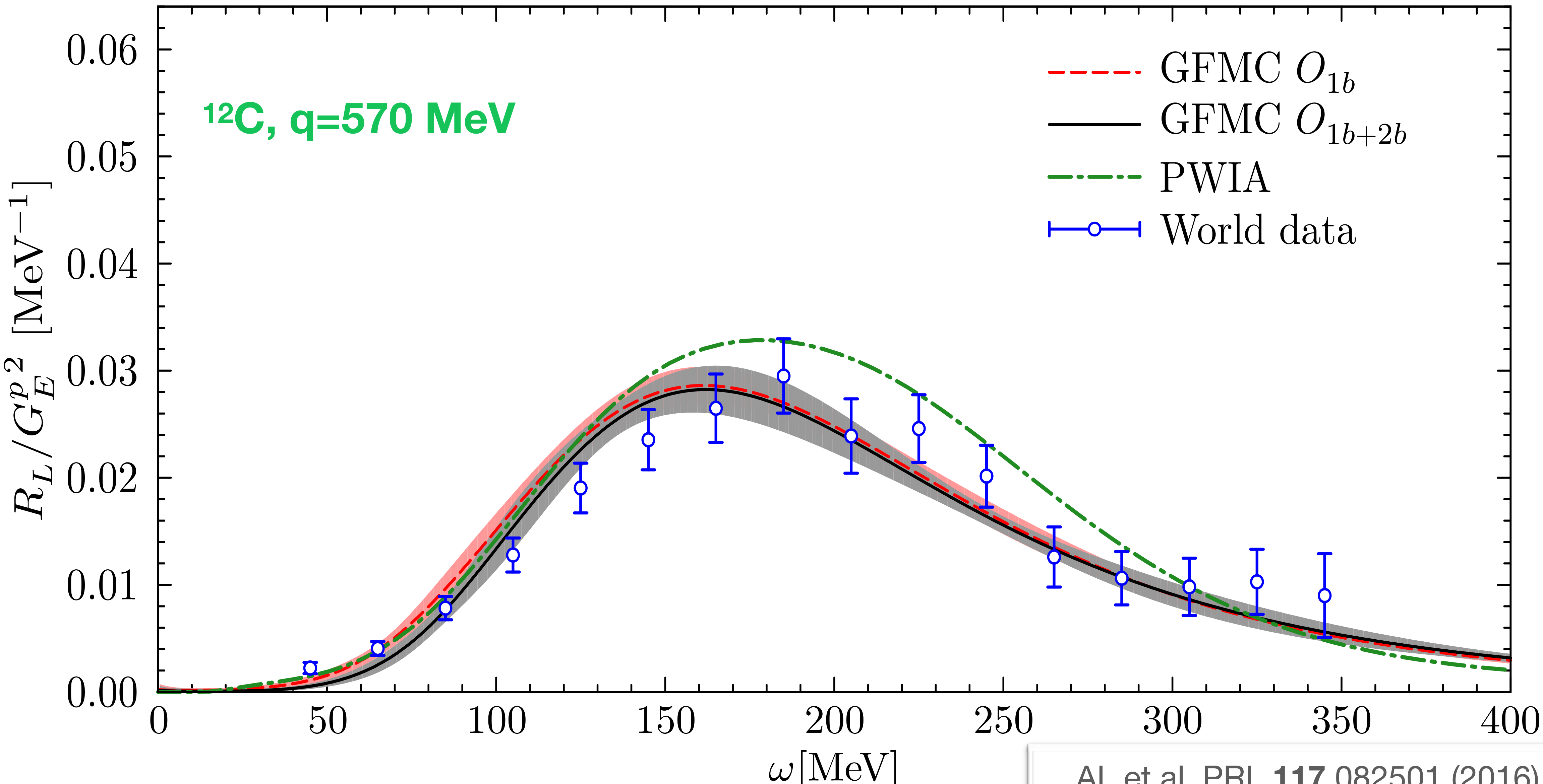


Sum rule → elastic FF^2 w/ increasing

Excellent agreement w/ EM (L & T) response in A=4,12
Lovato, 2015, PRL 2016

Electron Scattering from ^{12}C: Longitudinal Response

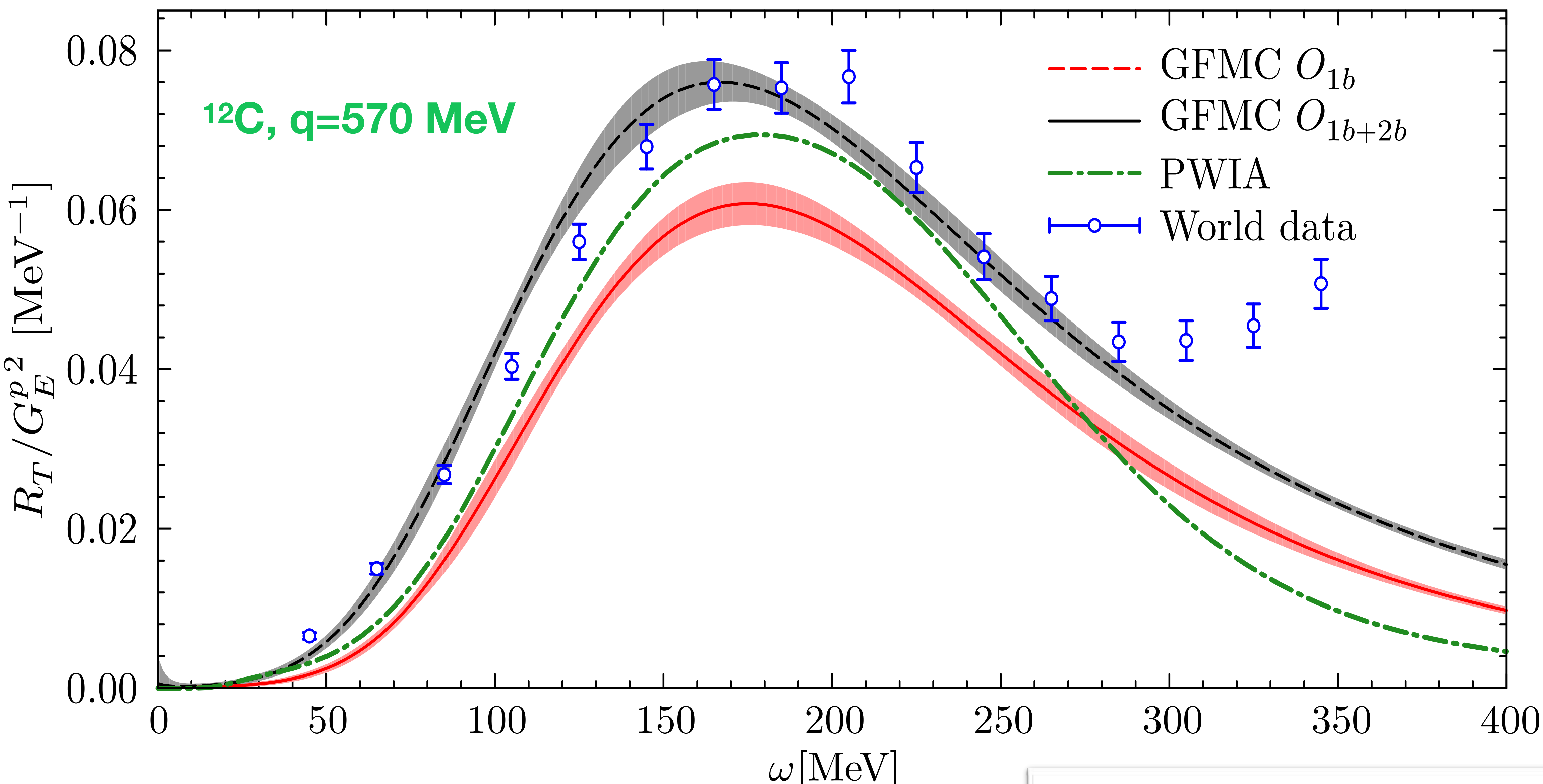
- We inverted the electromagnetic Euclidean response of ^{12}C
- Good agreement with data without in-medium modifications of the nucleon form factors
- Small contribution from two-body currents.



