Valence PDF of pion using quasi-PDF approach

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JLAB Theory Seminar

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What this talk is about

The central aspects of QCD are
asymptotic freedom at short distances and confinement at long distances

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(Mass of proton, finite T crossover etc.)

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Perturbation theory

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PDF+

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?

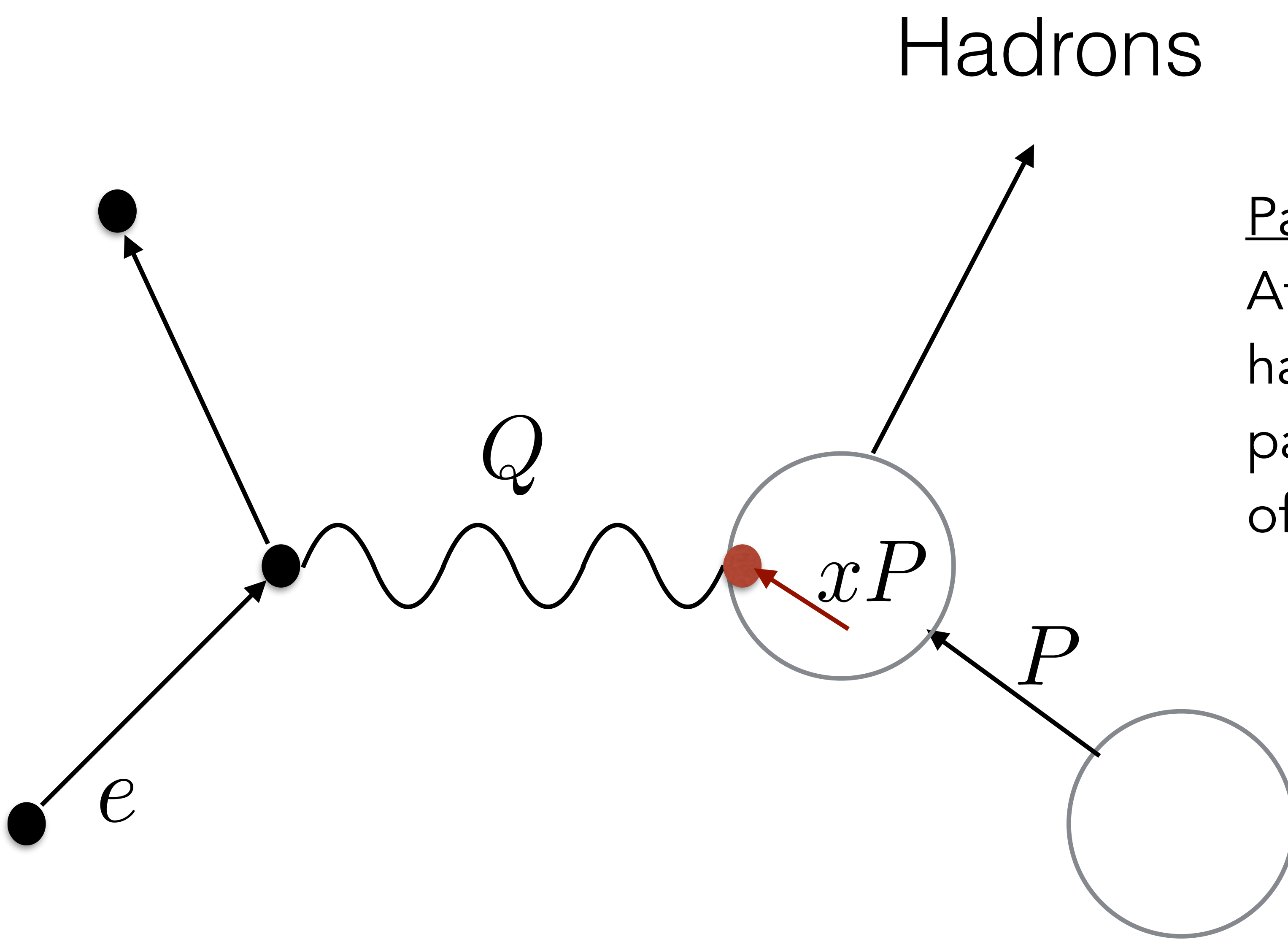
PDF+ ← Lattice

Perturbation theory

(Mass of proton, finite T crossover etc.)

Parton Distribution Functions

$Q^2 = -q^2 = \mathcal{O}(\text{few GeV}) \longrightarrow \alpha_S(Q^2) = \mathcal{O}(0.1) \longrightarrow$ Perturbative



Parton model: (Feynman '69)
At a high resolution Q^2, hadron = ensemble of massless partons each carrying fraction x of hadron momentum.

$$\sigma(eH \to eX) = \sum_i \int dx \; f_i(xP, Q^2) \; \sigma\left\{eq_i(xP) \to eq_i(xP+q)\right\}$$

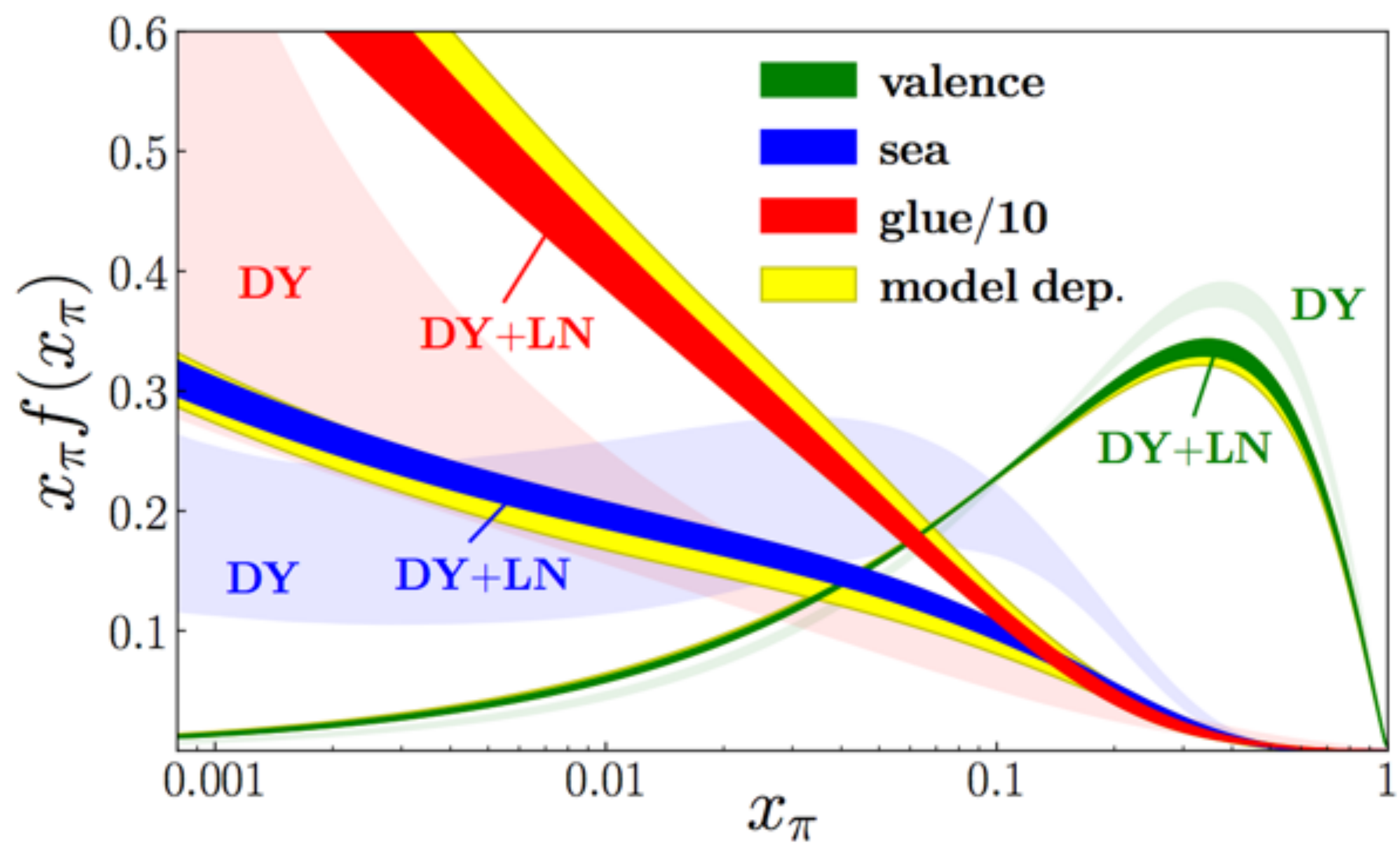
Valence PDF of $\pi^+(u\bar{d})$

We measure the valence PDF of charged pion:

$$f_{\text{valence}}(y,\mu) = f_u(x,\mu) - f_d(x,\mu)$$

Flavor non-singlet ➡ No mixing with glue and no disconnected fermion diagrams

P. C. Barry et al, 2018

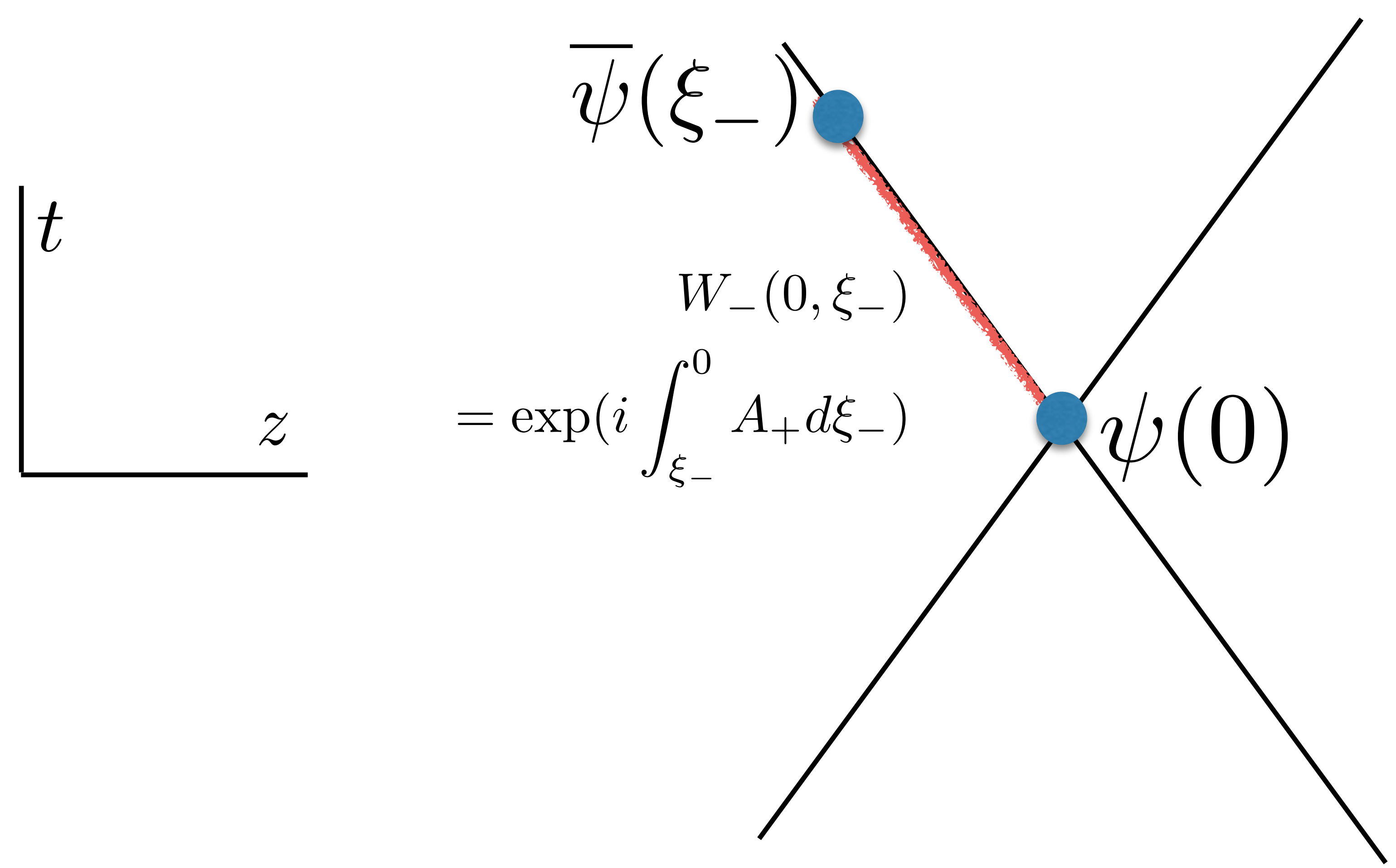


PDF as light-like separated quark-antiquark correlation

Field theoretic Gauge-invariant and Lorentz invariant construction: (Soper '77)

$$f(x) = \int \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle H(P)|\overline{\psi}(\xi_-)\gamma_+ W_-(0,\xi_-)\tau\psi(0)|H(P)\rangle$$

= "Number of on-shell massless partons with energy x P^+"



PDF as light-like separated quark-antiquark correlation (aka problem for lattice)

Projecting to hadron state is easy on lattice, but requires $t \to i\ t$

$$\lim_{\Delta t \to \infty} e^{-H_{\mathrm{QCD}} \Delta t} \hat{O}_h(t = 0, \mathbf{P}) |\Omega\rangle \propto |h(\mathbf{P}, E)\rangle$$

But presence of unequal time separation between $\psi(0)$ and $\psi(\xi_-)$ sandwiched between hadron states is a (sign) problem for Euclidean lattice.

PDF as light-like separated quark-antiquark correlation (aka problem for lattice)

Resolutions:

- Compute moments of PDF which are related by OPE to local operators. State of the art is 2nd moment.

  Martinelli and Sachrajda '88, W. Detmold et al, d > '01

- Quasi-PDF approach (this talk), pseudo-PDF and factorization of lattice cross-sections.

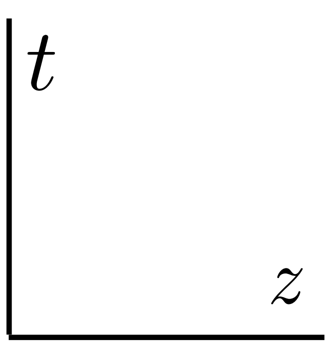
  X. Ji '13, A. Radyushkin '17, Ma and Qiu '17

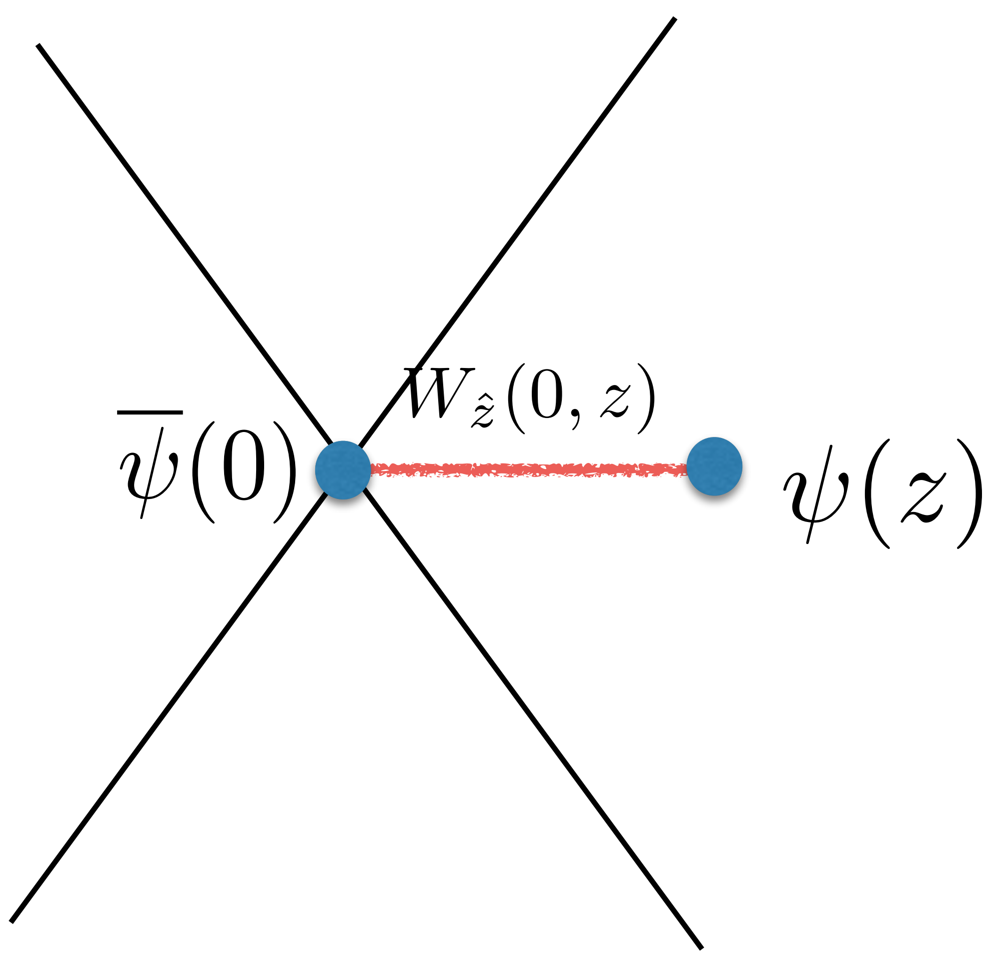
quasi-PDF approach to obtain PDF using Euclidean lattice

