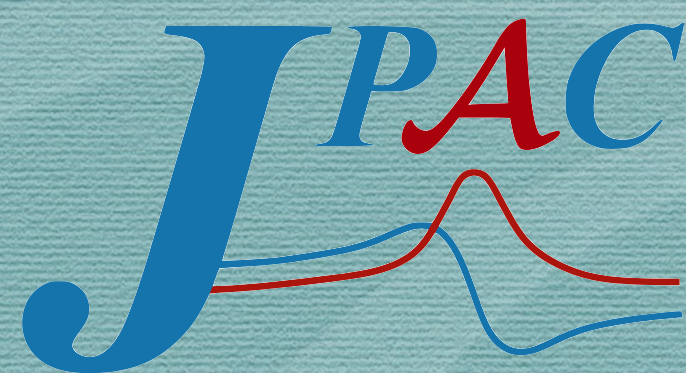




Regge phenomenology of the (charmless & bottomless) baryon spectrum

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Universidad Nacional Autónoma de México



Jorge A. Silva-Castro et al (JPAC) arXiv:1809.01954 (Master thesis)
CFR et al. (JPAC) PRD93 (2016) 074015

Instituto de
Ciencias
Nucleares
UNAM



Hadron spectroscopy

- * We study the hadron spectrum to gain insight on QCD
- * Typically we perform a partial wave analysis on data (exp and lat) to obtain the poles
- * Next, the poles need an interpretation $\rightarrow$ Resonances $\rightarrow$ masses (M_p) and widths (Γ_p)
- * And then we need to understand the dynamics that generate them
 $\rightarrow$ compact state/molecule//...
- * Usually states are studied on a one by one basis
- * The structure of the spectrum can provide insight

S matrix theory

- * Conservation of probability $\Rightarrow$ Unitarity
- * Particles $\leftrightarrow$ Antiparticles $\Rightarrow$ Crossing symmetry
- * Causality $\Rightarrow$ Analyticity
- * Additional symmetries: e.g. gauge, chiral