AL et al. PRL **117** 082501 (2016)

Electron Scattering from ^{12}C: Transverse Response

- We inverted the electromagnetic Euclidean response of ^{12}C
- Good agreement with the experimental data once two-body currents are accounted for
- Need to include relativistic corrections in the kinematics

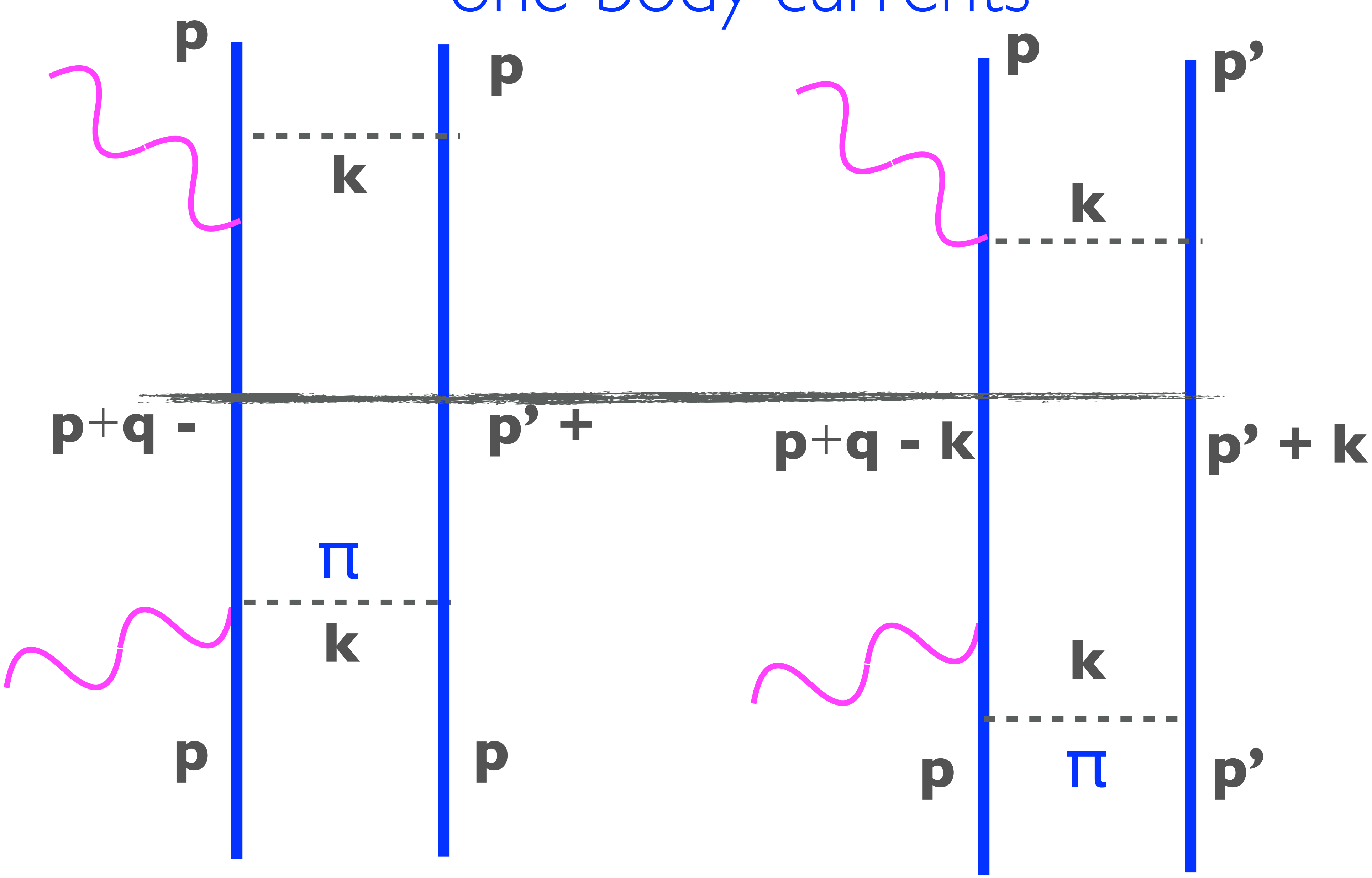


AL et al. PRL **117** 082501 (2016)

Why Y-scaling?

S. Pastore, et al., 2019

Even though two-nucleon currents are important, main contribution comes from interference w/ one-body currents



Overall strength enhanced, but through interference w/ same final states: similar shape

Why superscaling ?