Equal time correlation function that can be determined on lattice:

$$\tilde{q}(x) = \int \frac{dz}{4\pi} e^{-ixP_z z} \langle H(P_z, E) | \overline{\psi}(0) \gamma_\mu W_{\hat{z}}(0, z) \tau \psi(z) | H(P_z, E) \rangle$$

for $\mu = z \text{ or } t.$





quasi-PDF approach to obtain PDF using Euclidean lattice

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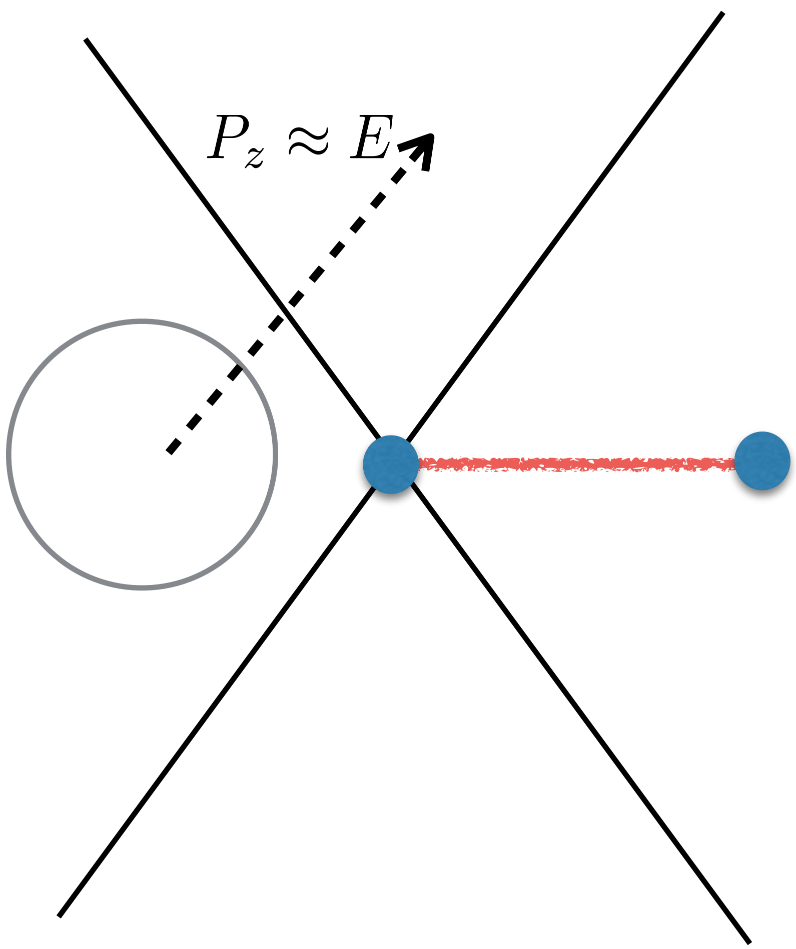
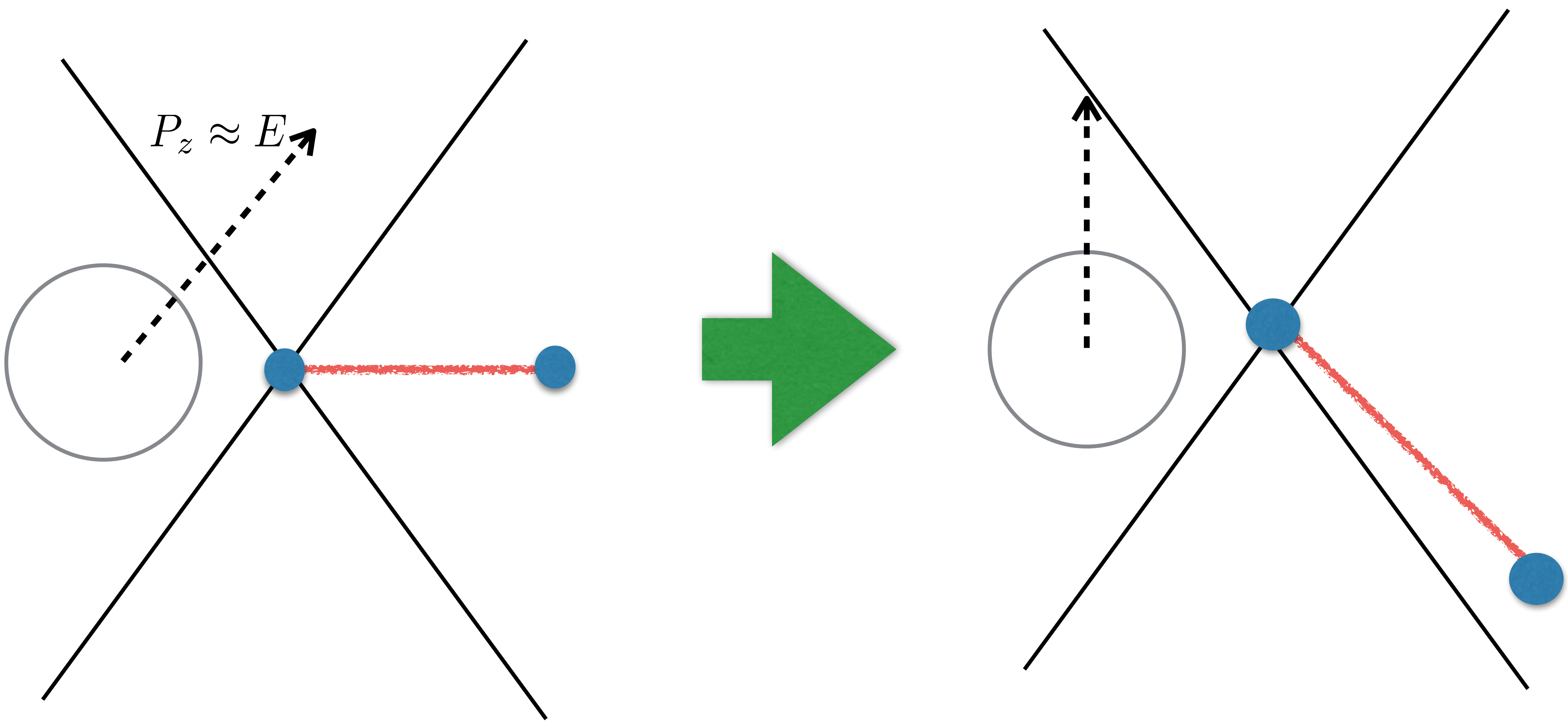
Rest frame of operator



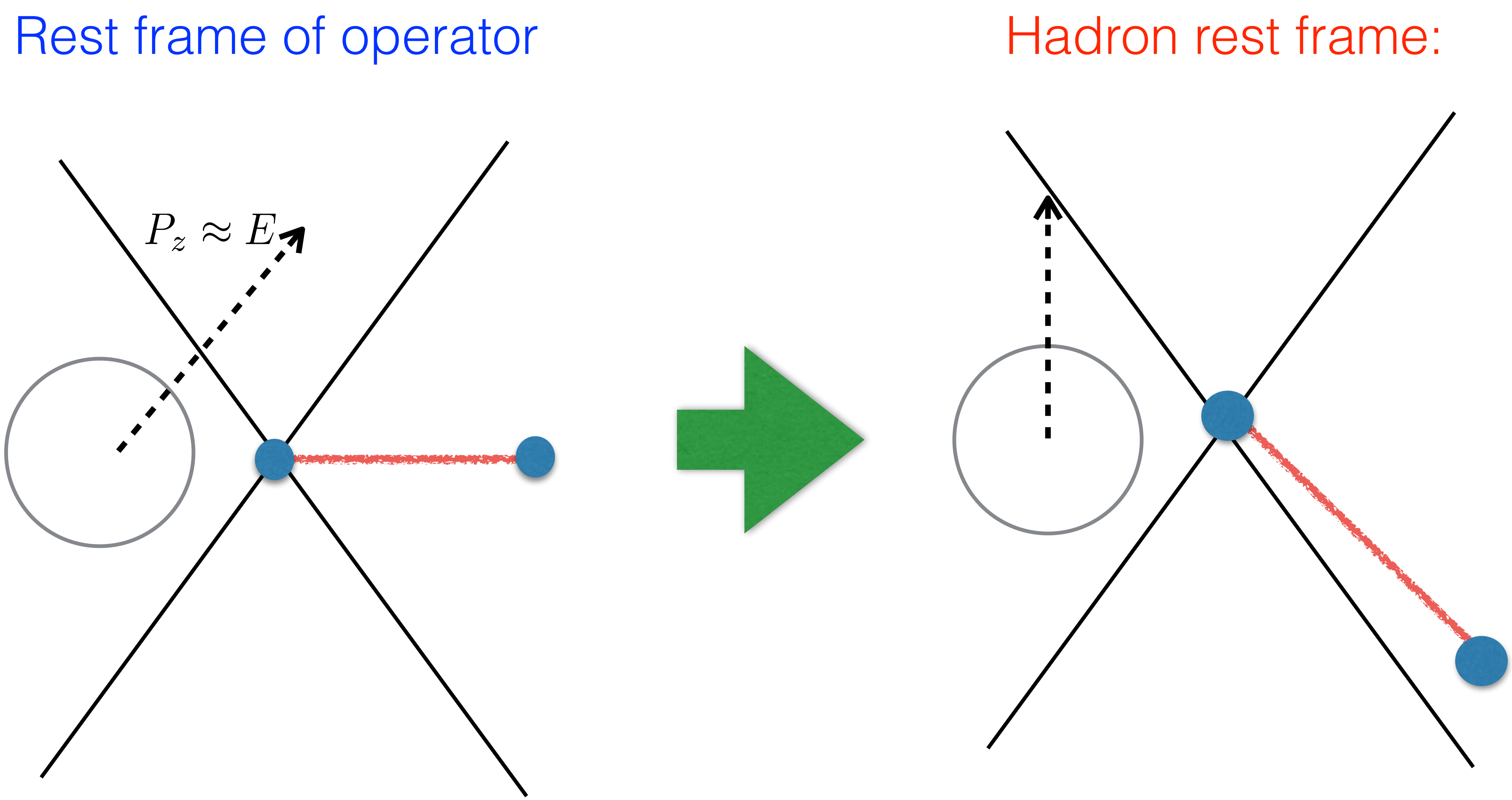
quasi-PDF approach to obtain PDF using Euclidean lattice

Rest frame of operator

Hadron rest frame:



quasi-PDF approach to obtain PDF using Euclidean lattice



$\boldsymbol{z}^2$ is Lorentz invariant. But the typical $\boldsymbol{z}$ contributing to Fourier transform at fixed $\boldsymbol{x}$ is $\mathbf{z}_{\text{typ}} \sim 1/\boldsymbol{P}_z$ and so $|\mathbf{z}_{\text{typ}}|$ is power suppressed.

Converse: Small $\boldsymbol{x}$ at fixed $\boldsymbol{P}_z$ ⟶ Larger $|\mathbf{z}_{\text{typ}}|$ ⇨ Effect of Λ_{QCD}, M, $\boldsymbol{z}^2$

Issue of limits

In 3+1d, PDF operator already is on the light-cone before regularization and renormalization.

On 4d lattice…

- one has finite lattice spacing ***a***
- At any finite ***a***, ***q(x)*** has to be renormalized at a scale P^R in a scheme that can be compared with experiments.
- Take ***a*** → 0 first, then $P_z \to \infty$

Perturbative matching

Not hopeless…

- Perturbative matching between ***q(x,P^R)*** in a regulator independent renormalization scheme at finite P_z to the infinite momentum MS-bar PDF ***f(x,μ^2)***

$$q(x;P_z,P^R)=\int_{-1}^{1}\frac{dy}{|y|}C\left(\frac{x}{y},\frac{\mu}{yP_z},\frac{P_\perp^R}{P_z^R},\frac{yP_z}{P_z^R}\right)f(x,\mu)$$

with the matching coefficient $C(\xi)=\delta(1-\xi)+\alpha_S(\mu)C^{(1)}(\xi)$

X. Ji '13, Stewart and Zhao '17

Perturbative matching

Since C is perturbative, it is universal for PDF of all hadrons.

So, it is computed using (gauge-fixed) quark PDF and qPDF with IR regulator $p^2 < p_z^2$ (in 3+1d).

$$q_{\rm quark}(x) = f_{\rm quark}(x) + \alpha_S(\mu)\left(q^{(1)}_{\rm quark}(x) - f^{(1)}_{\rm quark}(x)\right)$$

Collinear sing. cancel

$$= \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_z}, \frac{P_\perp^R}{P_z^R}, \frac{yP_z}{P_z^R}\right) f_{\rm quark}(x,\mu)$$

Due to the resemblance to a computation of corrections to EFT, the above matching is called “Large momentum effective theory” (LaMET).

Work-flow

Work-flow

Compute bare qPDF operators

$$h^b_{\gamma_t}(z, P_z, a)$$

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$$h^b_{\gamma_t}(z, P_z, a)$$

↓ Renormalize at $(P^R_z, P^R_\perp)$

$$h^R_{\gamma_t}(z, P_z, P^R_z, P^R_\perp)$$

Work-flow

Compute bare qPDF operators

$$h^{b}_{\gamma_t}(z, P_z, a)$$

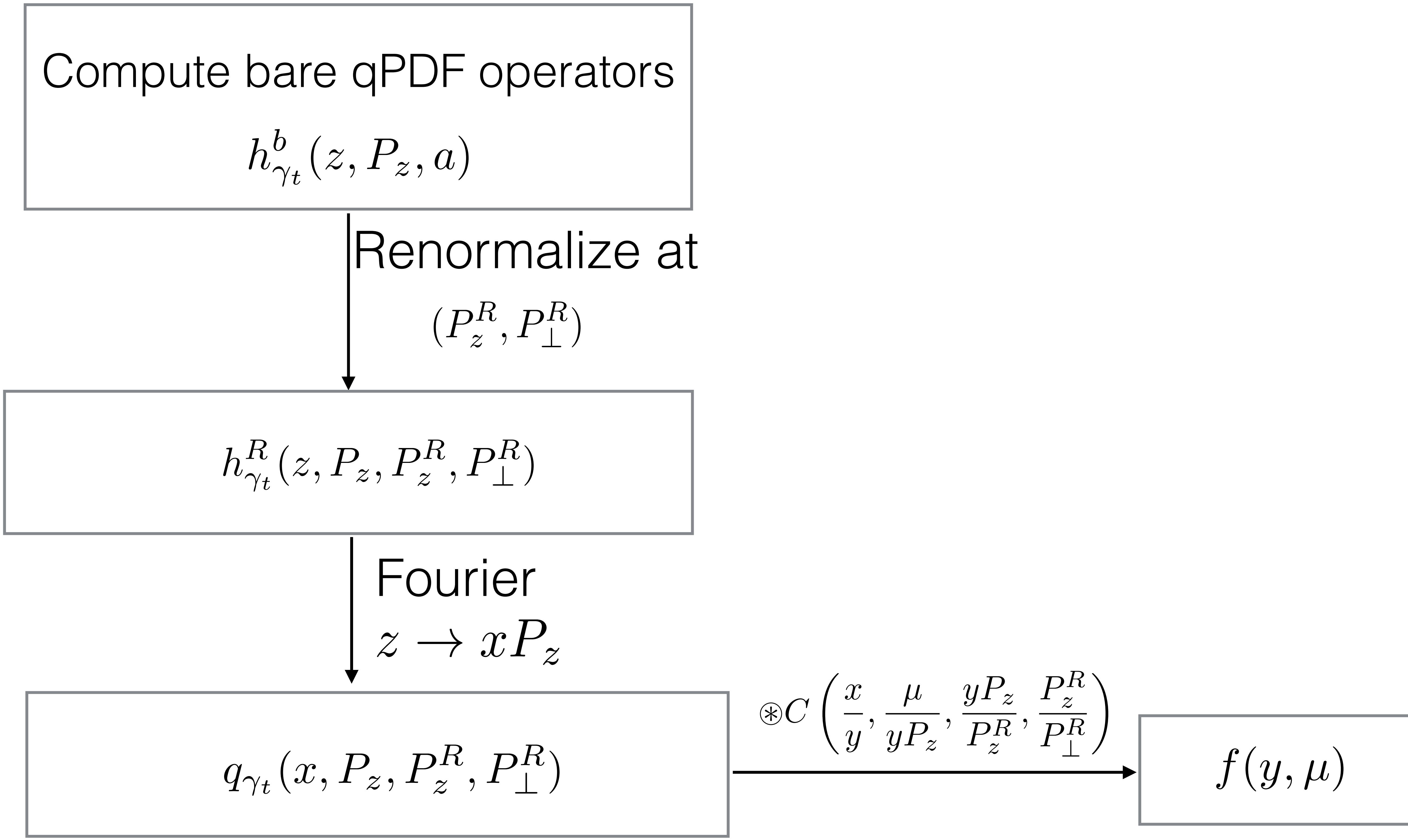
↓ Renormalize at $(P_z^R, P_\perp^R)$

$$h^{R}_{\gamma_t}(z, P_z, P_z^R, P_\perp^R)$$

↓ Fourier $z \to xP_z$

$$q_{\gamma_t}(x, P_z, P_z^R, P_\perp^R)$$

Work-flow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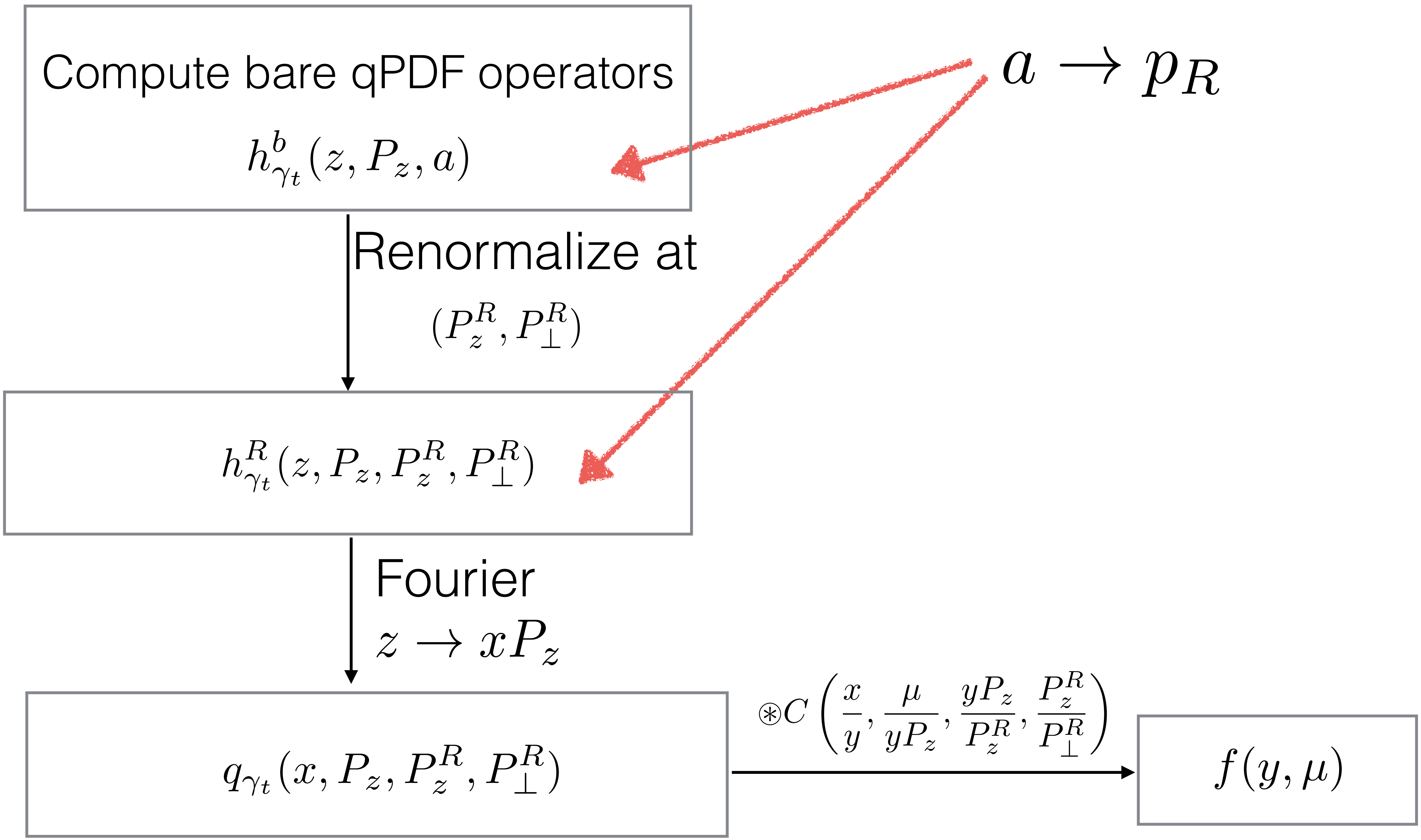
Renormalize at
$(P_z^R, P_\perp^R)$
$h^R_{\gamma_t}(z, P_z, P_z^R, P_\perp^R)$
Fourier
$z \to xP_z$
$q_{\gamma_t}(x, P_z, P_z^R, P_\perp^R)$
$\circledast C\left(\frac{x}{y}, \frac{\mu}{yP_z}, \frac{yP_z}{P_z^R}, \frac{P_z^R}{P_\perp^R}\right)$
$f(y, \mu)$