Analytic structure

e.g. $pn \rightarrow pn$

$$a_\ell(s + i0) - a_\ell(s - i0) = 2i\rho(s + i0)a_\ell(s + i0)a_\ell(s - i0)$$

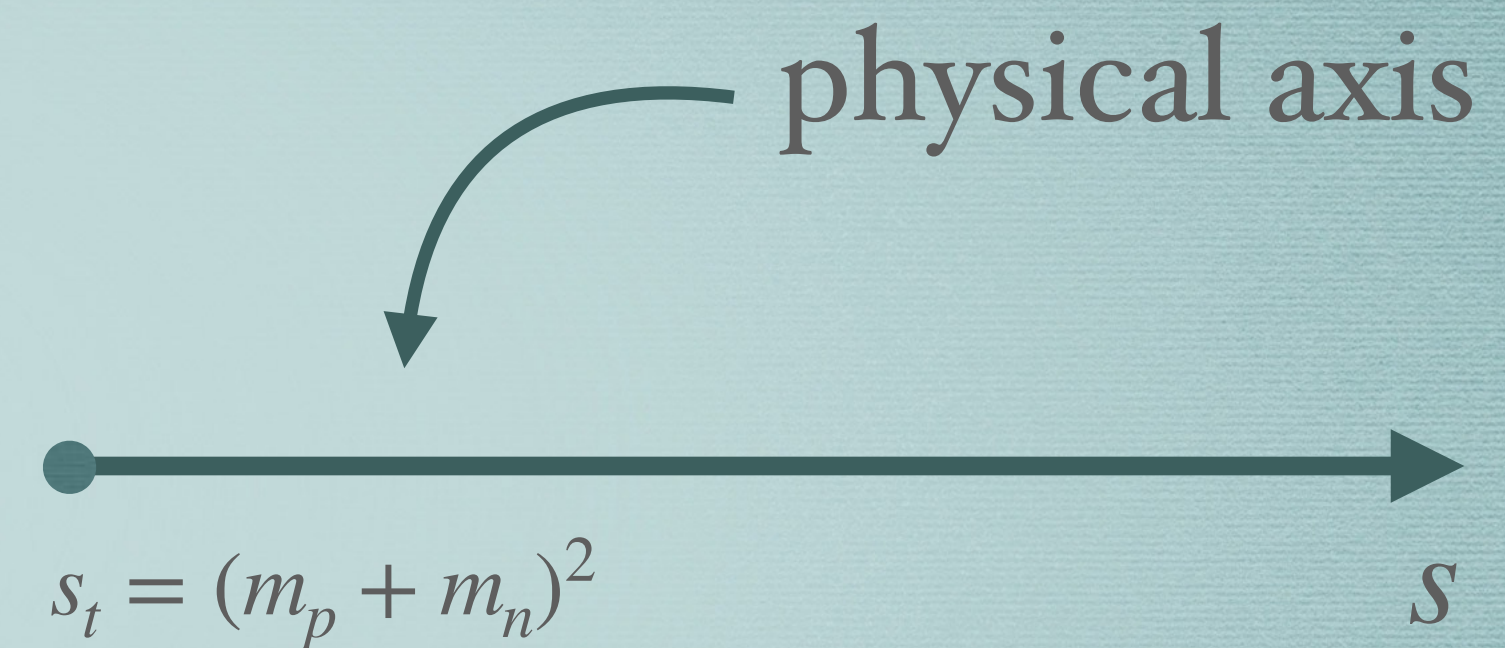
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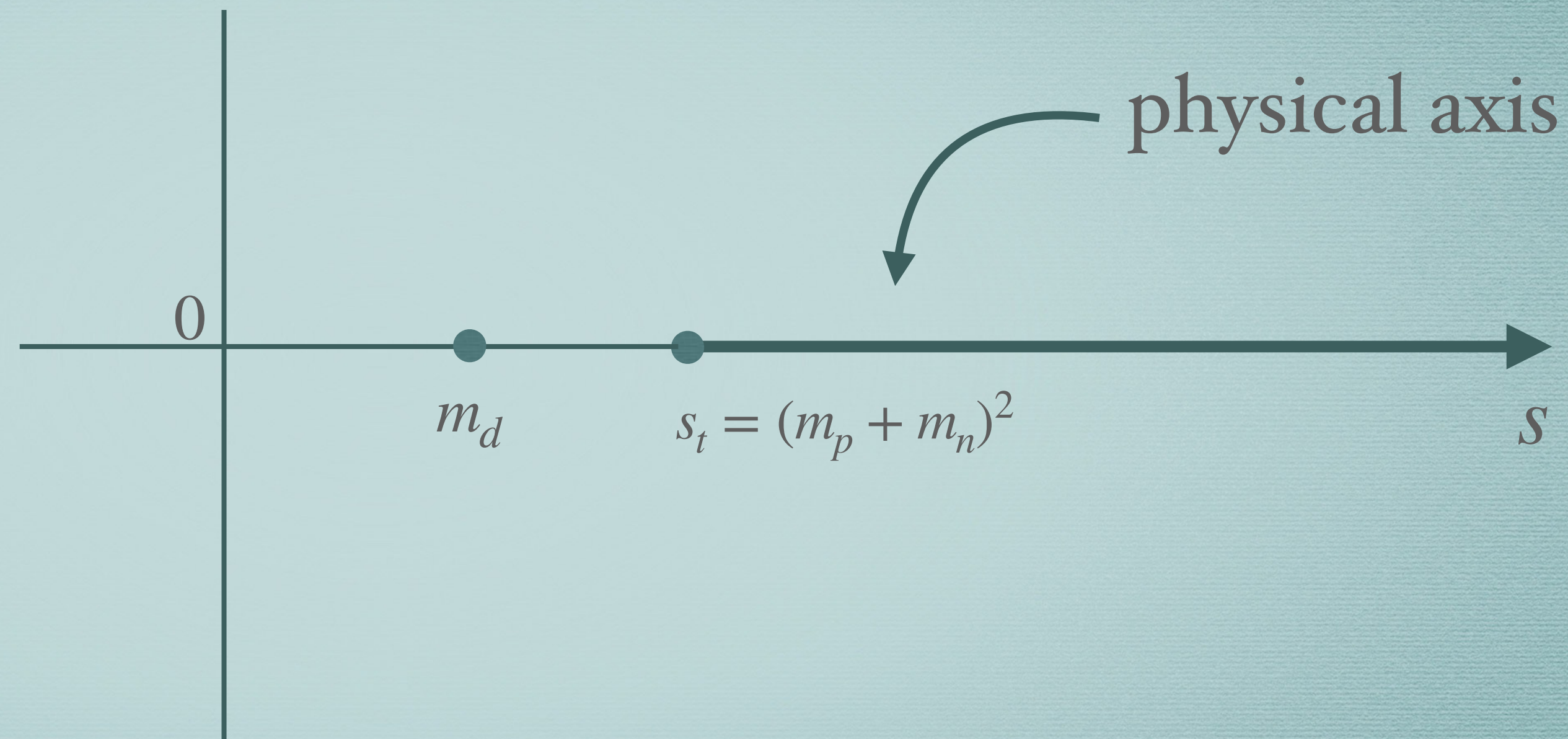