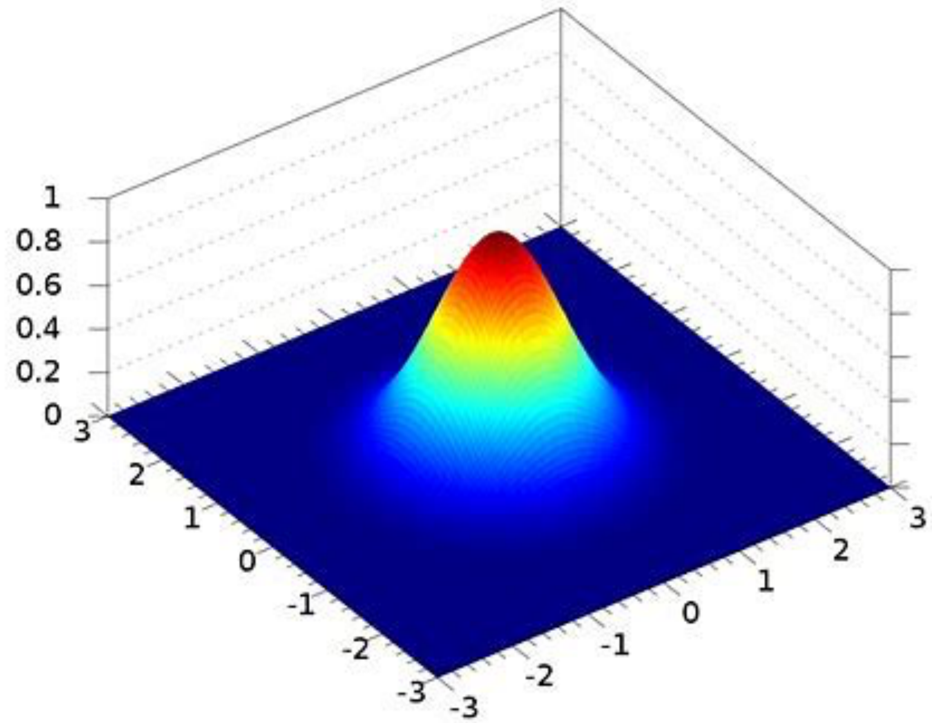
For nuclei w/ N~Z, bulk density is very similar: nuclear saturation at ~0.16 fm^{-3}

also pair densities very similar for A>12 nuclei

Inclusive Scattering at Quasi-Elastic energies and momenta is a nearly local operator

Free particle propagator:

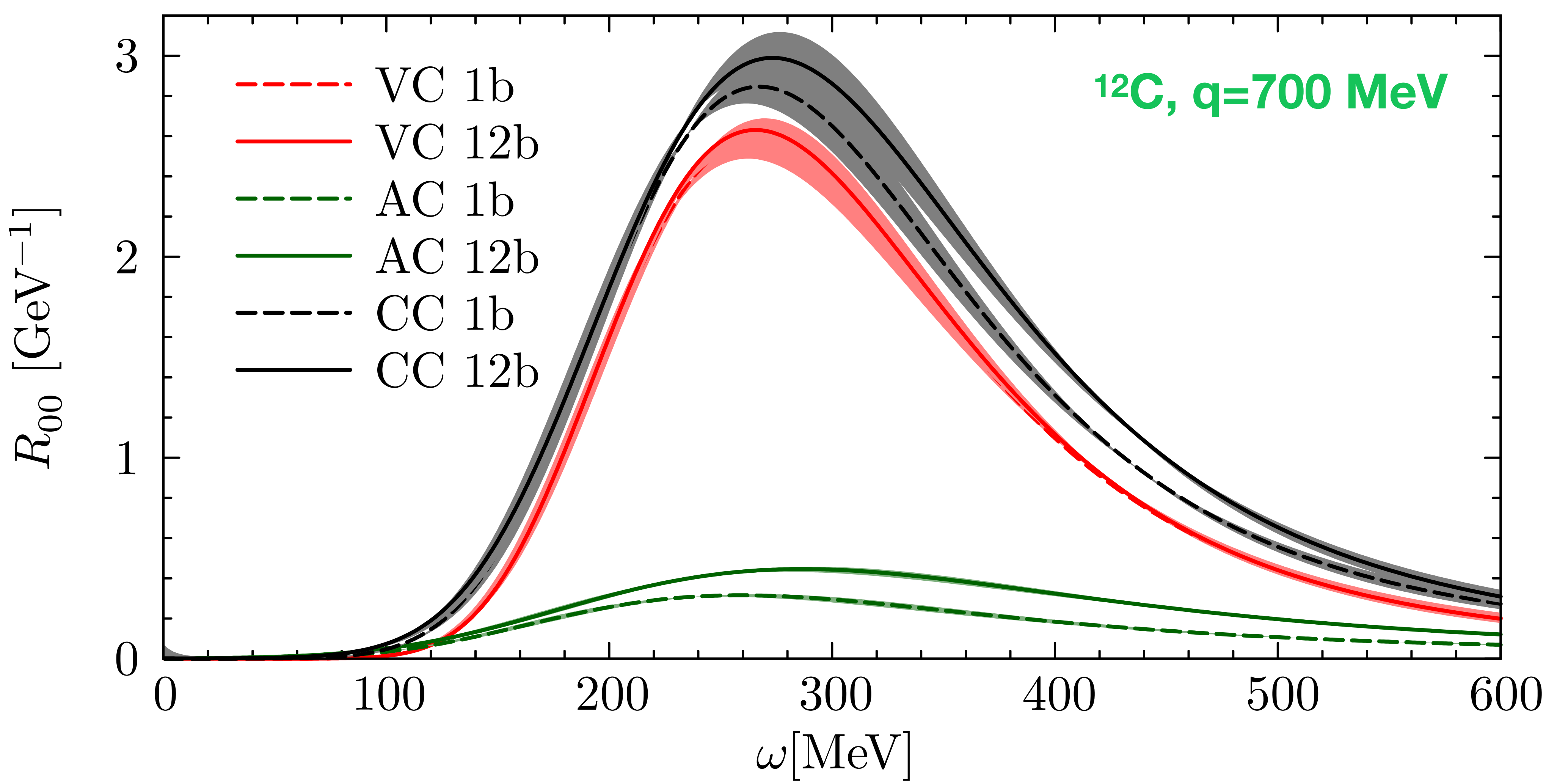
$$\exp[-(r-r')^2/(4\frac{\hbar^2}{2m}\delta\tau]$$