Work-flow



Compute bare qPDF operators
$h^b_{\gamma_t}(z, P_z, a)$
$a \to p_R$
Renormalize at
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Fourier
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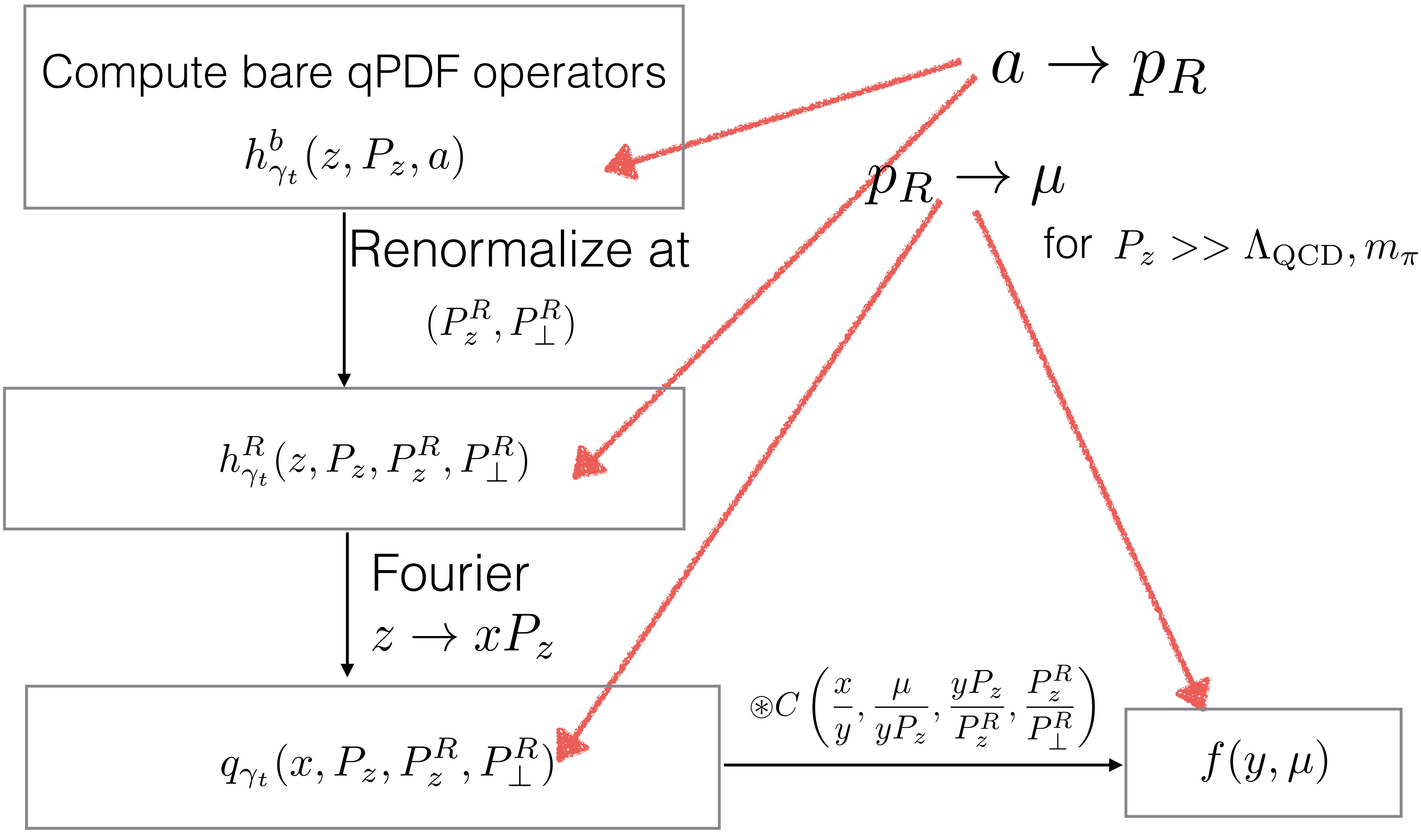

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$f(y, \mu)$
$a \to p_R$
$p_R \to \mu$
for $P_z >> \Lambda_{\rm QCD}, m_\pi$

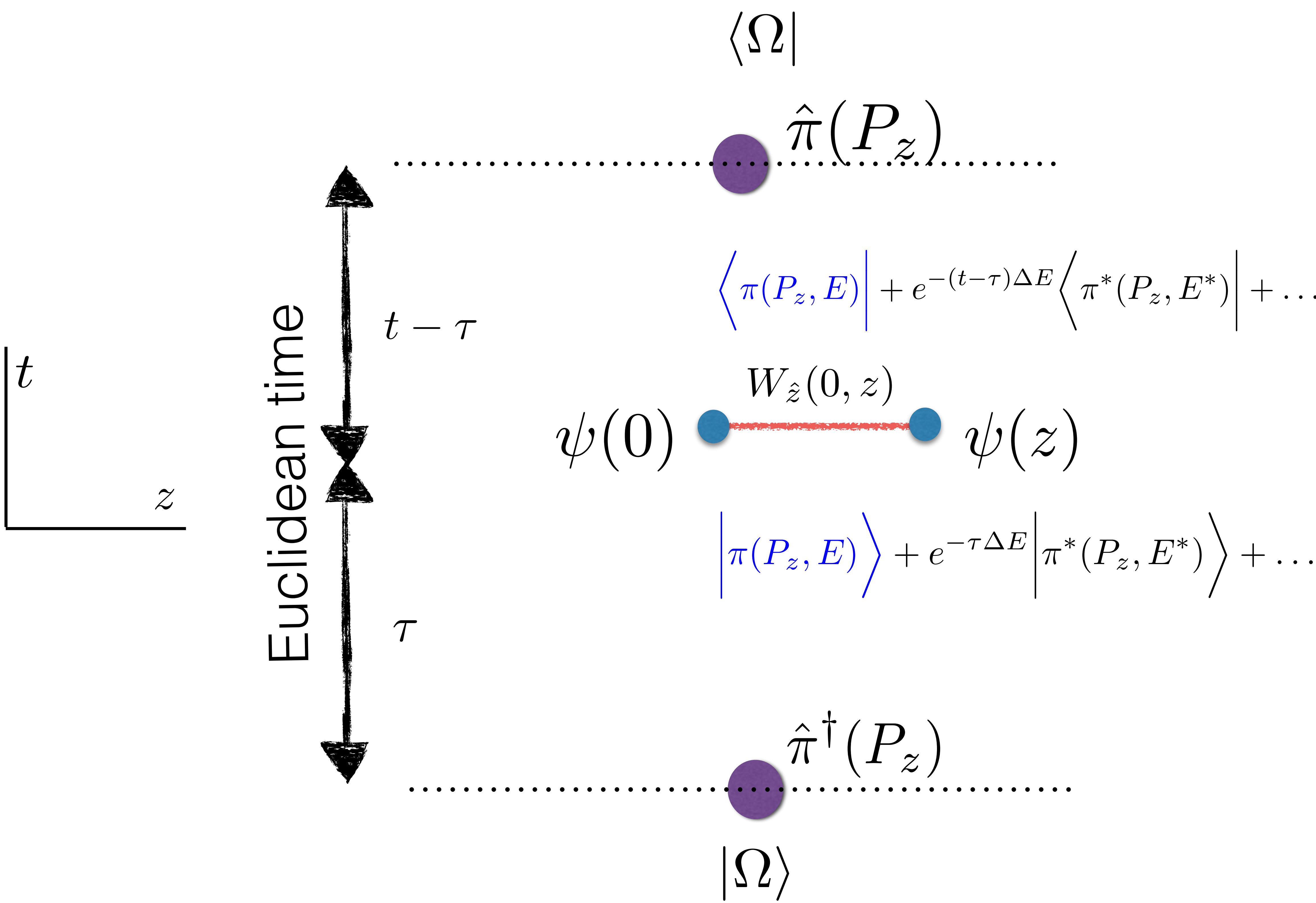
Computing the bare quasi-PDF

Simulation details

- HISQ sea quark from HotQCD gauge field ensemble
- 1-HYP smeared Wilson-Clover valence quark tuned to 300 MeV pion
- Lattice spacing a=0.06 fm (=3.28 GeV)
- 1-HYP smeared Wilson line
- Volume: 3.84 x 2.88^3 fm^4 ⇒ $M_\pi L = 4.4$

Glossary:
sea quark: (noun) det(D) used in Monte Carlo.
valence quark: (noun) D^{-1} used in propagators.
HYP : (Abbr.) A procedure to suppress UV lattice-like gluons.

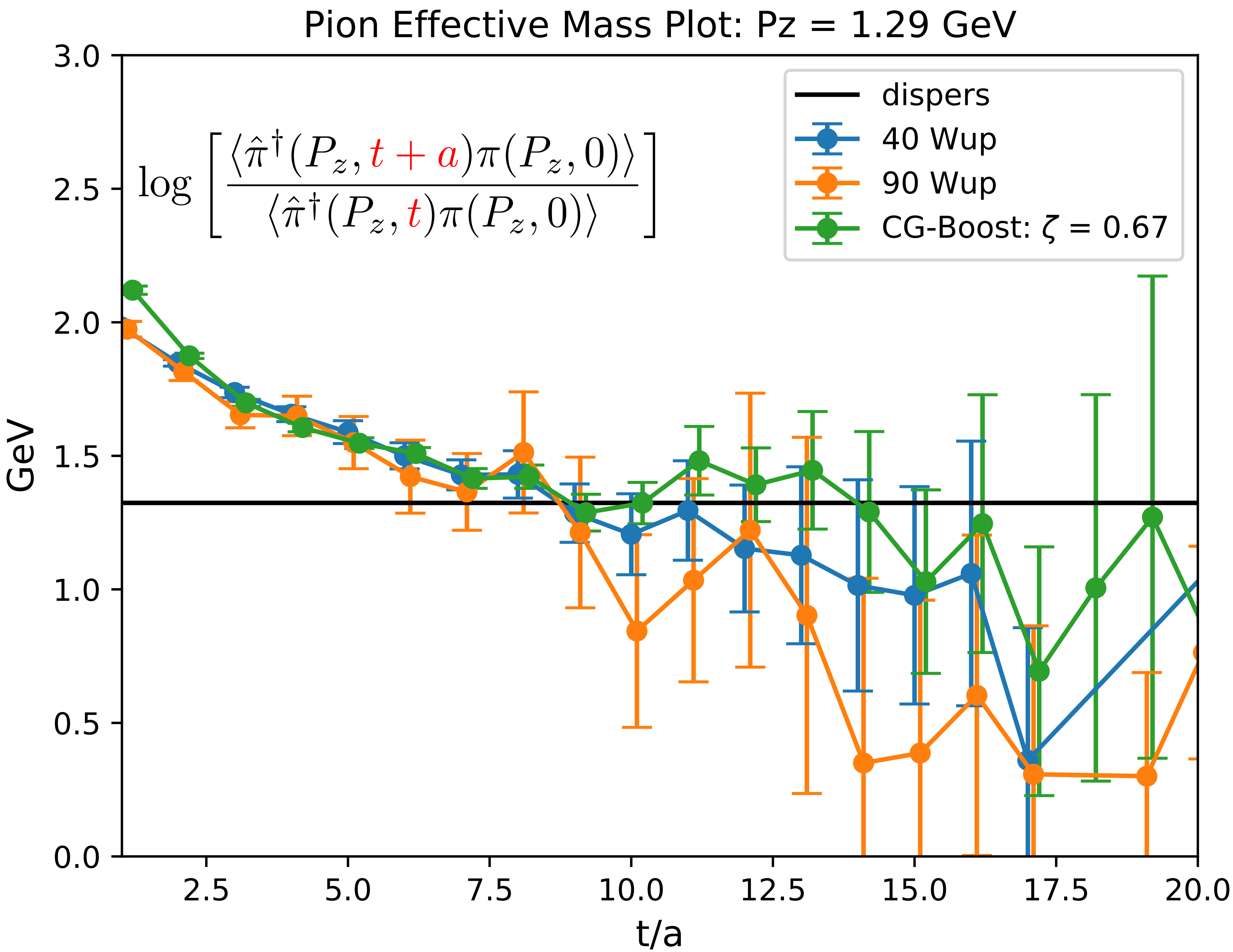
Set up of the 'measurement'



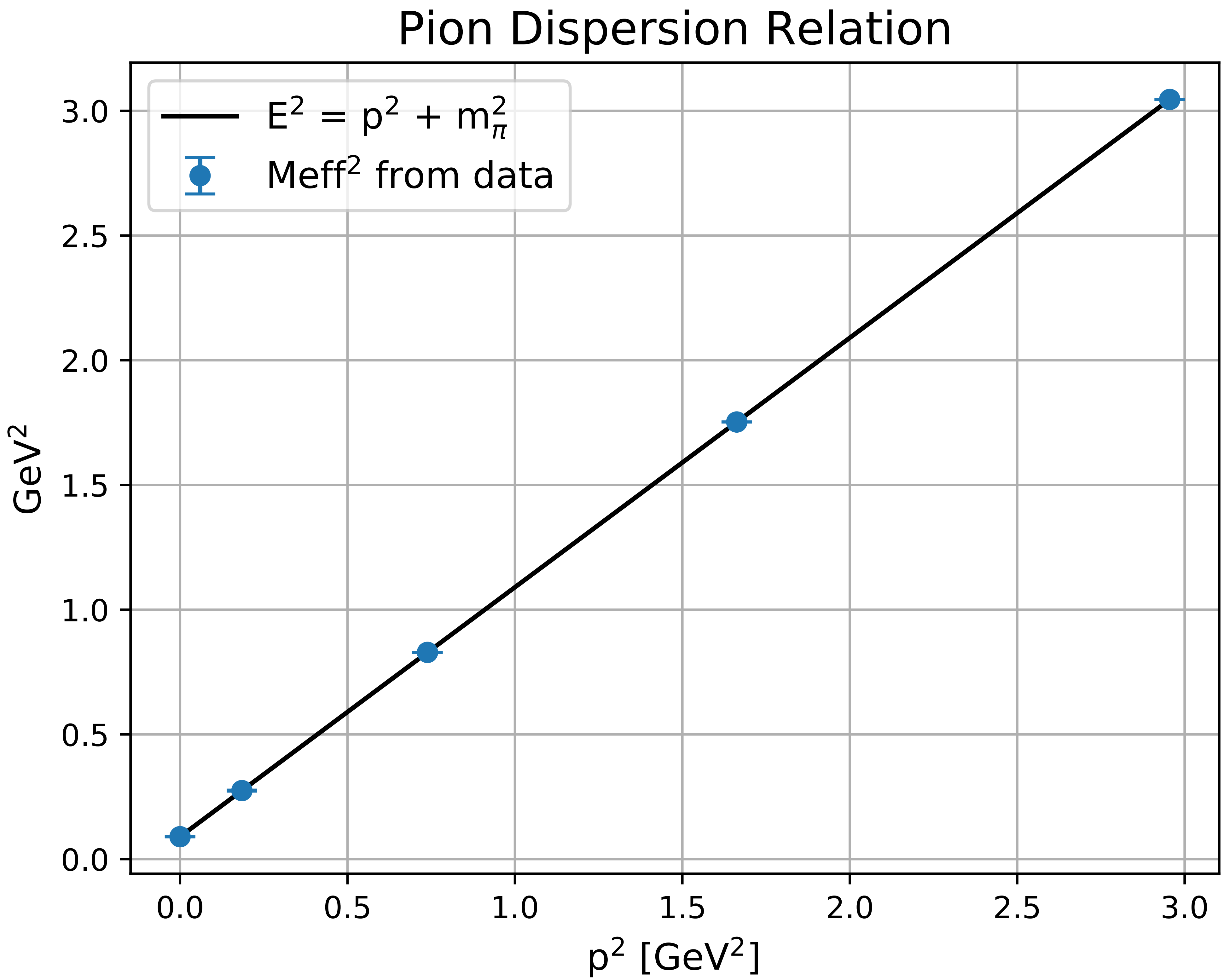
Choice of the creation operator $\hat{\pi}(P_z)$ is important