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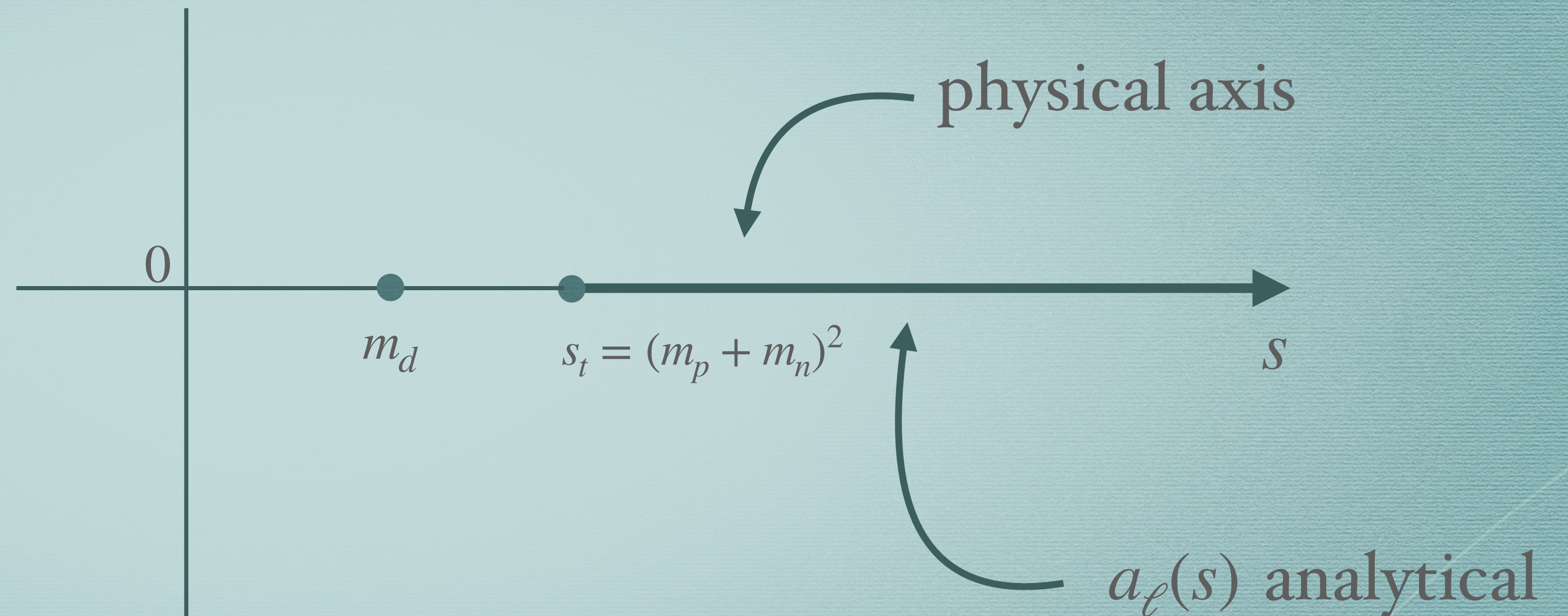


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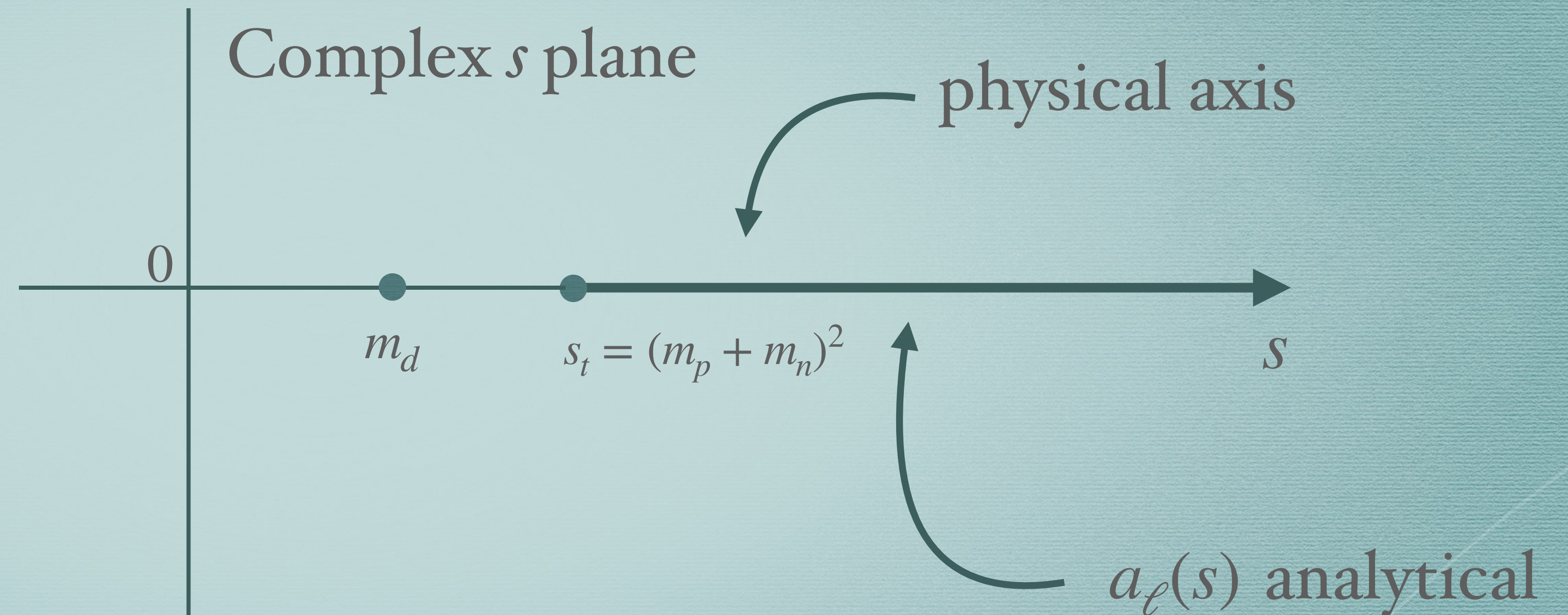