at **δτ** = 1/100 MeV; r-r' ~ 1.1 fm
at **δτ** = 1/ 50 MeV; r-r' ~ 1.6 fm

^{12}C charged-current responses

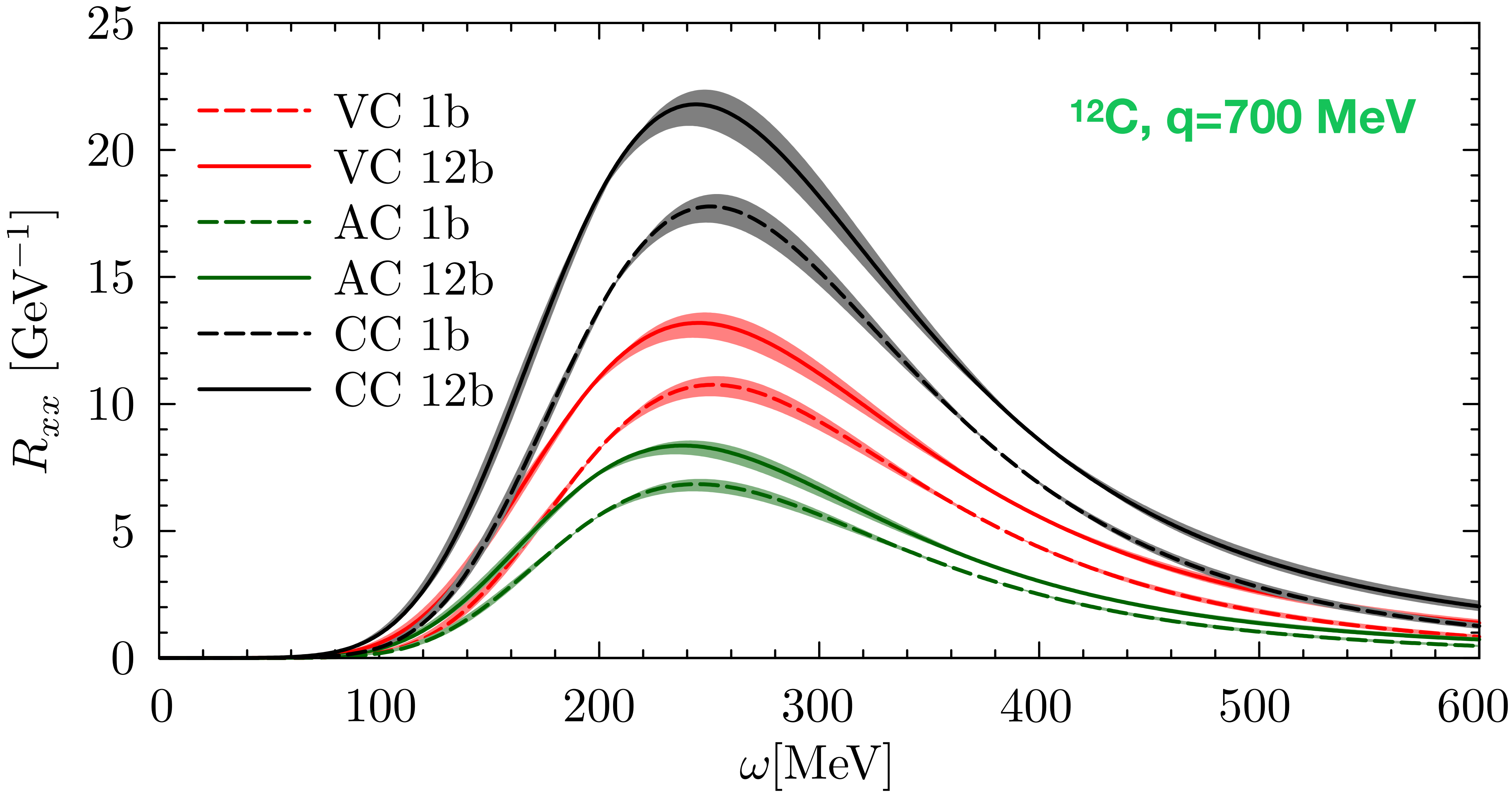
- We recently computed the charged-current response function of ^{12}C
- Calculations from q= 200 - 700 MeV/c



Alessandro Lovato, 2019

^{12}C charged-current responses

- We recently computed the charged-current response function of ^{12}C
- Two-body currents have a sizable effect in the transverse response, both in the vector and in the axial contributions
- Calculations from q = 200 - 700 MeV/c



Alessandro Lovato, 2019

Towards Exclusive Scattering and Larger Nuclei

Ground-state nuclei: doable with some approximations

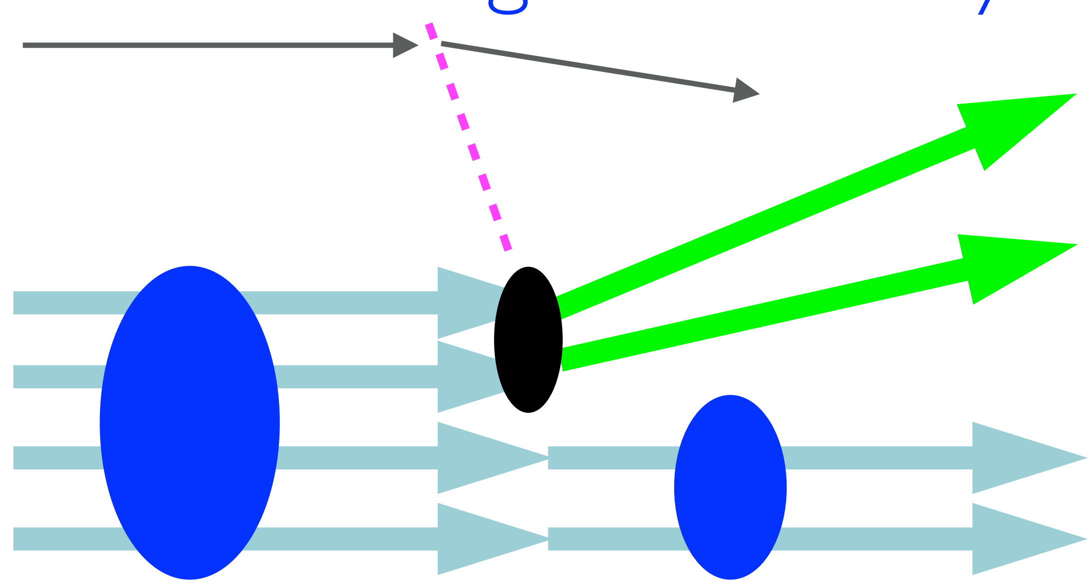
Propagation: ^{12}C GFMC calculations to $\tau \sim 0.1$ MeV^{-1}

Each particle propagates ~ 3 fm

Sign problem much worse in Ar than Carbon

Any fermion interchange in the system contributes to the noise

How much information can we get from very short ***real*** times?



Short Time Approximation: Towards real-time dynamics

Saori Pastore, et al, 2019

$$R^O(q,\omega) \;=\; \frac{\int d\Omega_q}{4\pi}\sum_f \langle\Psi_0|\mathcal{O}^\dagger(\mathbf{q})|\Psi_f\rangle\langle\Psi_f|\mathcal{O}(\mathbf{q})|\Psi_0\rangle\delta(E_f-E_0-\omega),$$

$$R^O(q,\omega) \;=\; \frac{\int d\Omega_q}{4\pi}\int\frac{dt}{2\pi}\exp[i\omega t]\langle\Psi_0|\mathcal{O}^\dagger(\mathbf{q},t')\exp[-iHt]\mathcal{O}(\mathbf{q},t=0)\Psi_0\rangle,$$

At short time evolution can be described as a product of NN propagators

$$\langle\mathbf{R}',\sigma',\tau'|\exp[-iHt]|\mathbf{R},\sigma,\tau\rangle \approx \langle\mathbf{R}',\sigma',\tau'|\prod_i \exp[-iH_i^0 t]\frac{\mathcal{S}\prod_{i<j}\exp[-iH_{ij}t]}{\prod_{i<j}\exp[-iH_{ij}^0 t]}|\mathbf{R},\sigma,\tau\rangle$$

Evaluate as a sum of matrix elements of NN states embedded in the nucleus

Incoherent sum of single nucleon currents

$$\sum_{q,Q,J,L,S,T} \langle\Psi_0|\,\mathbf{j_i}^\dagger\,|\psi_{NN}(q,Q)\rangle\,\langle\psi_{NN}(q,Q)|\,\mathbf{j_i}\,|\Psi_0\rangle\,\delta(E_f-E_i-\omega)$$