$\pi(x_0) = u(x_0)\gamma_5\bar{d}(x_0)$...choose quark sources $\psi(x_0,t) \sim \int d^3k e^{ikx_0} e^{-\sigma^2 \frac{(k-\zeta P)^2}{2}} \tilde{\psi}(0)$

Bali et al '16



Lattice dispersion relation matches continuum for all P_z

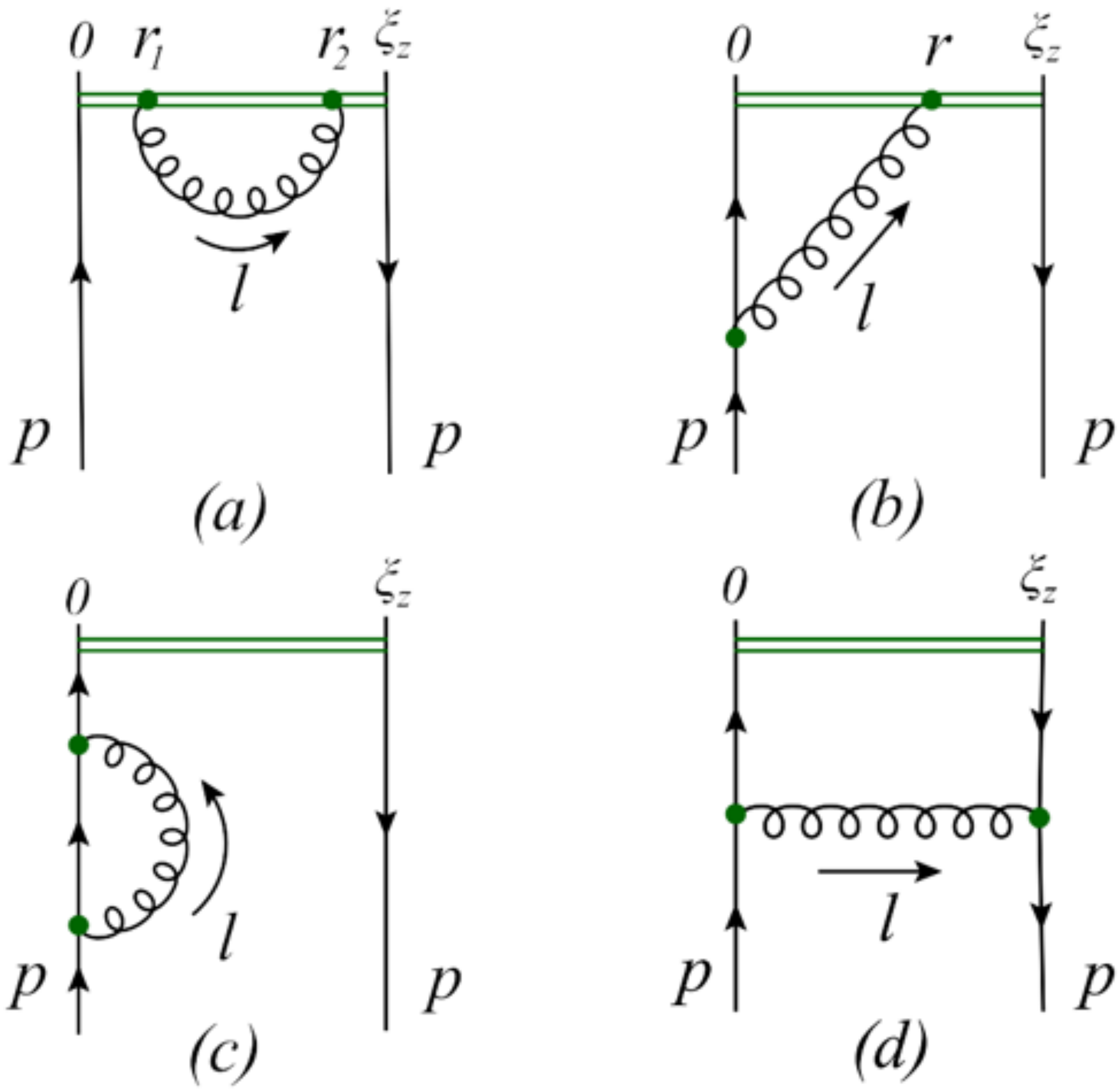


Satisfy the hierarchy $P_z \gg M_\pi, \Lambda_{\text{QCD}}$ & $P_z \ll$ UV lattice scales

Renormalization

Renormalizability of bi-local quark bilinear

- Real-space quasi-PDF operator can be multiplicatively renormalized with a factor **Z(z)** (Ishikawa et al ’17)



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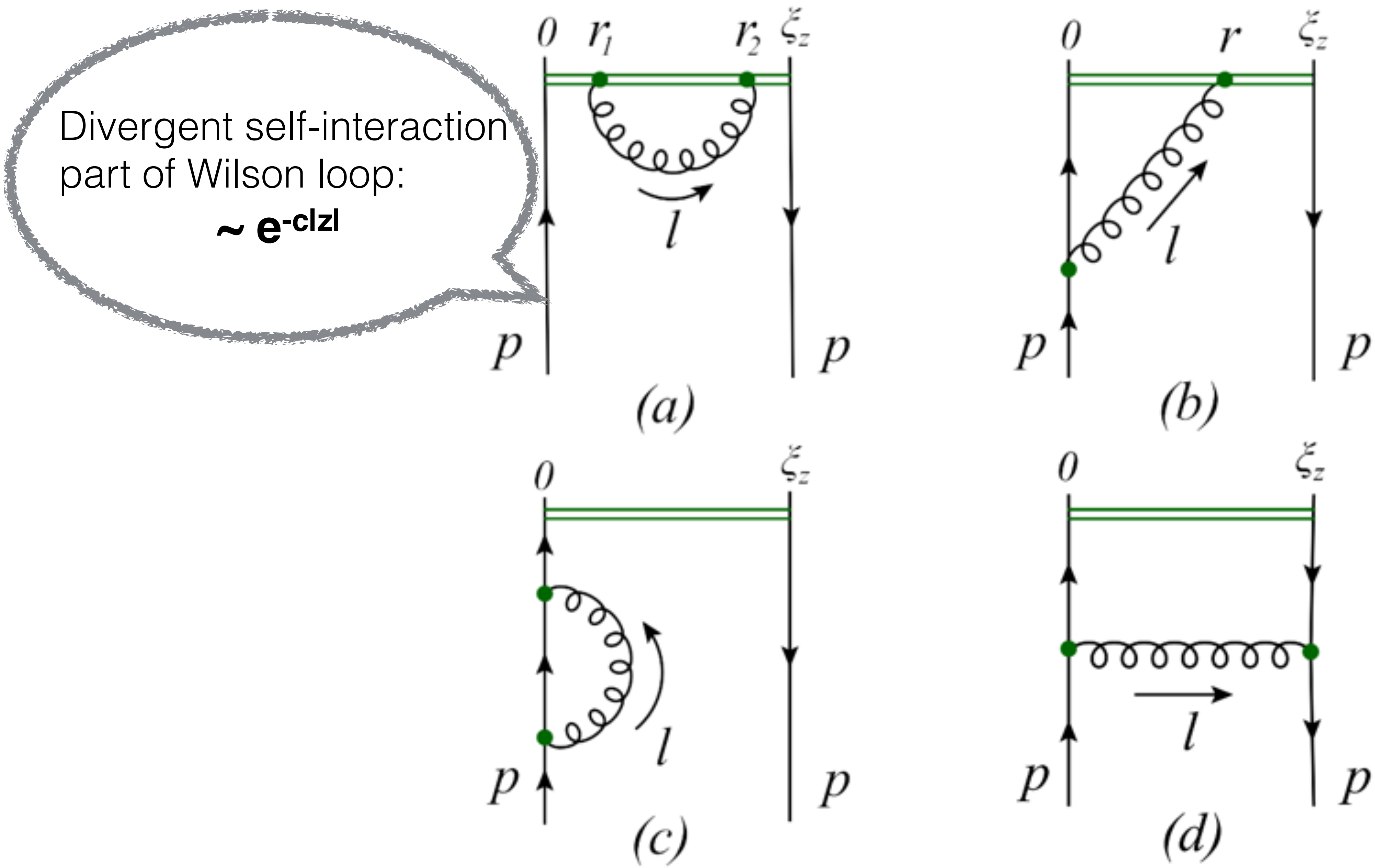

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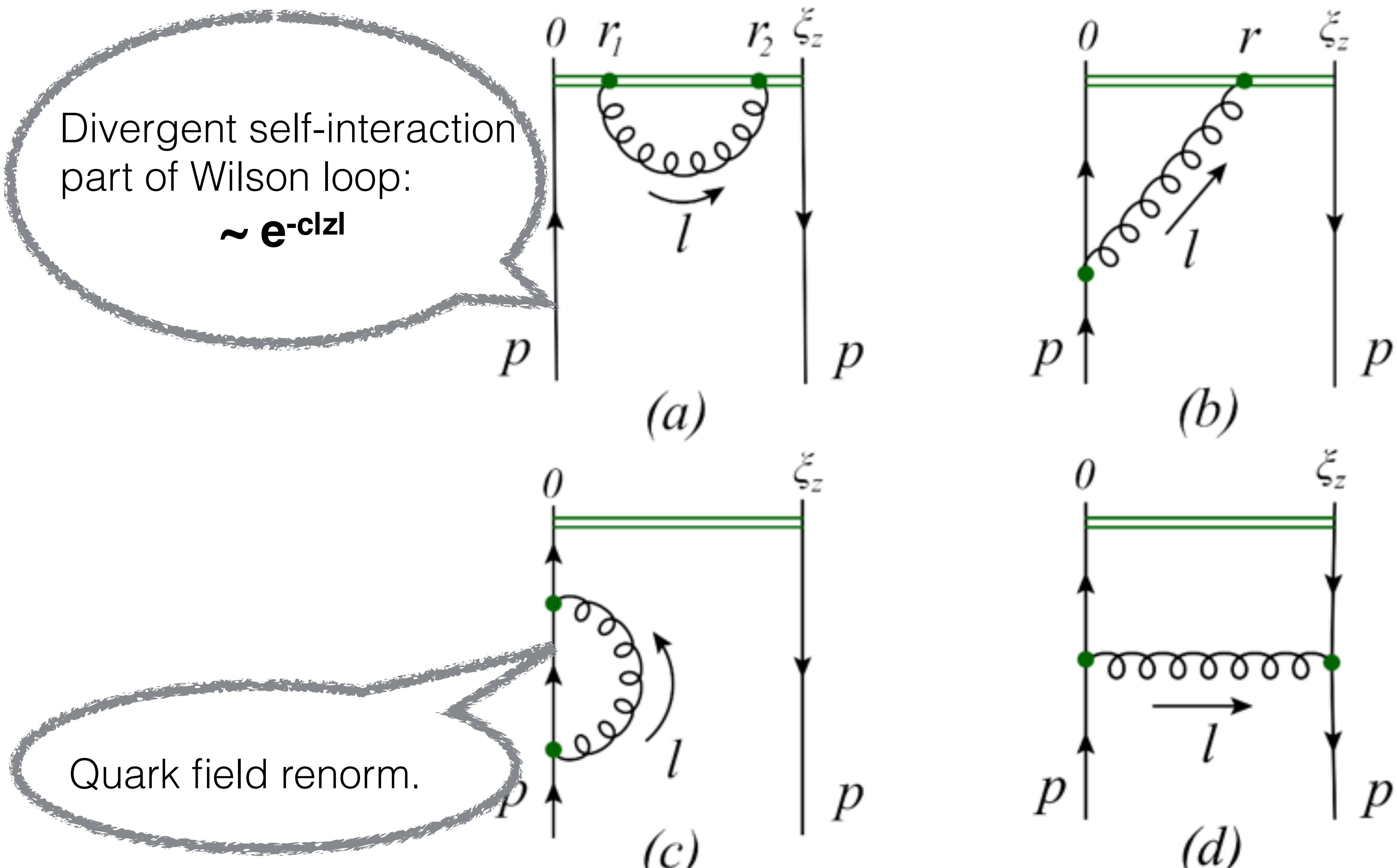

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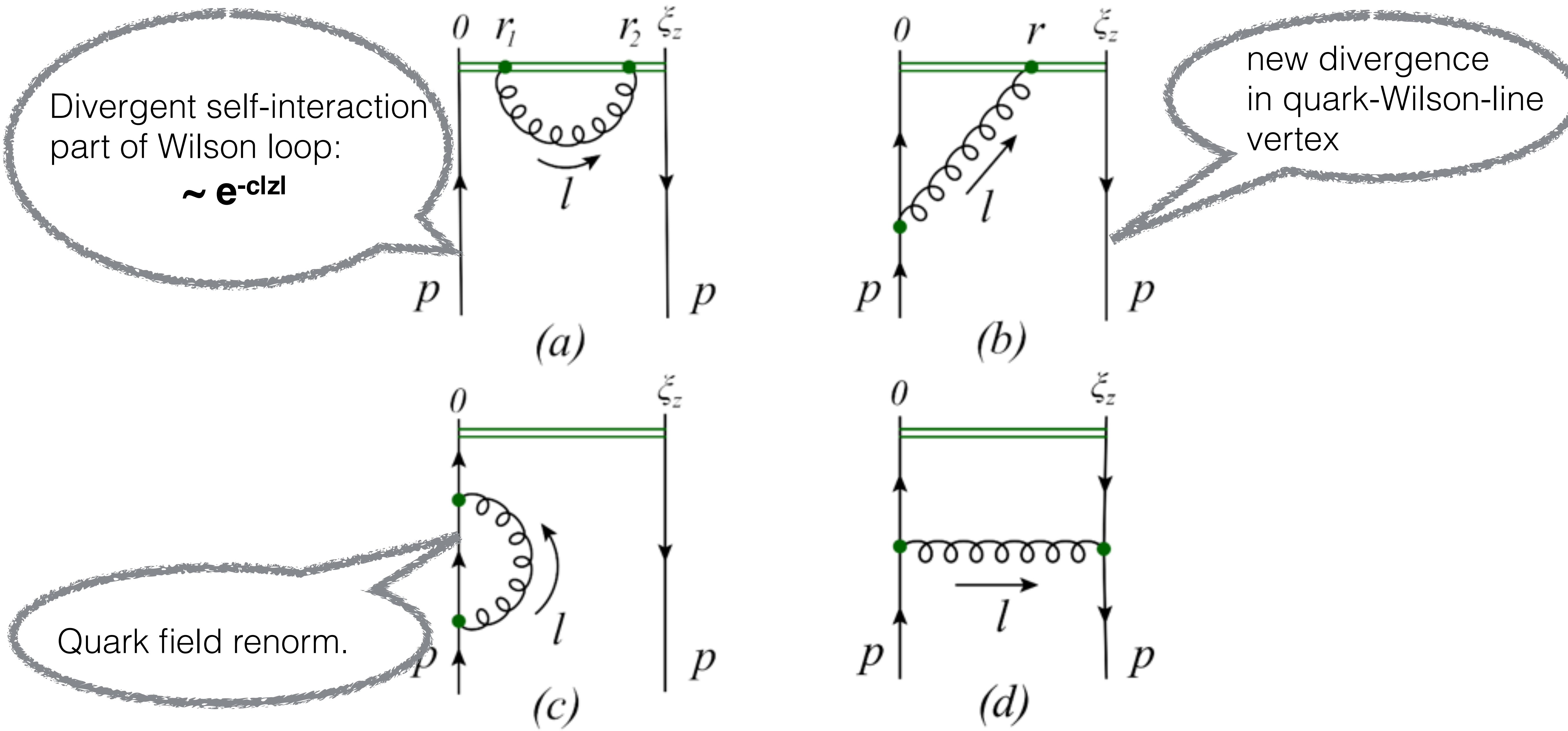

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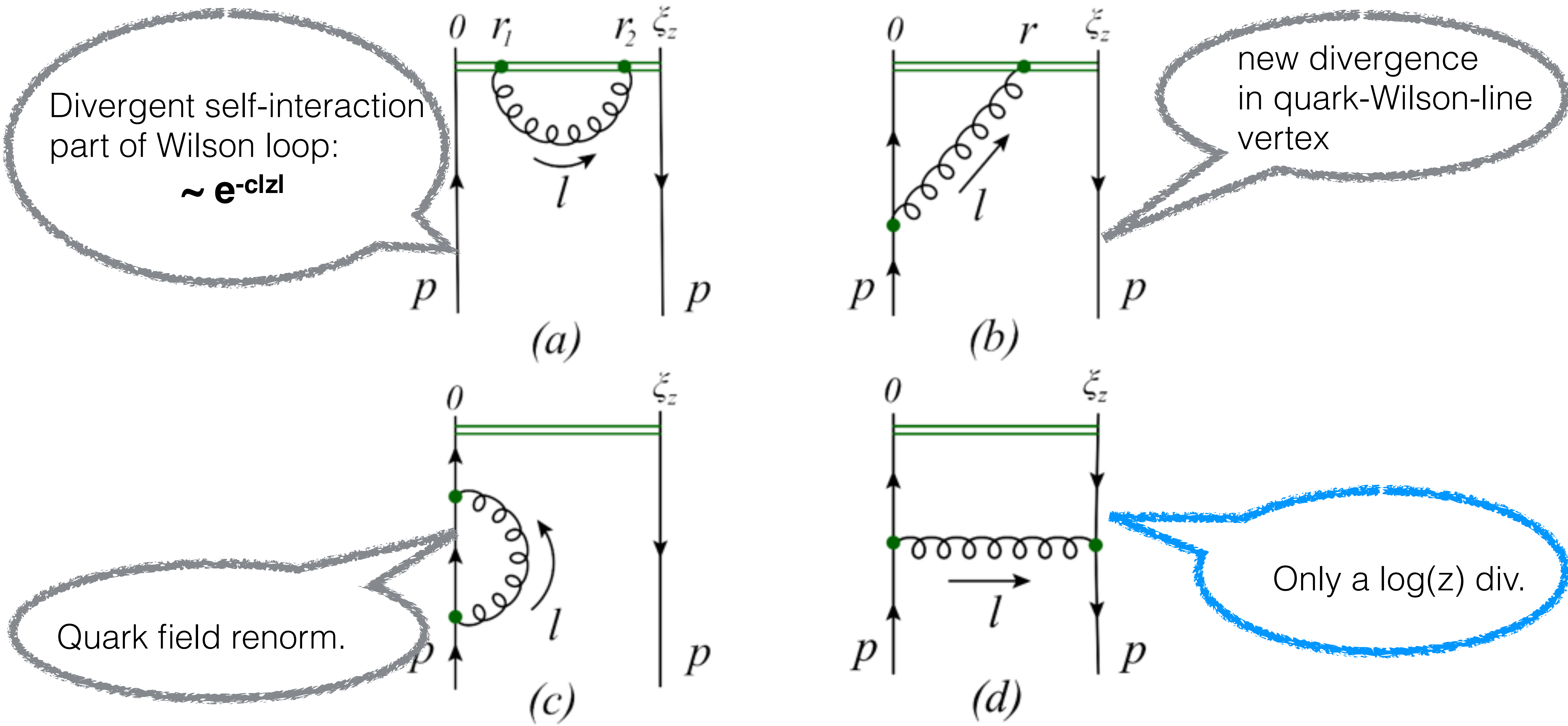