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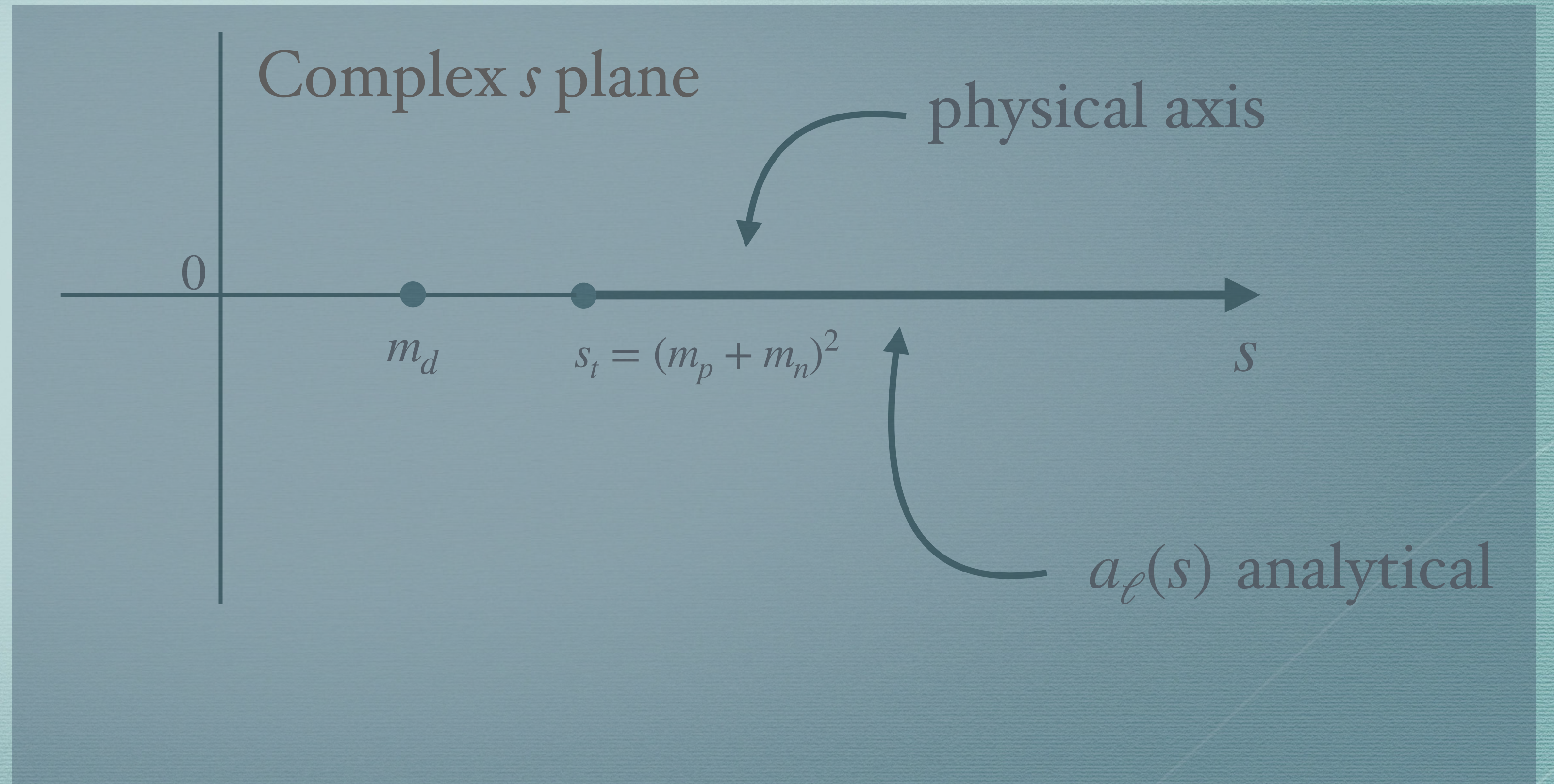