Interference of 1- and 2-nucleon currents

$$\sum_{q,Q,J,L,S,T} \langle\Psi_0|\,\mathbf{j_{ij}}^\dagger\,|\psi_{NN}(q,Q)\rangle\,\langle\psi_{NN}(q,Q)|\,\mathbf{j_i}\,|\Psi_0\rangle\,\delta(E_f-E_i-\omega)$$

Diagonal 2-nucleon currents

$$\sum_{q,Q,J,L,S,T} \langle\Psi_0|\,\mathbf{j_{ij}}^\dagger\,|\psi_{NN}(q,Q)\rangle\,\langle\psi_{NN}(q,Q)|\,\mathbf{j_{ij}}\,|\Psi_0\rangle\,\delta(E_f-E_i-\omega)$$

Short Time Approximation: Towards real-time dynamics

Saori Pastore, et al, 2019

$$R^O(q,\omega) \;=\; \frac{\int d\Omega_q}{4\pi} \int \frac{dt}{2\pi} \exp[i\omega t] \langle \Psi_0 | \mathcal{O}^\dagger(\mathbf{q}, t') \exp[-iHt] \mathcal{O}(\mathbf{q}, t=0) \Psi_0 \rangle,$$

At short time evolution can be described as a product of NN propagators

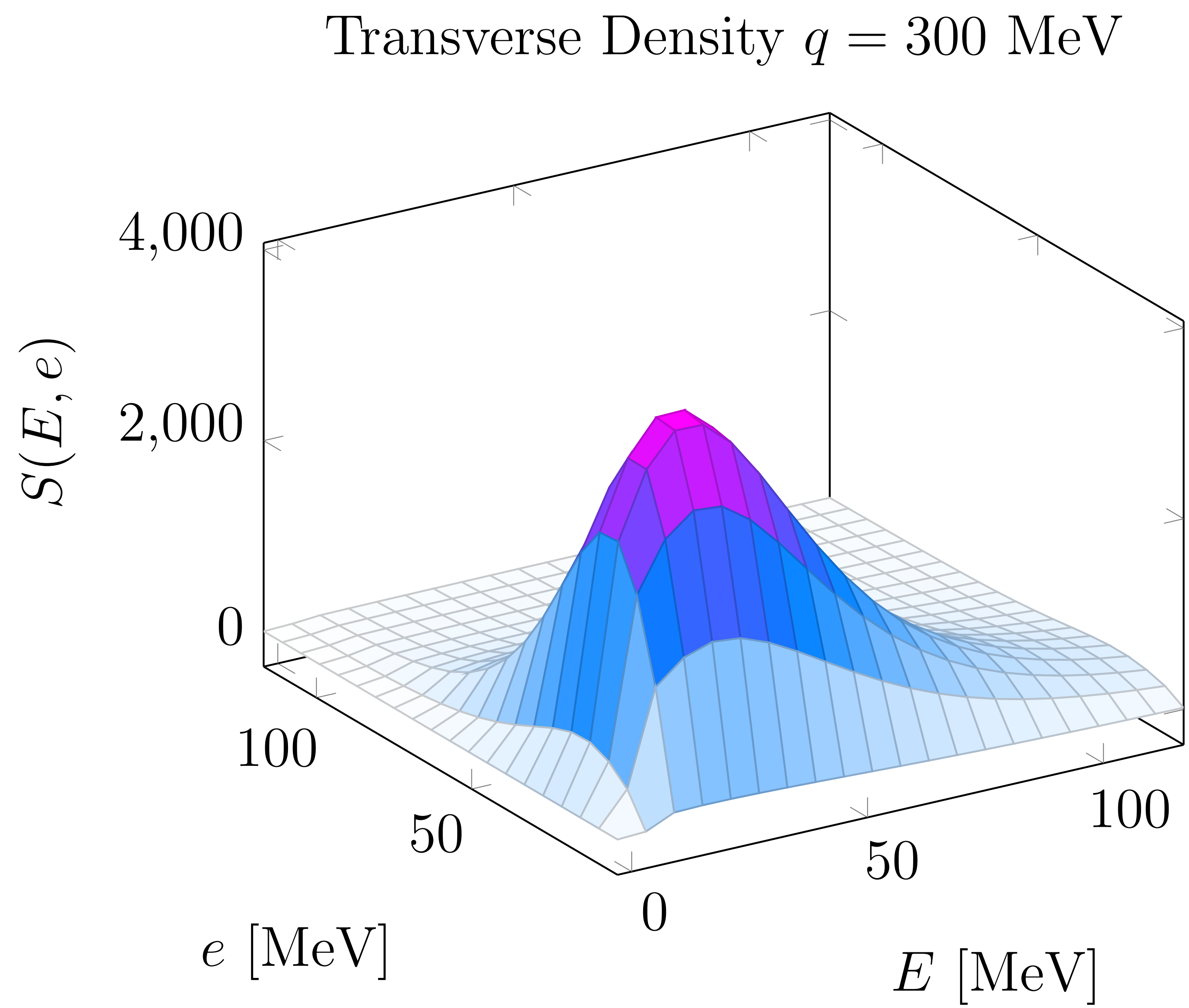
$$\langle \mathbf{R}', \sigma', \tau' | \exp[-iHt] | \mathbf{R}, \sigma, \tau \rangle \approx \langle \mathbf{R}', \sigma', \tau' | \prod_i \exp[-iH_i^0 t] \frac{\mathcal{S} \prod_{i<j} \exp[-iH_{ij} t]}{\prod_{i<j} \exp[-iH_{ij}^0 t]} | \mathbf{R}, \sigma, \tau \rangle$$

Evaluate as a sum of matrix elements of NN states embedded in the nucleus

A set of two-nucleon off-diagonal density matrix elements:

- Calculate for each operator and each q
- Incorporates: Exact sum rule nearly exact energy-weighted sum rule
- Incorporates full Pauli principal (A-nucleon ME)
- Information on the 2-nucleon quantum state right after the vertex
  - couple with semi-classical event generators