Renormalization conditions

Renormalization conditions

The renormalizability means:

$$h^R_{\gamma_t}(z; P_z, P^R) = Z_{\gamma_t\gamma_t}(z; P^R) \cdot h^b_{\gamma_t}(z; P_z, a)$$

renormalized hadron qPDF

bare hadron qPDF

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renormalized hadron qPDF

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Renormalization scheme independent conditions:

$$Z_{\gamma_t\gamma_t}(z; P^R) \cdot h^b_{quark}(z; P = P^R, a) = h_{free}(z; P^R)$$

barequark qPDF in full QCD

Tree level value

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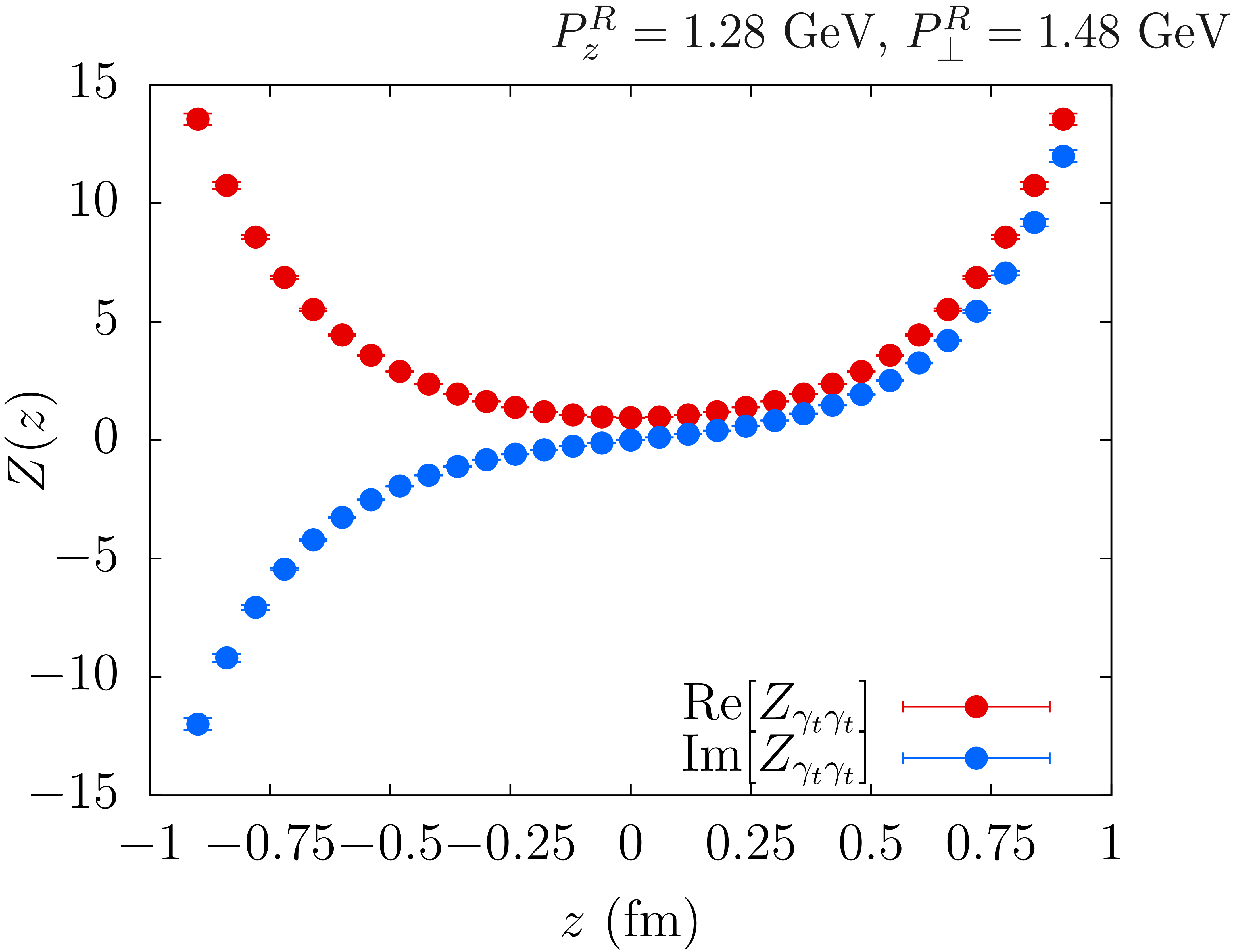
barequark qPDF in full QCD

Tree level value

Implementable in lattice as well as pert. theory with off shell quark with $P^2 > 0$

Renormalization Constants Including Self-Energy



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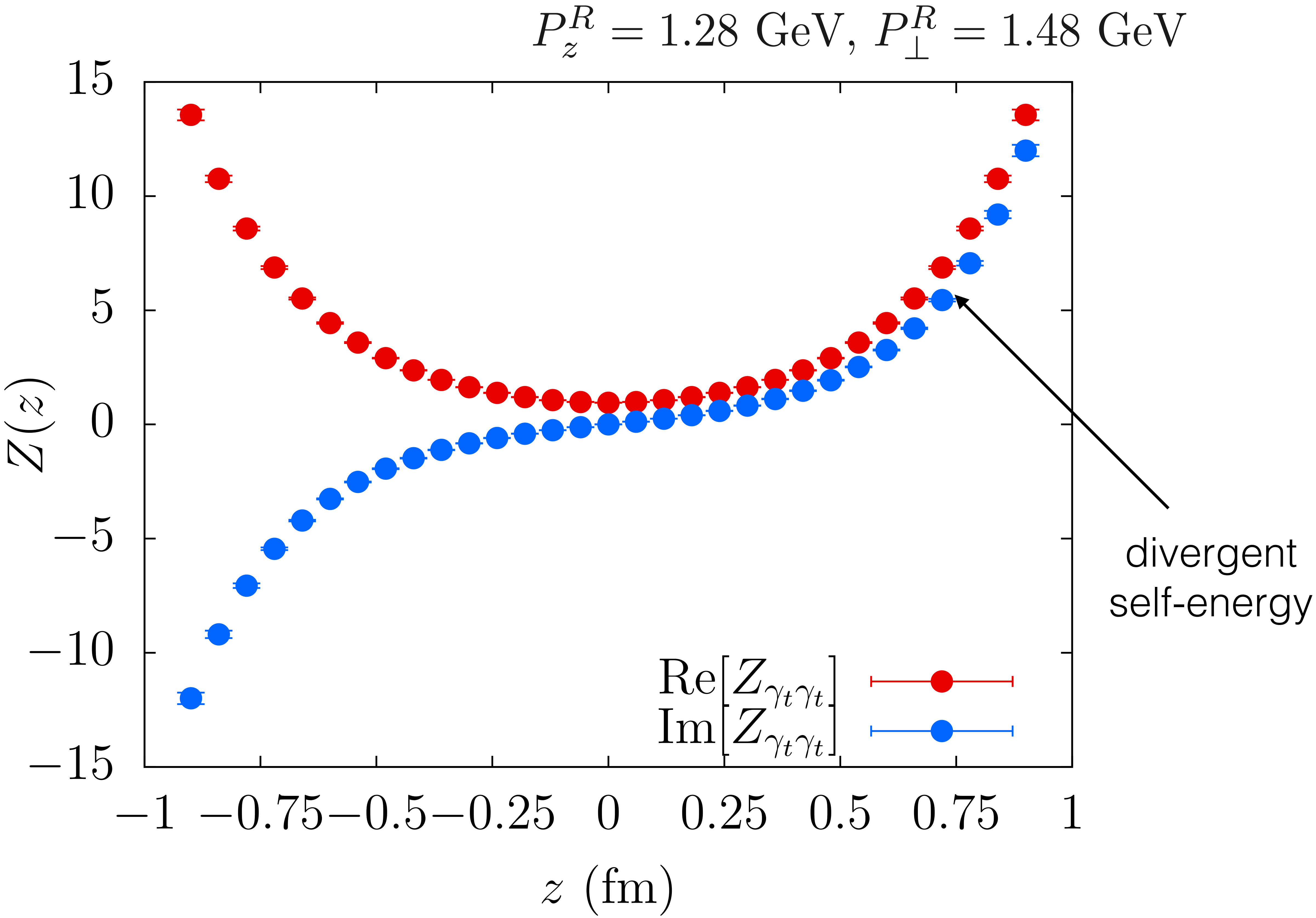
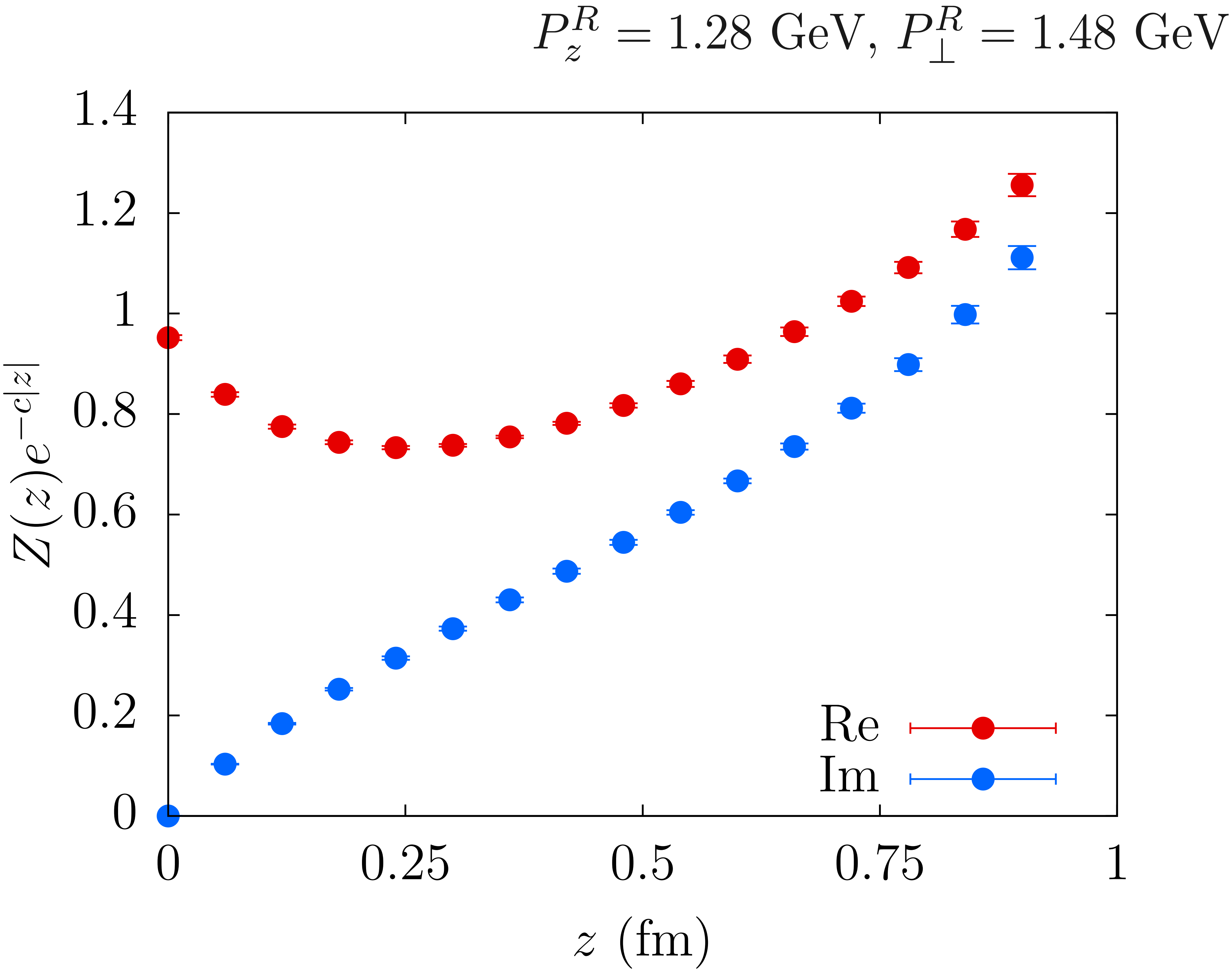


Renormalization constants excluding self-energy is O(1)



Comparison between lattice and perturbative quark qPDF

We compare the lattice and 1-loop *running* of quark PDF away from renormalization point:

$$\frac{h^R_{quark}(z; P \neq P^R, P^R)}{h^R_{quark}(z; P = P^R, P^R)} = 1 + \alpha_S F(z, P, P^R) \text{ + negligible NLO(?)}$$

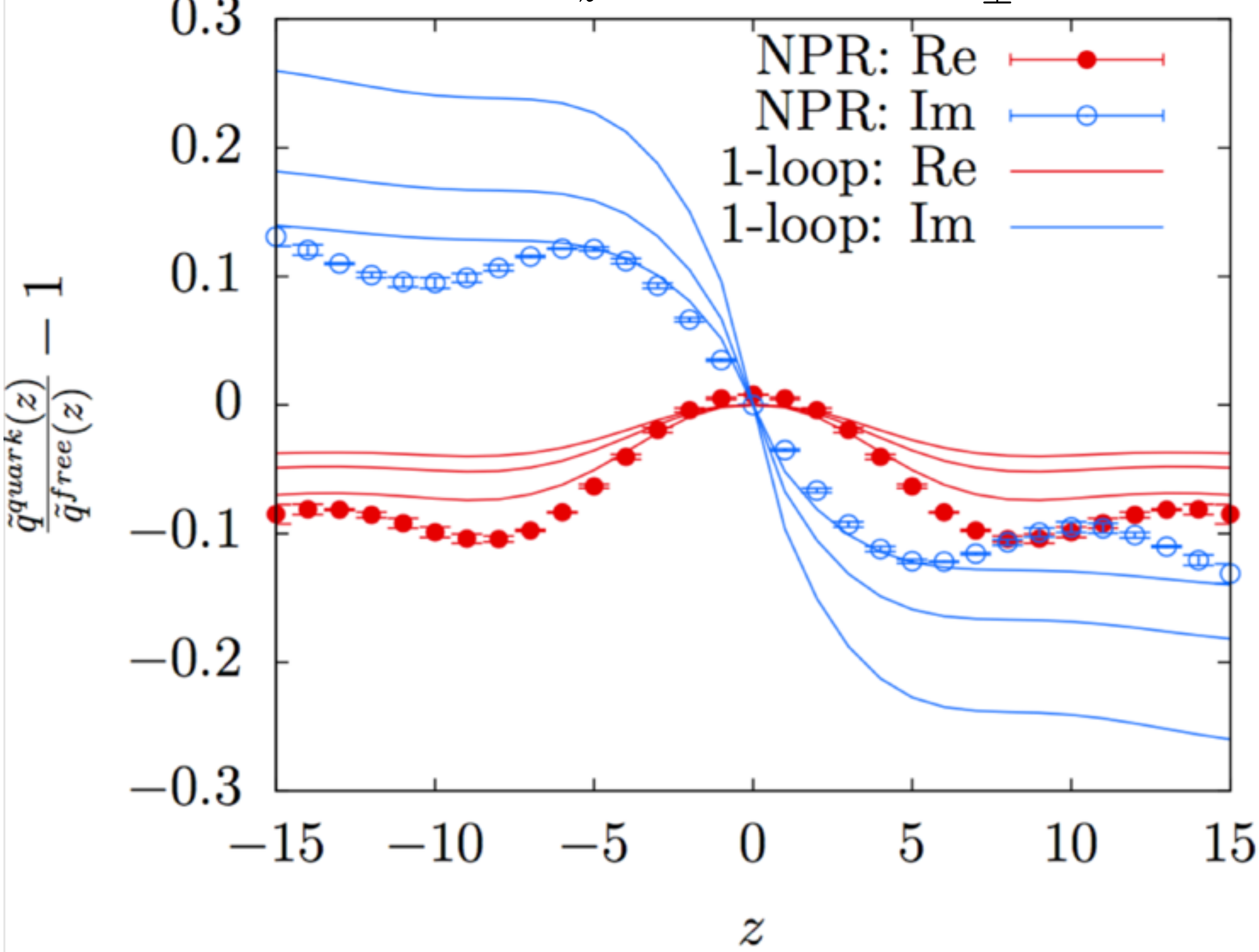
with the Renormalization condition fixing the value :