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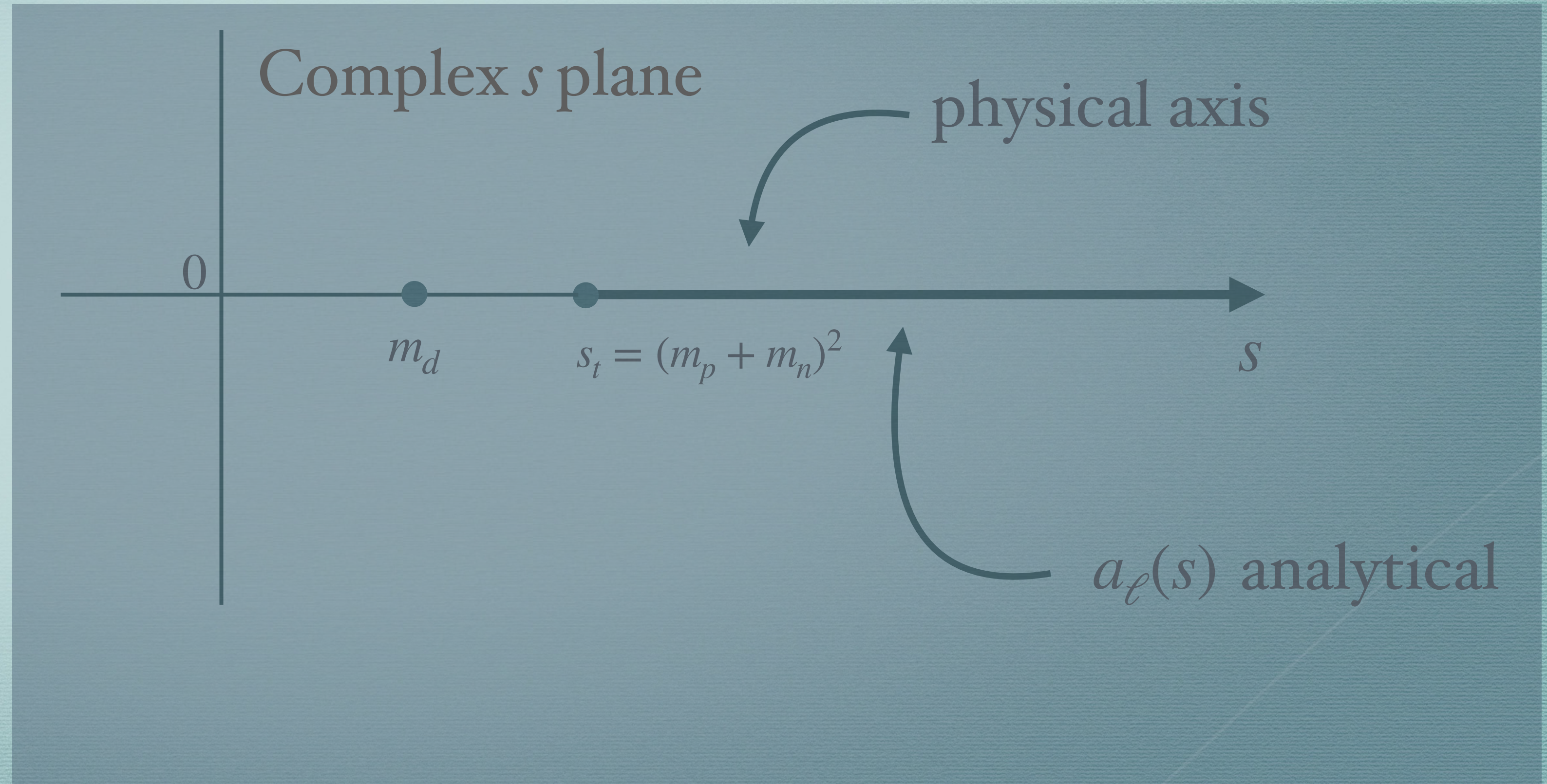
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