Response Densities

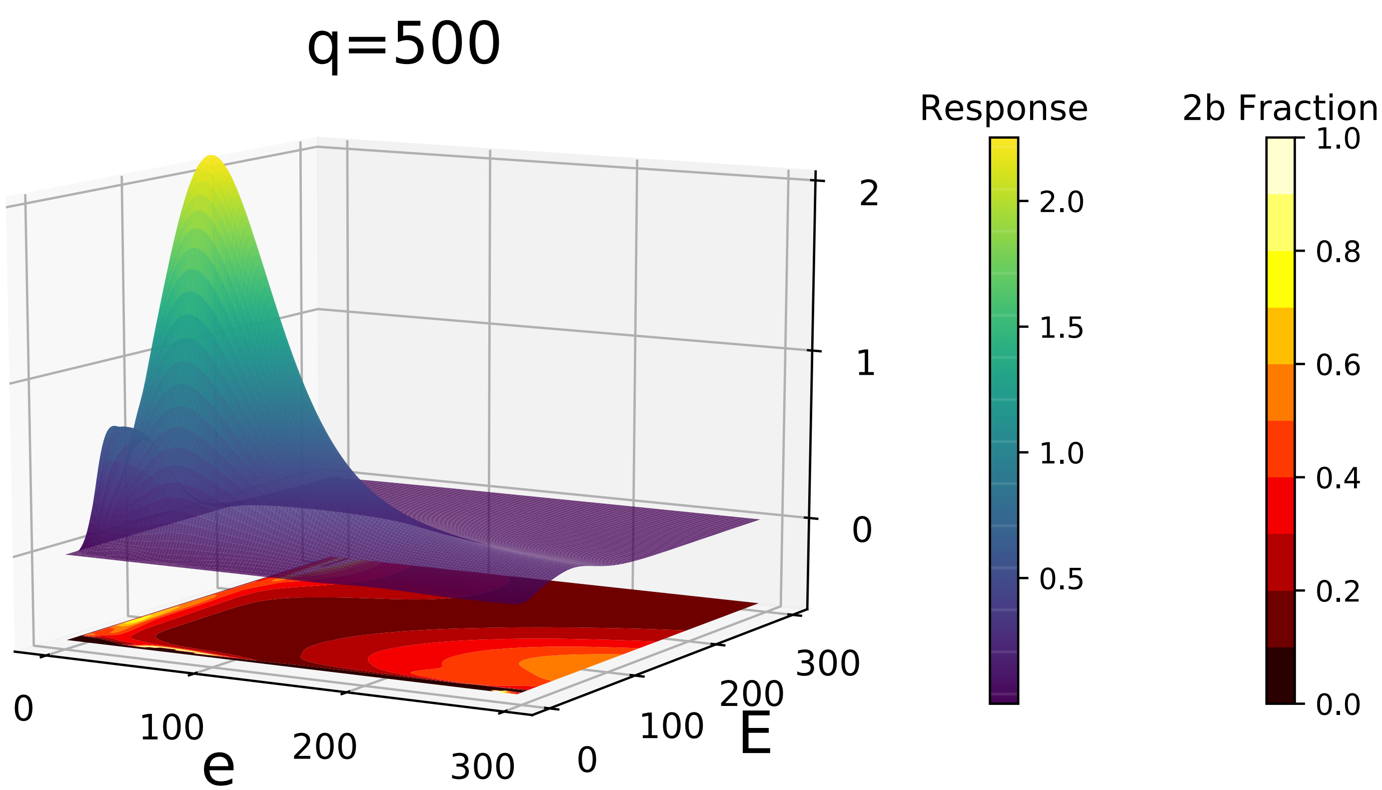


- Calculate individual response densities as a function of CM and relative energies of the struck pair