$$h^R_{\rm quark}(z; P = P^R, P^R) = h_{\rm free-quark}(z; P = P^R)$$

Comparison between lattice and perturbative quark qPDF

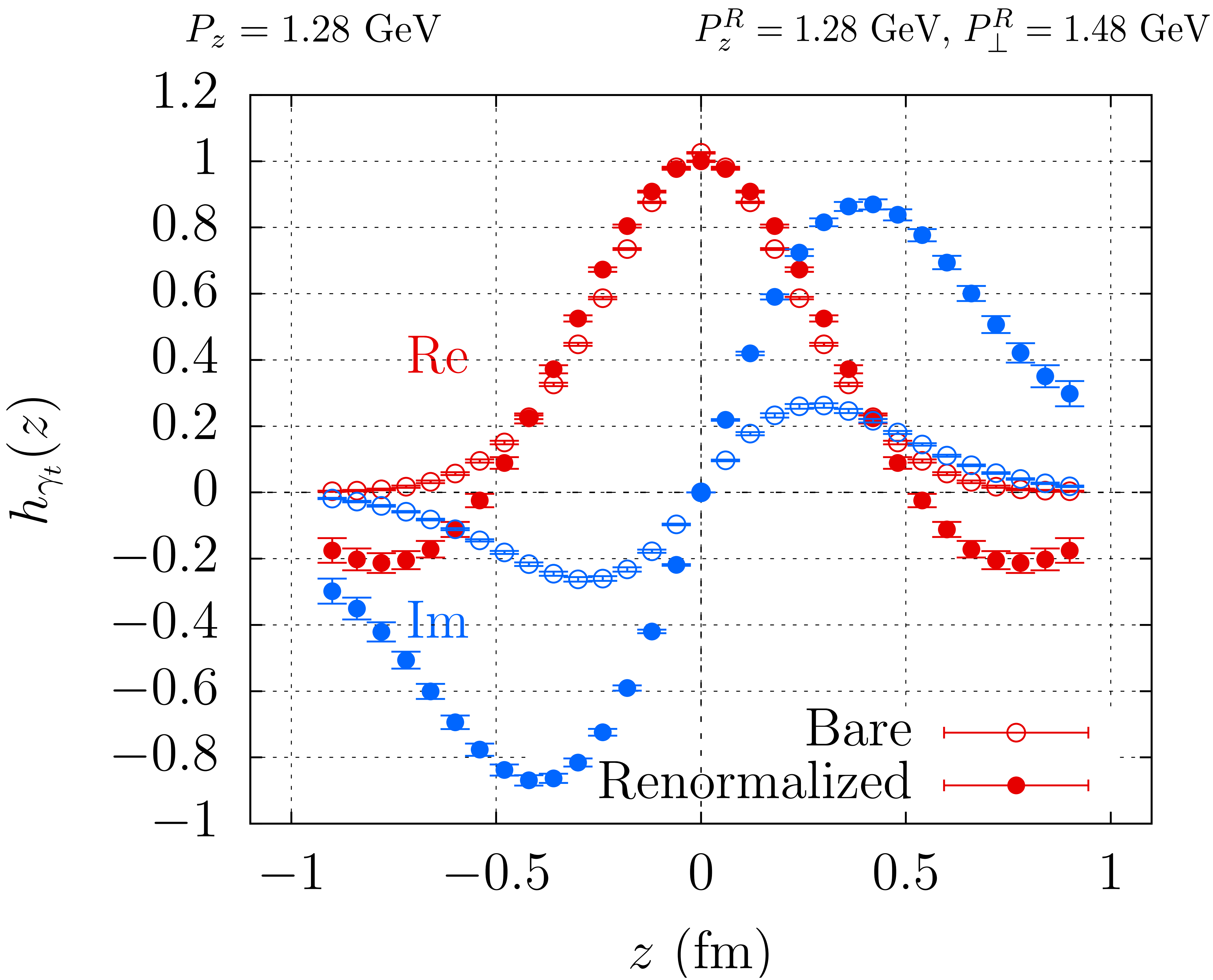
A generic case: $P_z = 1.28$ GeV, $P_\perp = 1.48$ GeV

$P_z^R = 1.28$ GeV, $P_\perp^R = 2.22$ GeV

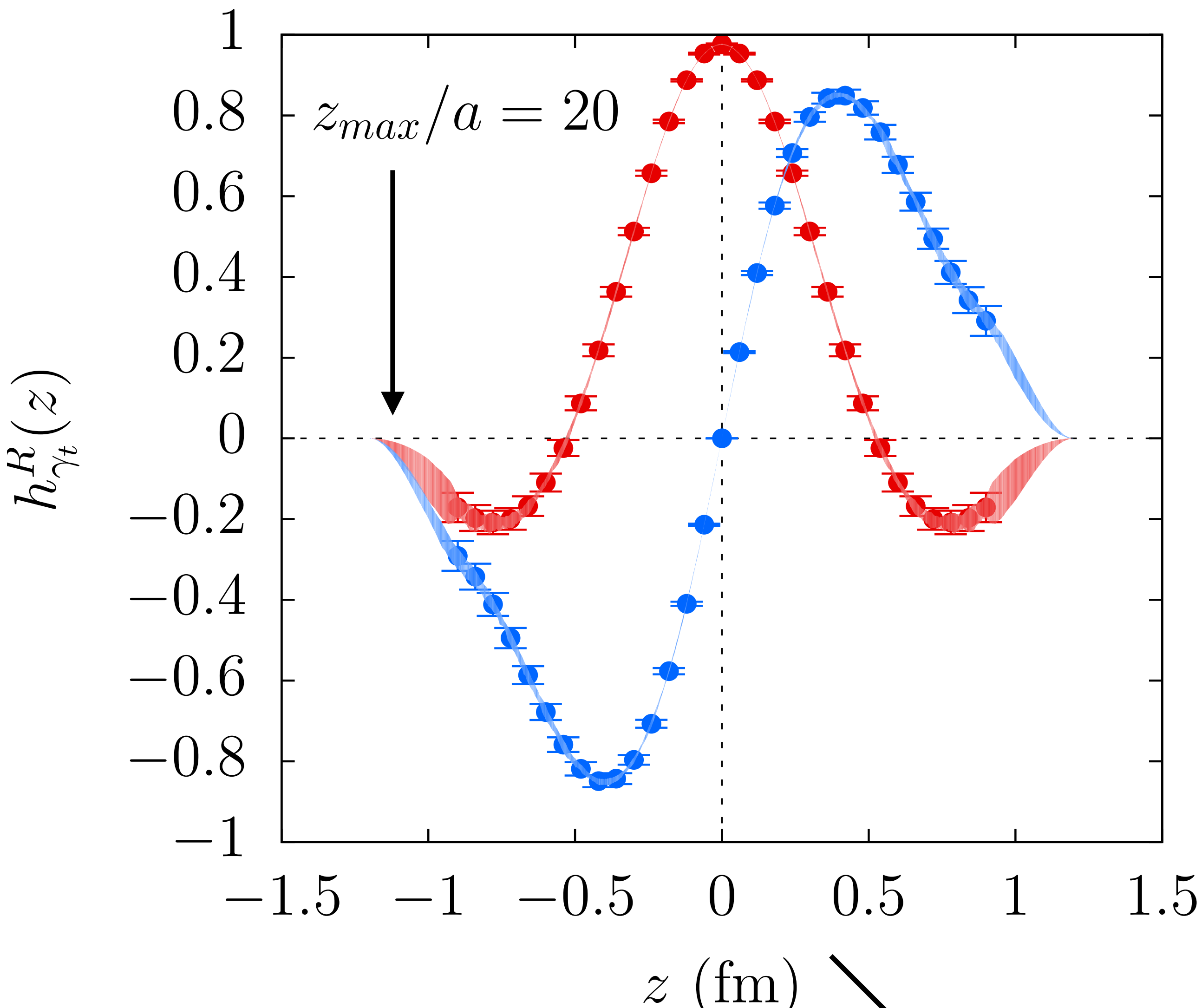


Matching to pion PDF

Real-space pion qPDF



Pion qPDF from real-space to Fourier

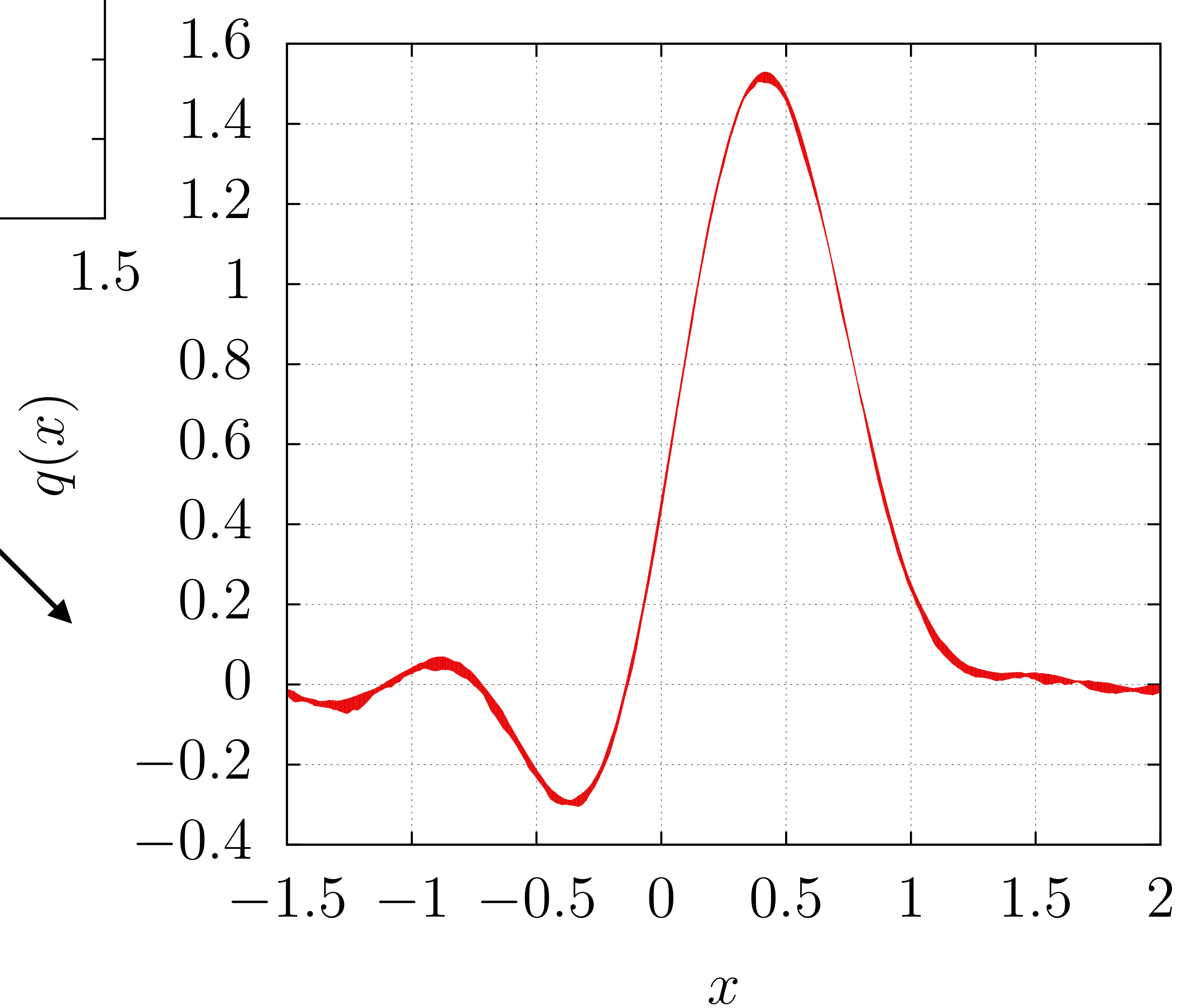


Quantify the unknown:

Cubic spline from z= $-z_{\max}$ to $+z_{\max}$

Vary $z_{\max}$

Fourier



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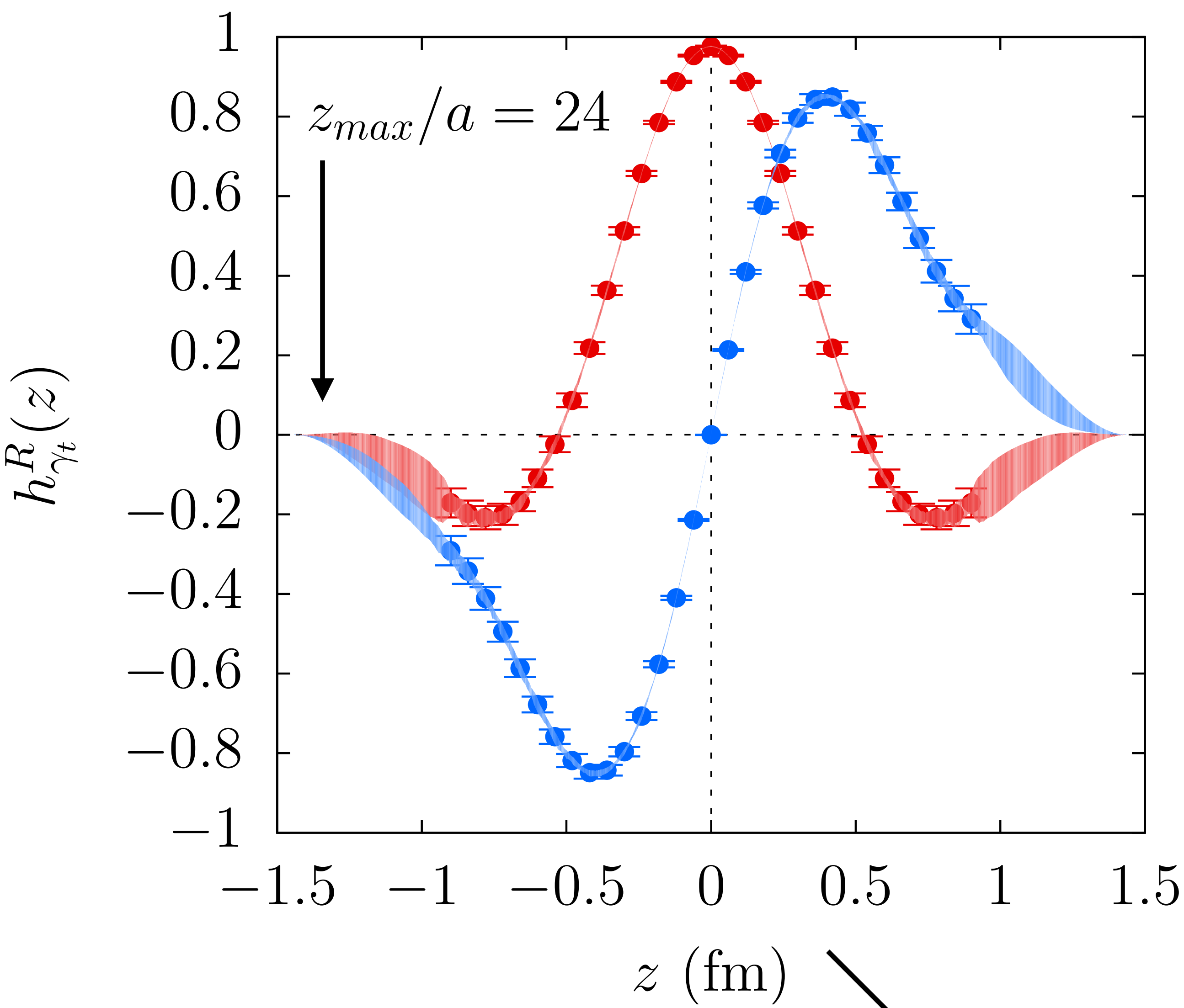
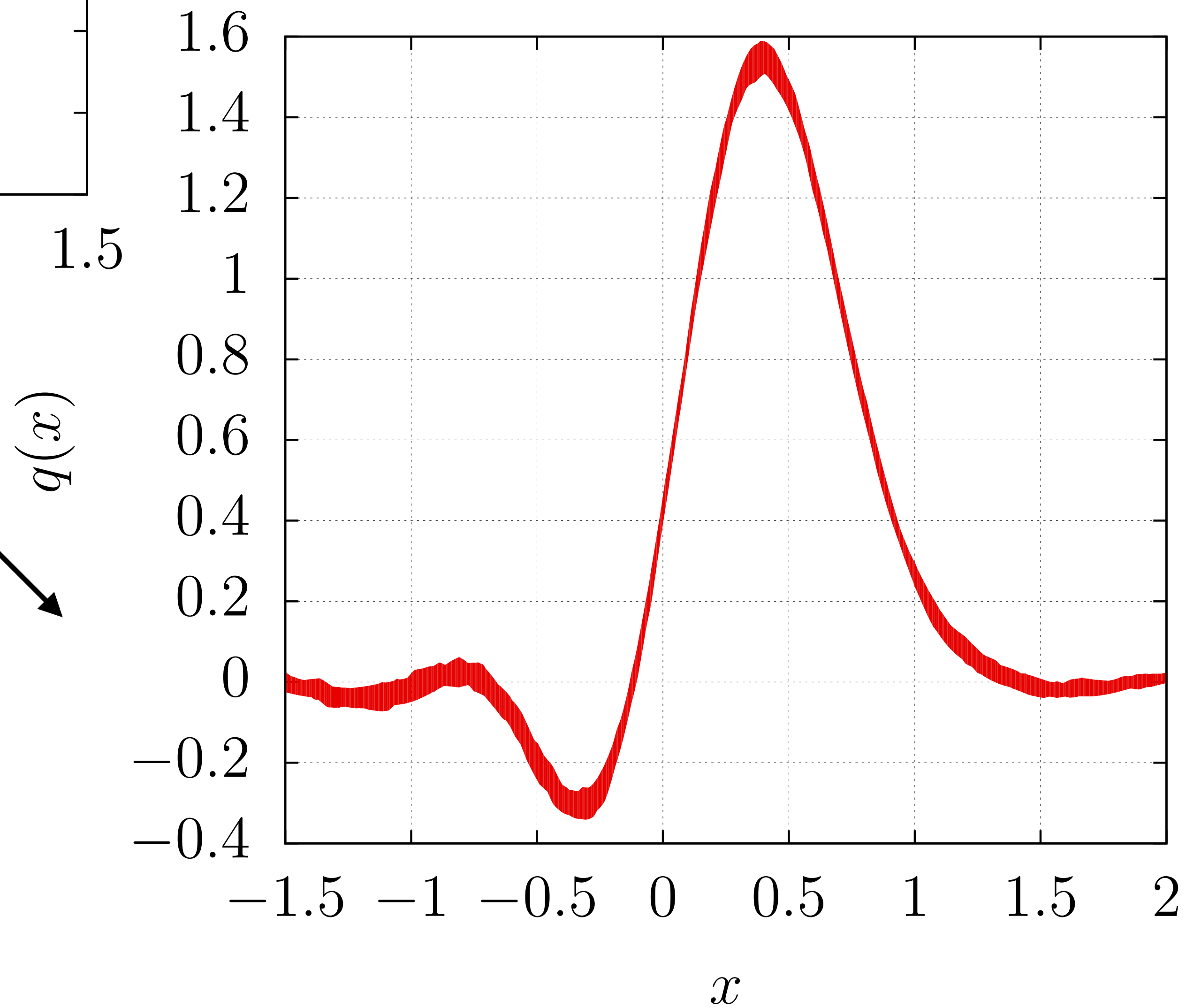
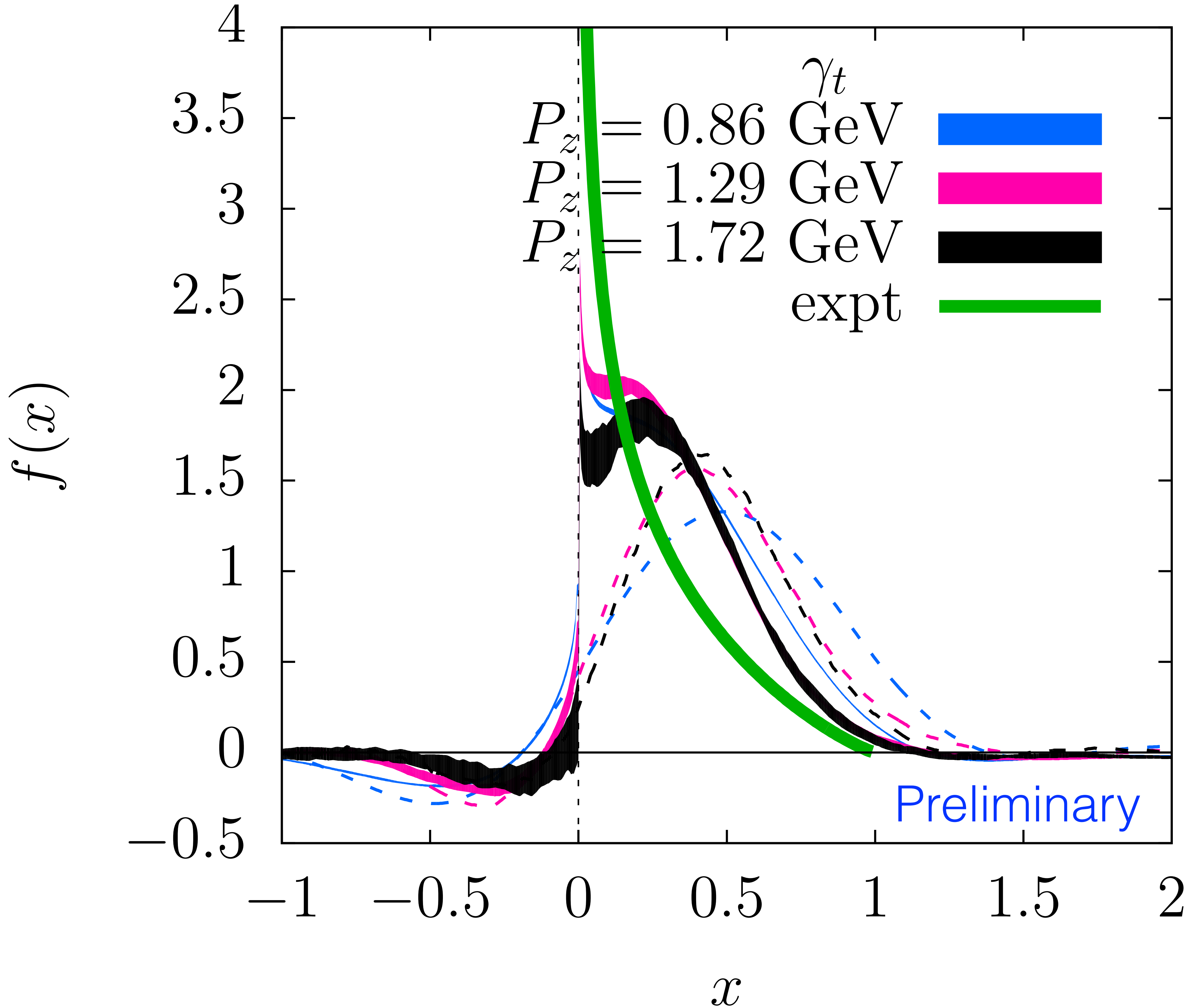