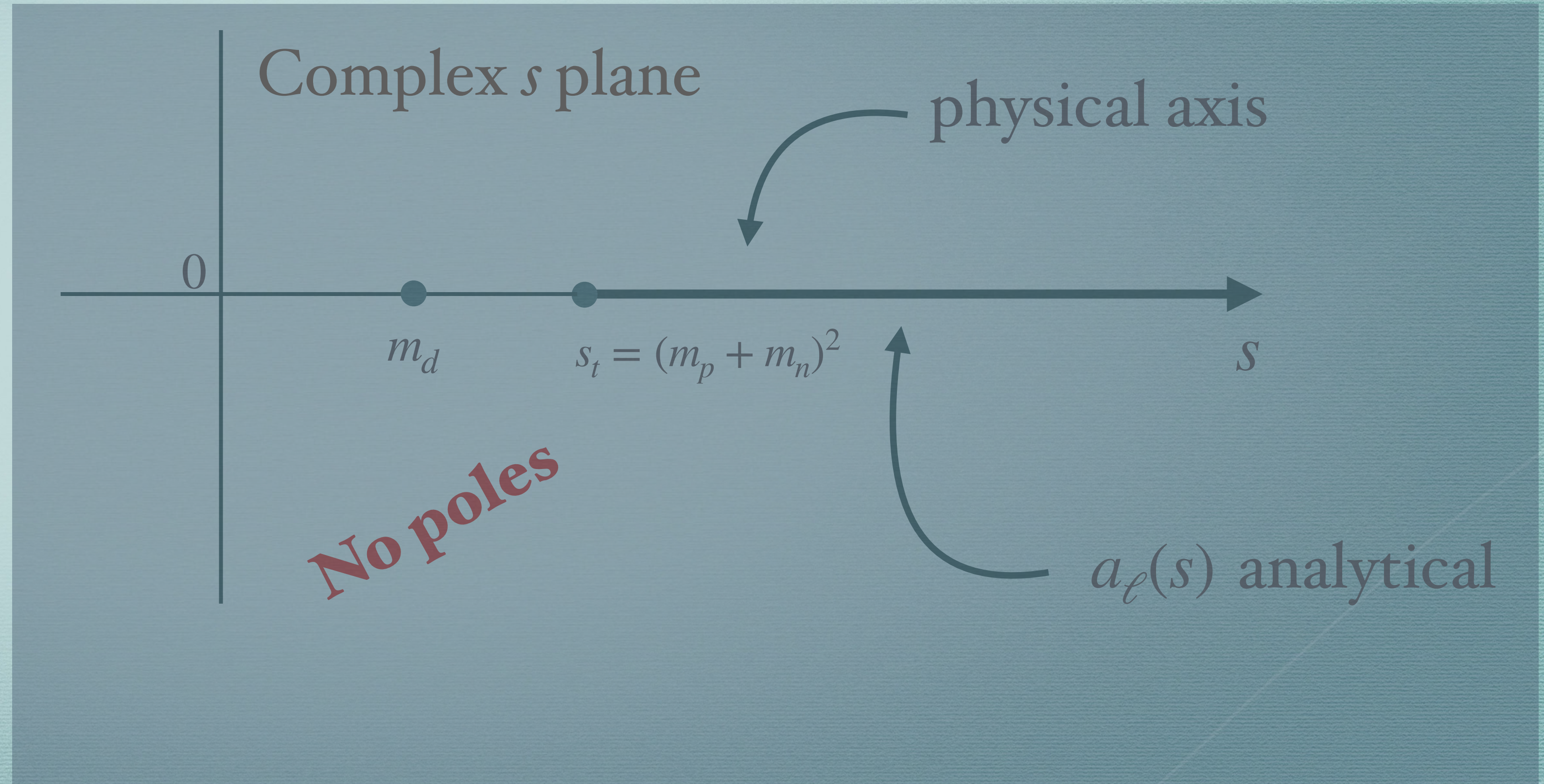
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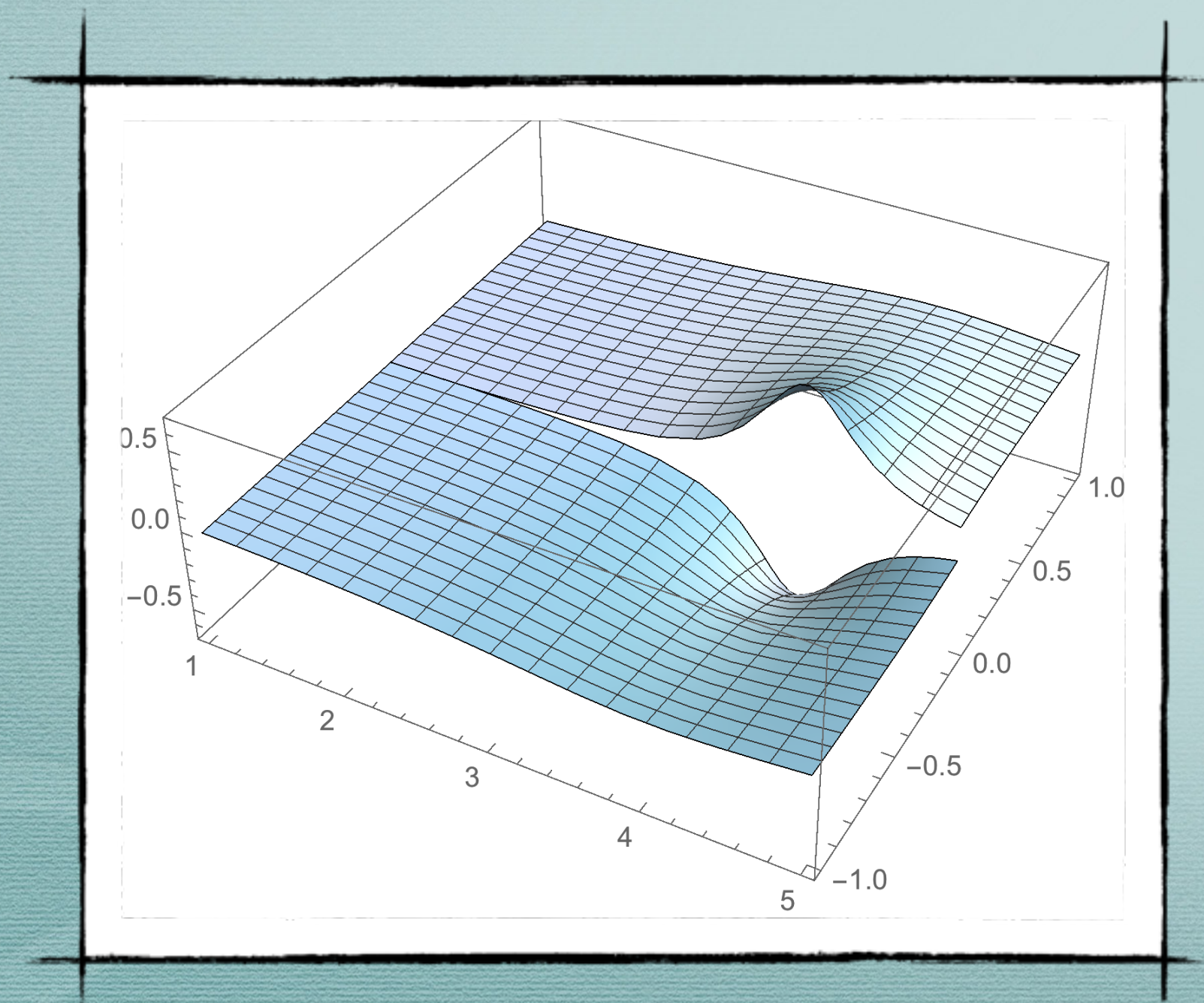