- The integral over surfaces w/ constant e+E gives full response

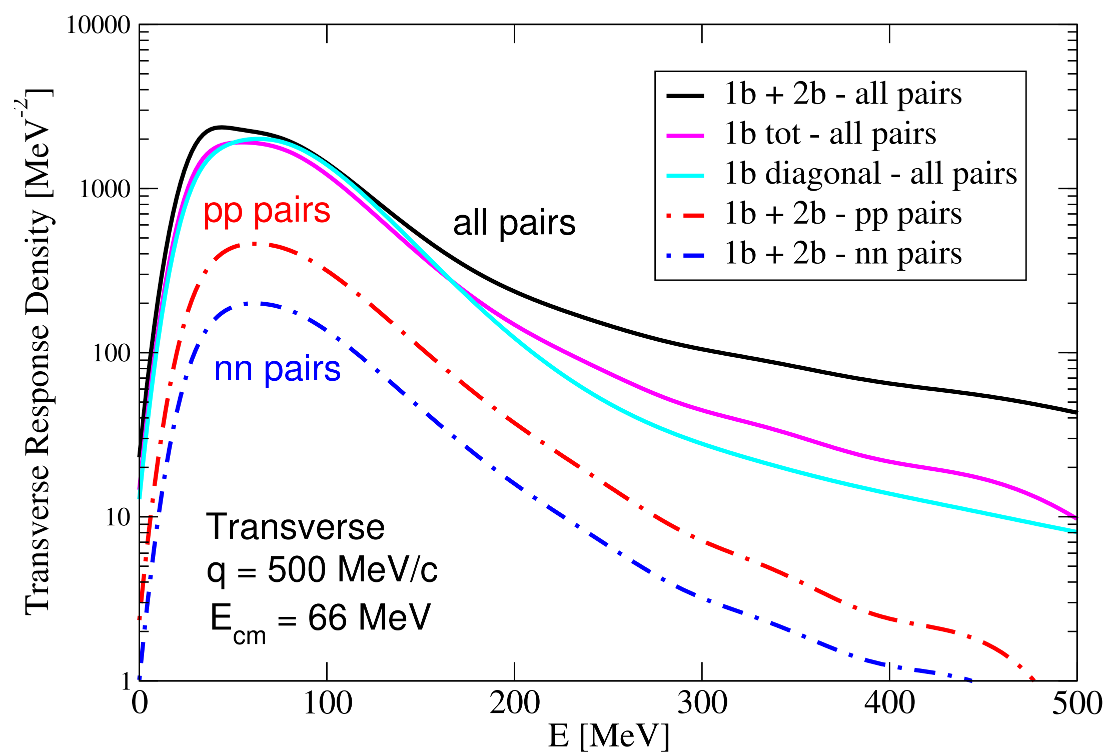
Response Densities

Fraction of Transverse response that include a 2N current



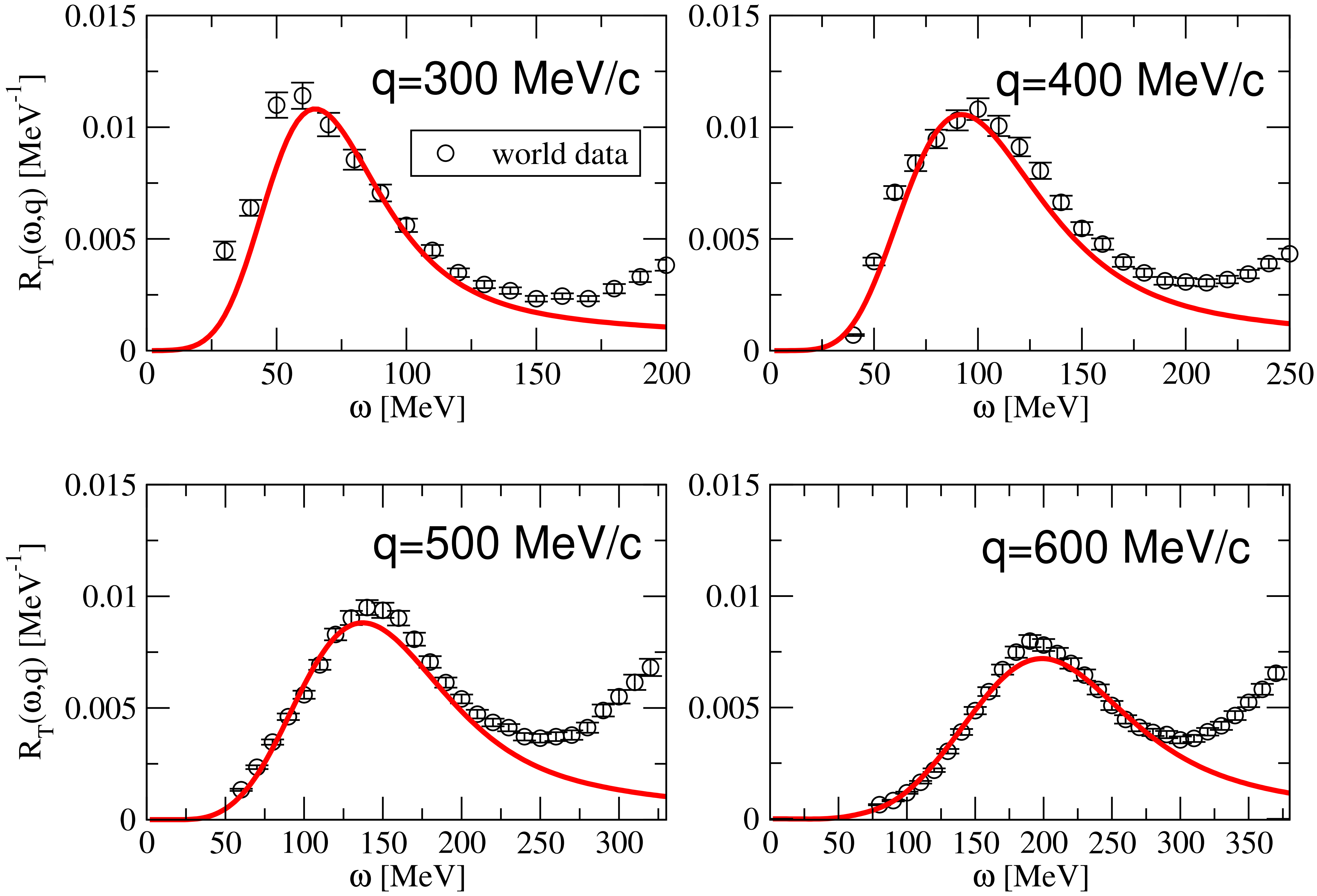
Large impact of 2-body currents at high relative energy

np vs. pp, etc.

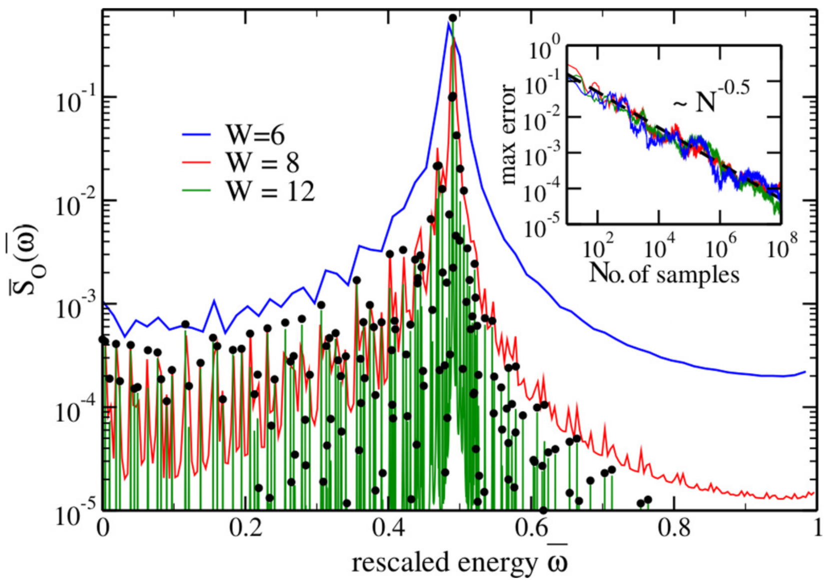


np vs pp in back-to-back kinematics

Comparison to Data (A=4)



Beyond the short-time approximation: Quantum Computing



Algorithm requires:

- ground-state preparation,
- coupling to current
- real time propagation (short)

Roggero, JC; PRC 2019

Highly Simplified Lattice Problem

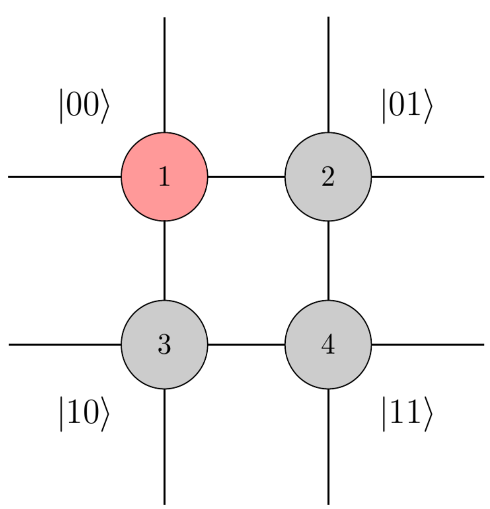
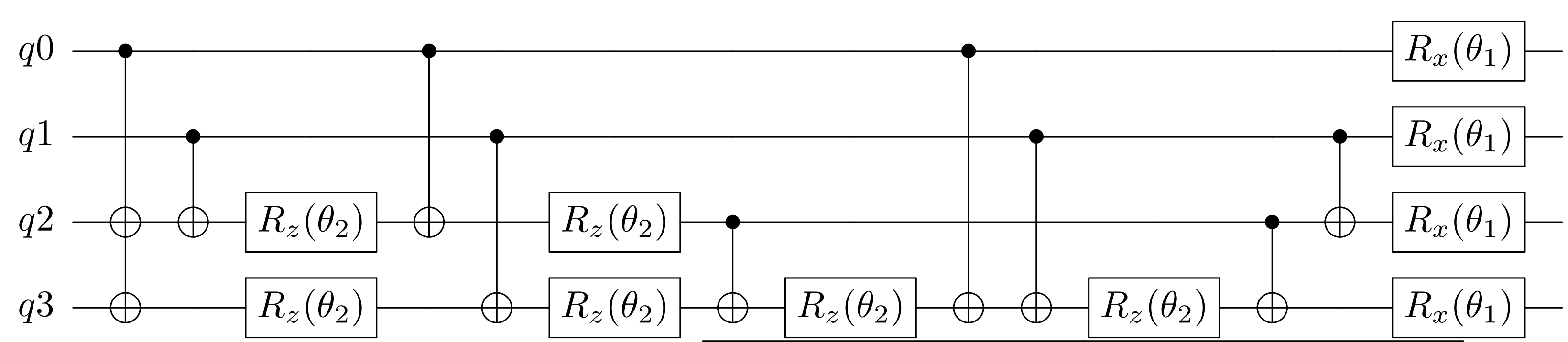


FIG. 6. Qubit mapping for a single fermion.