Shape seems to be robust under z_{max} variations

Fourier



Pion PDF at $\mu^2 = 10\ \mathrm{GeV}^2$ at different boosts P_z



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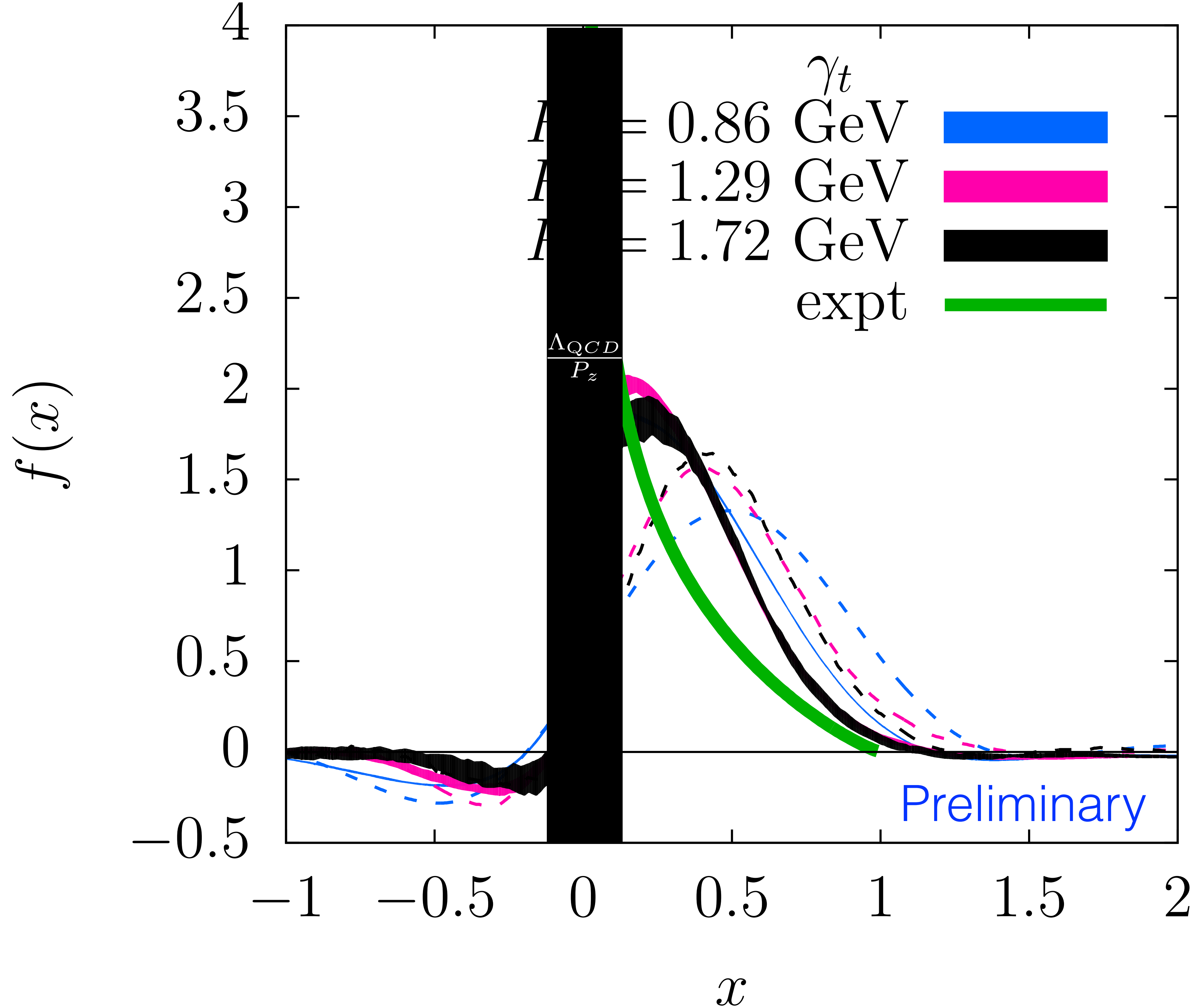
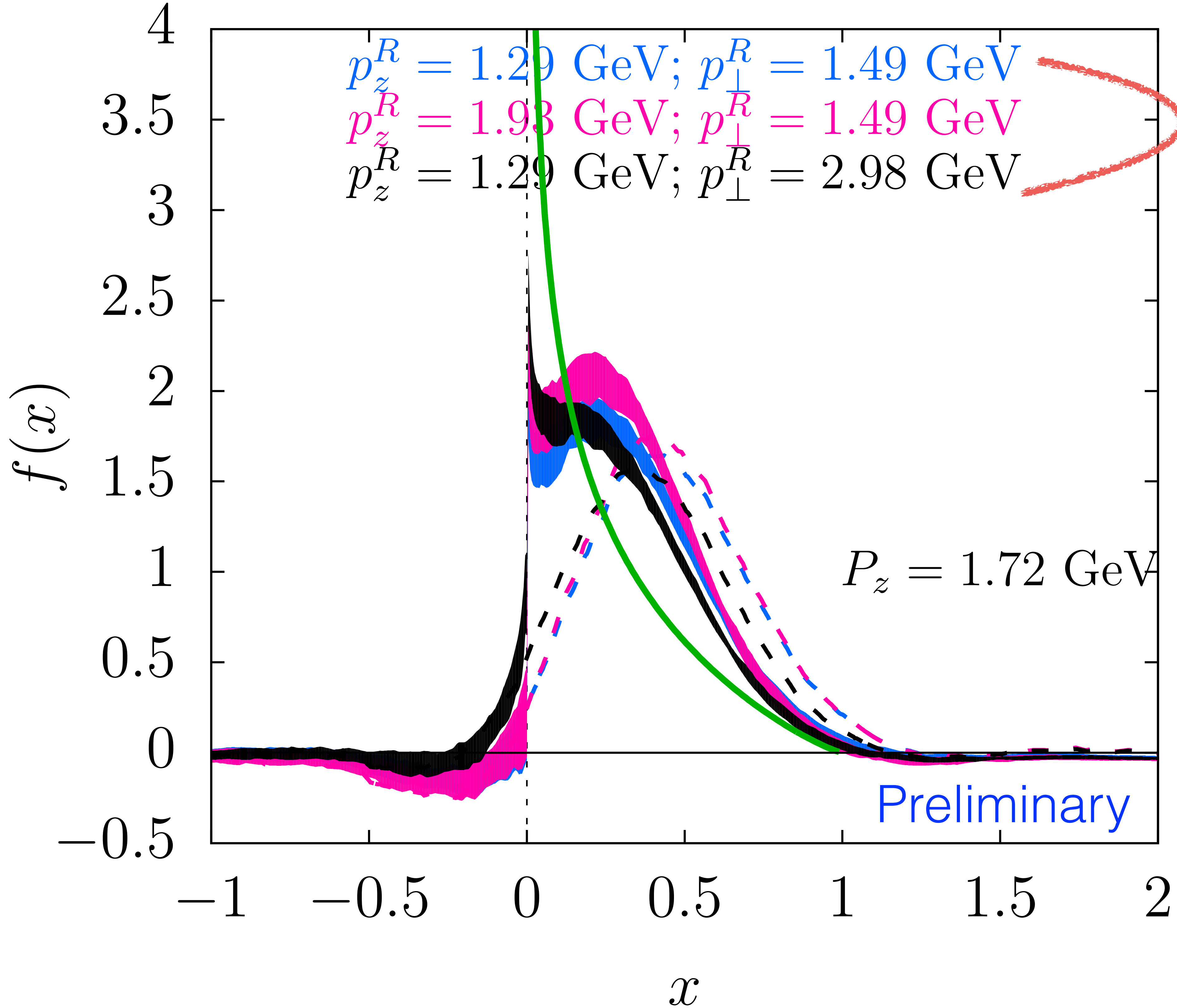
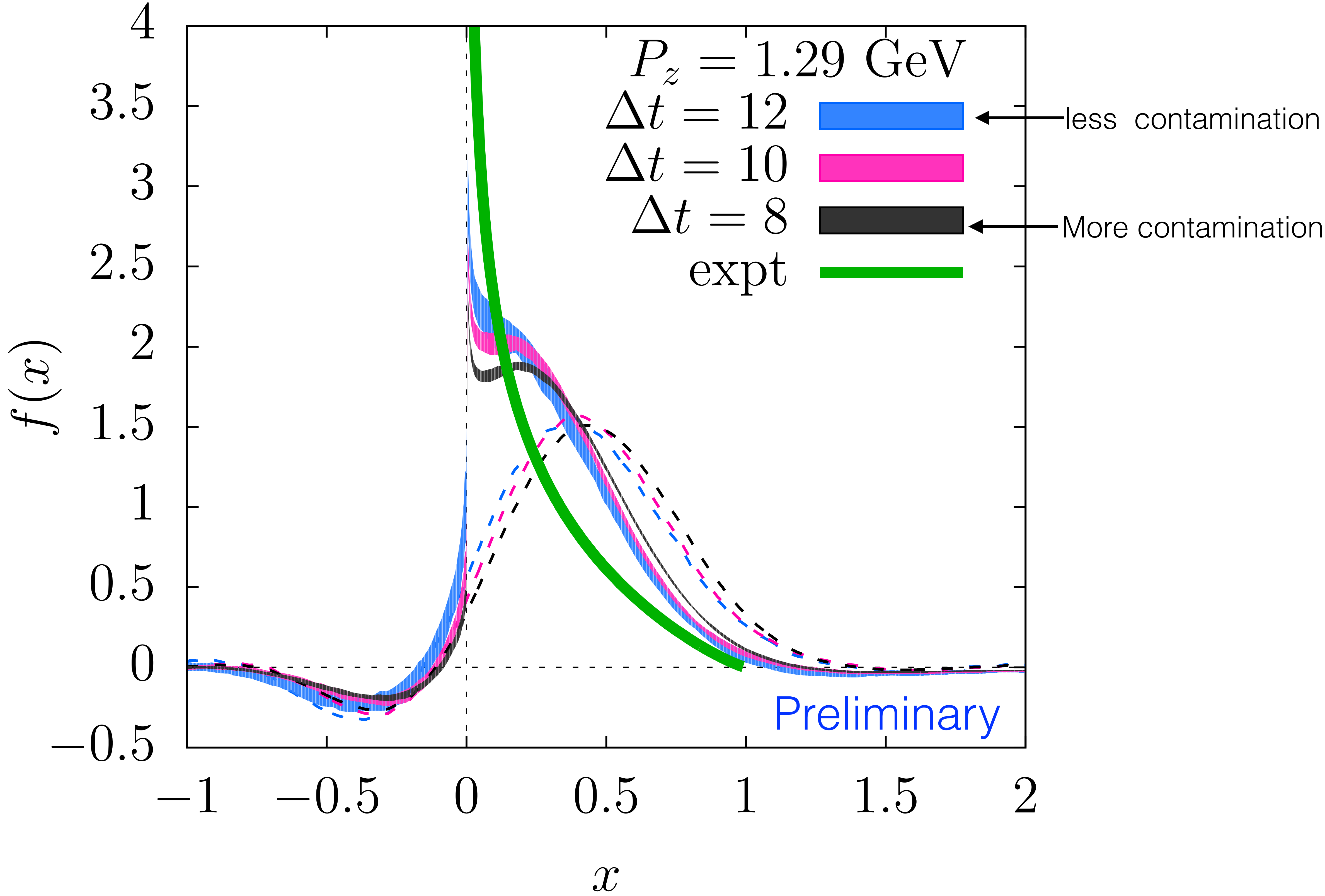


In principle, PDF should not depend on renormalization point of qPDF… but there is some dependence

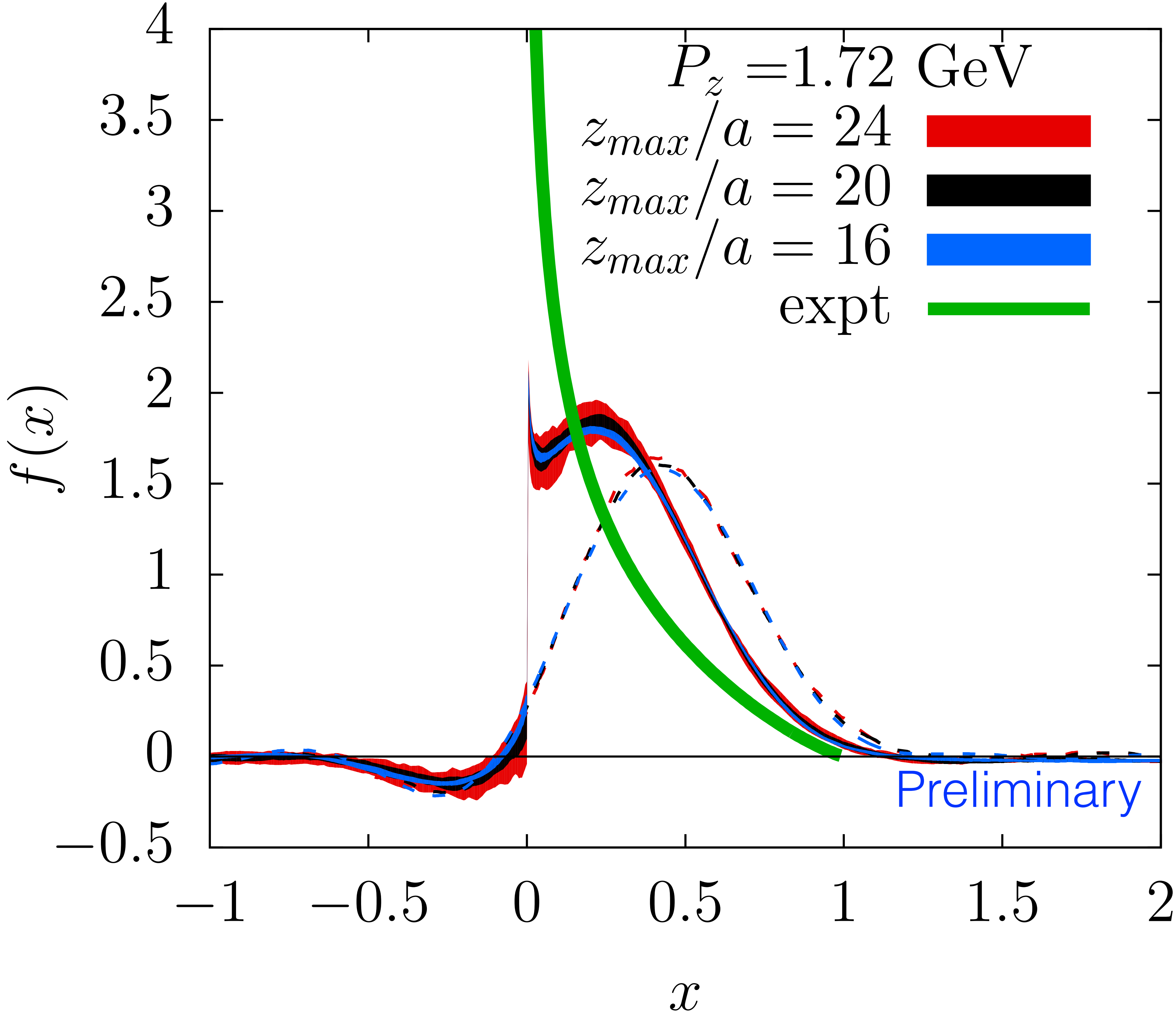


Different renormalization points with everything else fixed.