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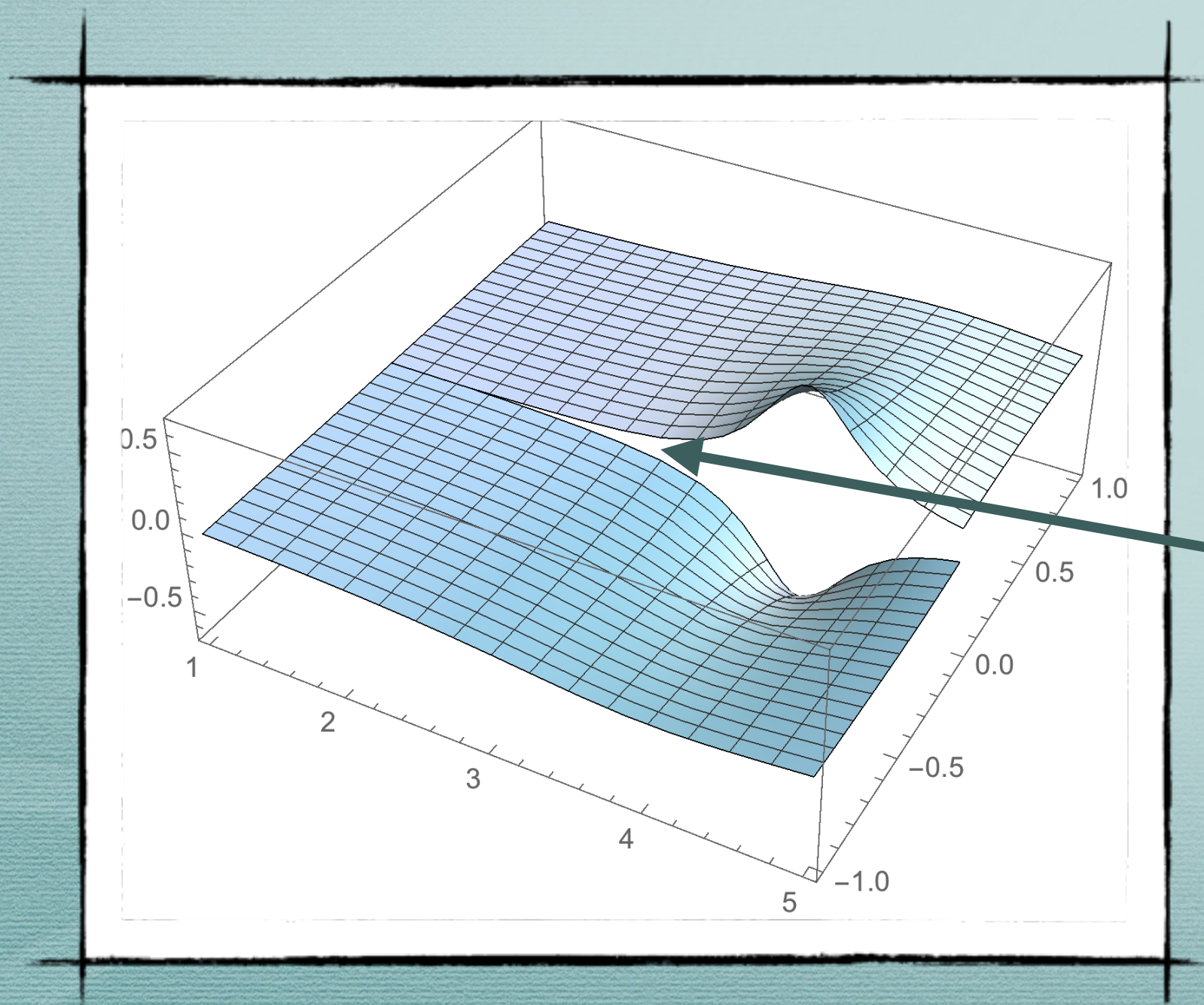