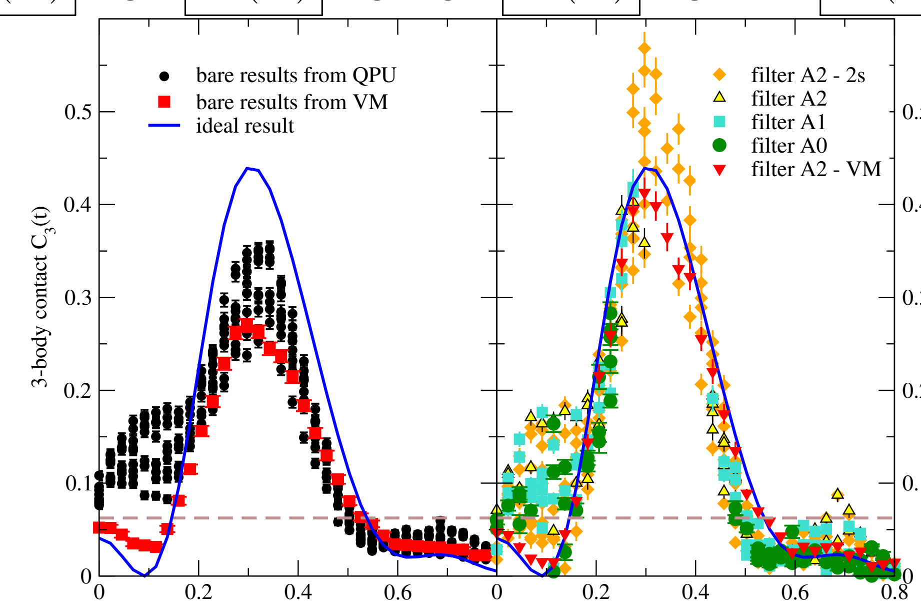
Circuit for Time Evolution



Scaling with problem size, highly simplified problem on actual QC

Roggero, et al, arXiv: 1911.06368

STA is only one (nontrivial) time step



Conclusions

EW processes on nuclei at the q ≧ k_F
are important, even sometimes at low energy

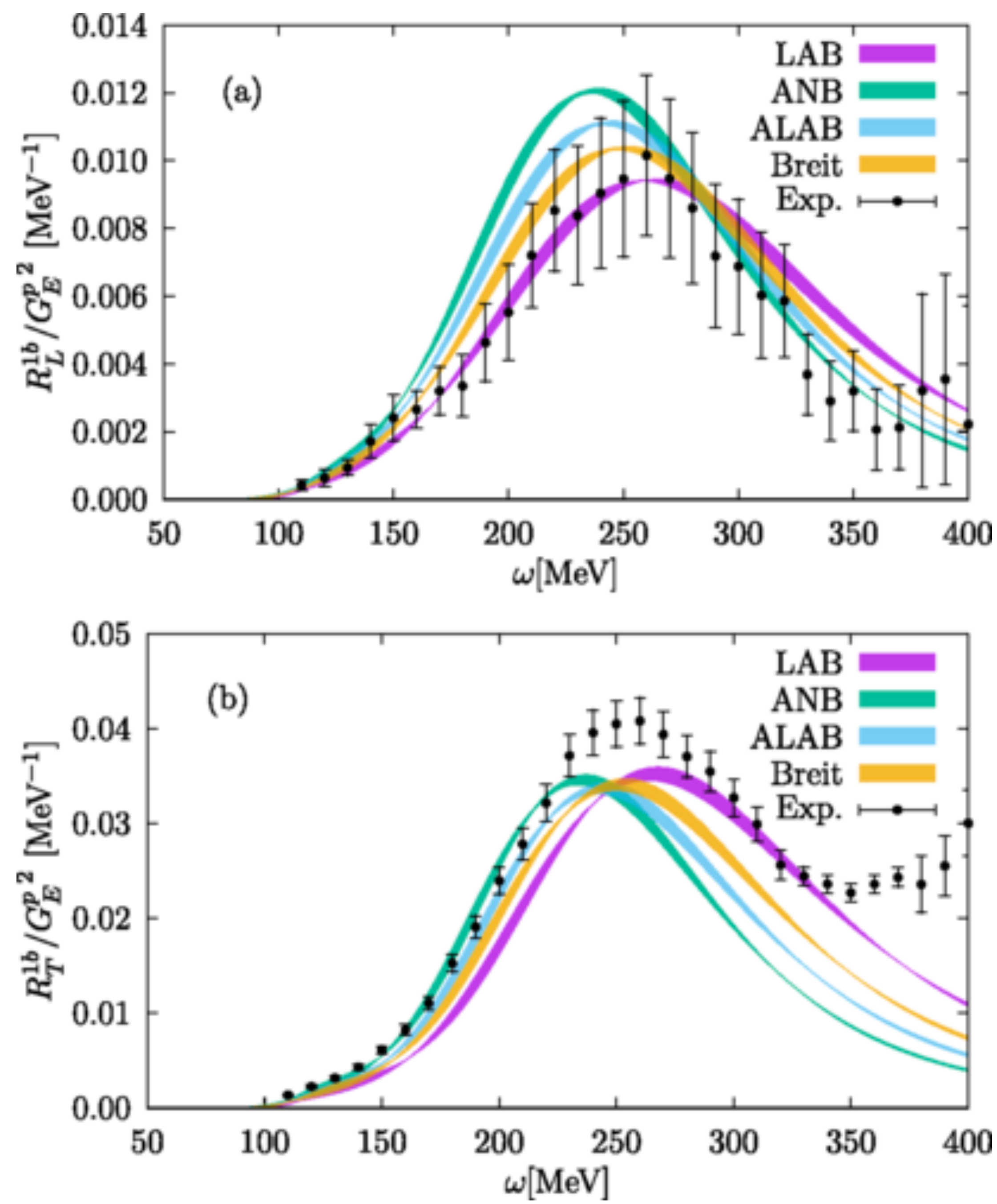
electron/neutrino scattering
electron and neutrinos in astrophysics
beta decay and double beta decay
0+ to 0+ beta decay

Good description w/ realistic nuclear interactions
and currents

Real-time dynamics is important

Future directions

- Larger Nuclei
- Relativistic few-nucleon dynamics
- Pion Production (Noemi Rocco, et al) requires NN inelastic processes can we match to lattice
- Quantum to Classical Transition can we match to generators
- Quantum Computing: even a short coherence time may be valuable.



Noemi Rocco, et al (2018)