Effect of excited states in pion channel



Slight changes to long-distance part of h(z) has little effect



An analysis of “experimental quasi-PDF”

Pion PDF data from expt → RI-MOM qPDF at Pz

(Barry et al,
PRL 121, 152001)

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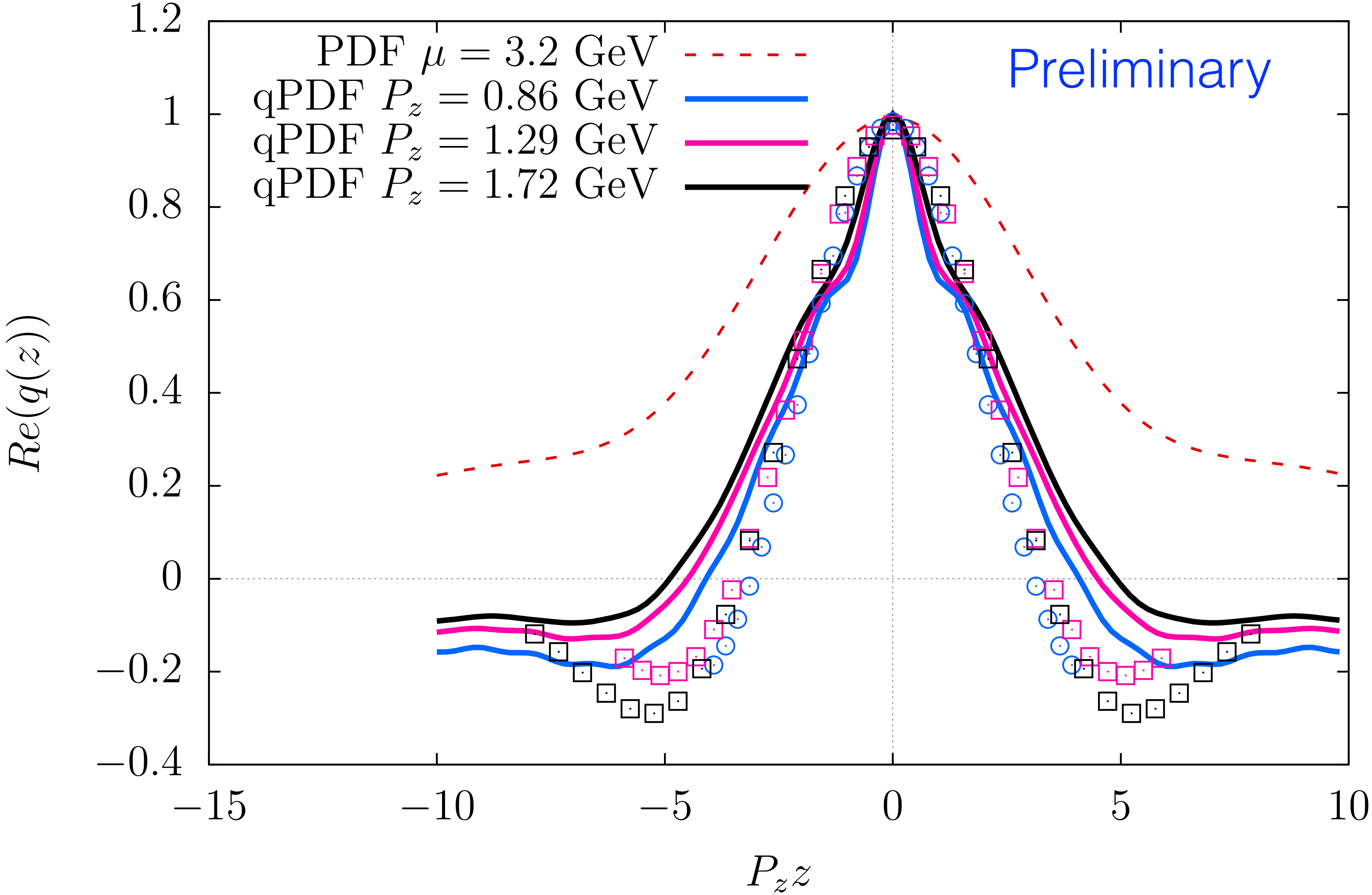
Pion PDF data from expt (Barry et al, PRL 121, 152001) → RI-MOM qPDF at Pz



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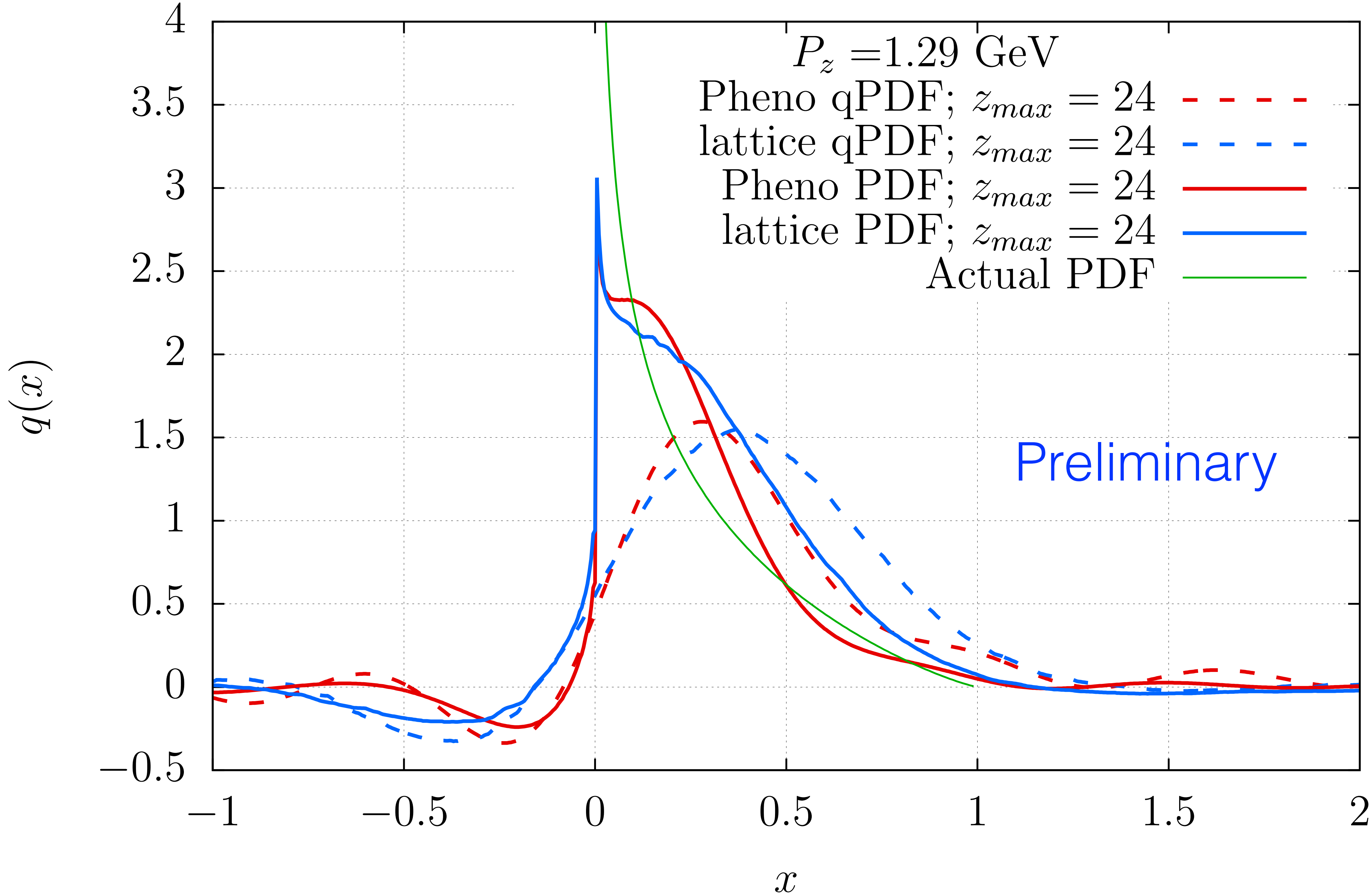

Lattice data is more like the “pheno-lattice" data with z_{max} cut

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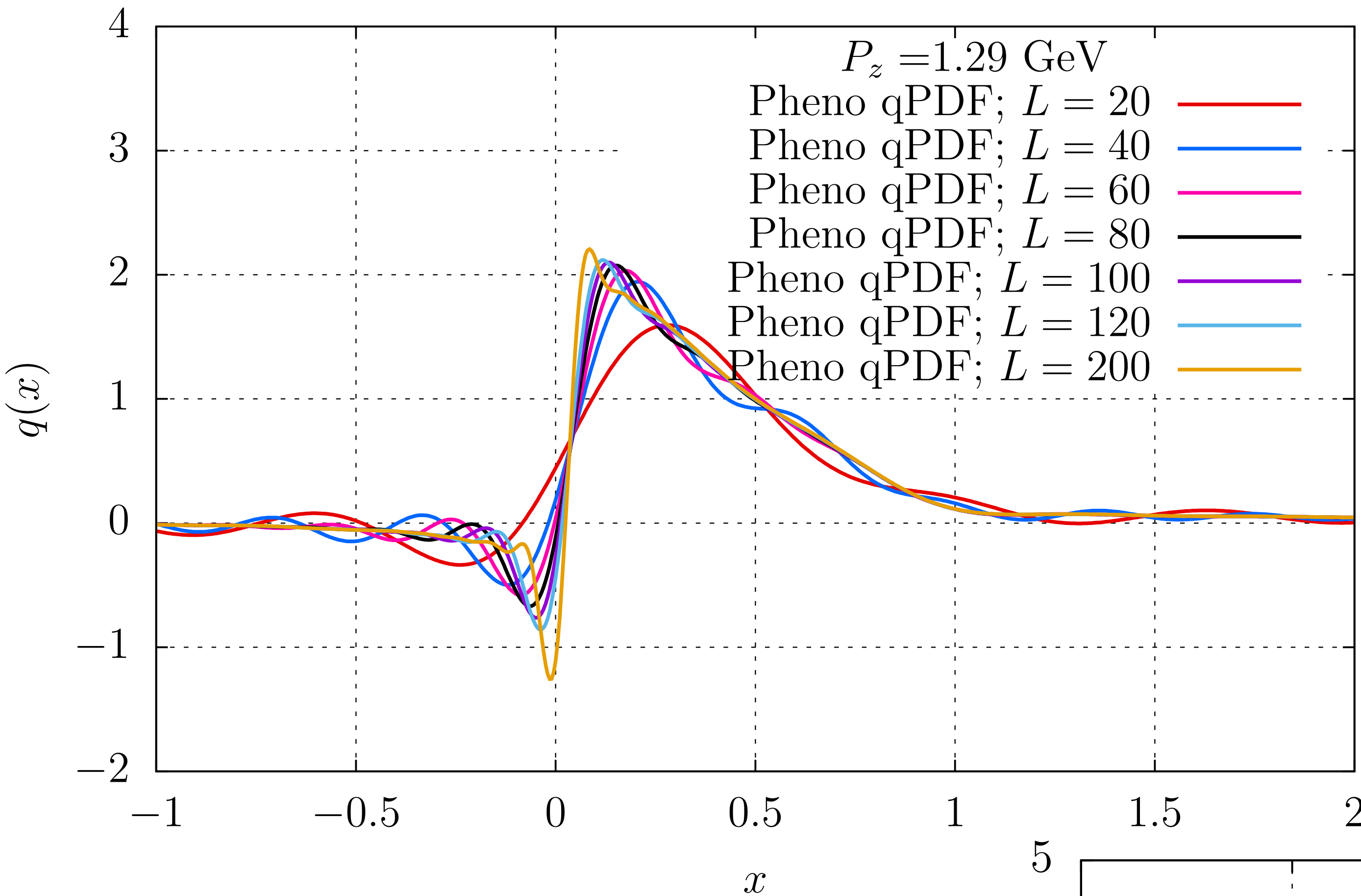
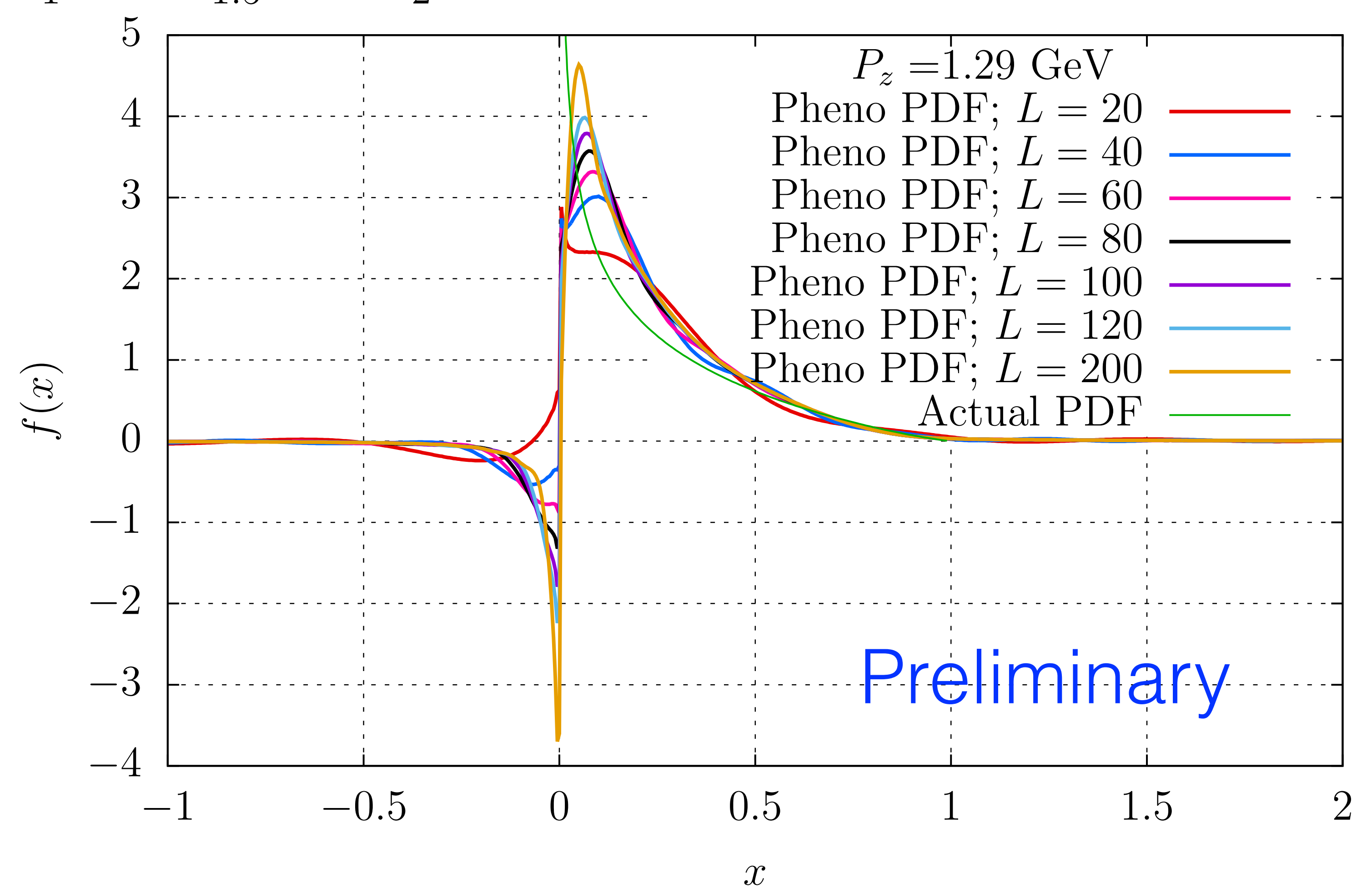


Increase the number of lattice sites L over which “pheno-lattice" data is determined on.



Conclusions

- We studied valence pion quasi-PDF using HISQ sea quarks and Wilson-Clover valence quarks.
- We investigated the validity of 1-loop renormalization in describing NPR and found qualitative agreements between the two.
- We matched the pion qPDF to the PDF at mu=3.2 GeV. Though matching suppressed values above x>1, there is still discrepancy with phenomenological result (pion mass? *long distance not well accounted for?* Even larger P_z is needed?)
- Under investigation: (*being done*) removing the effect of source-sink separation, generate data at larger z, (*will be done*) towards continuum including a=0.04 fm ensemble