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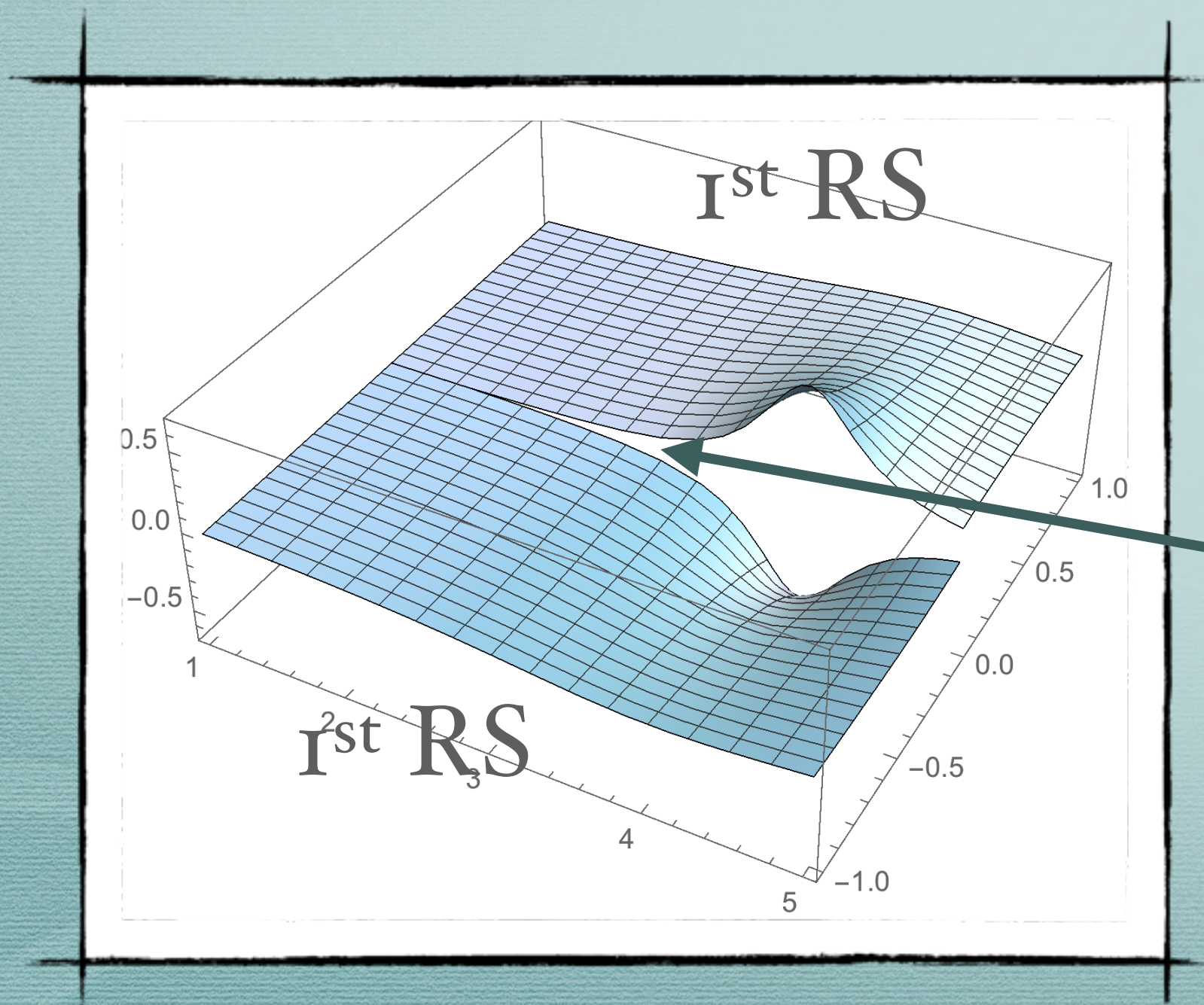
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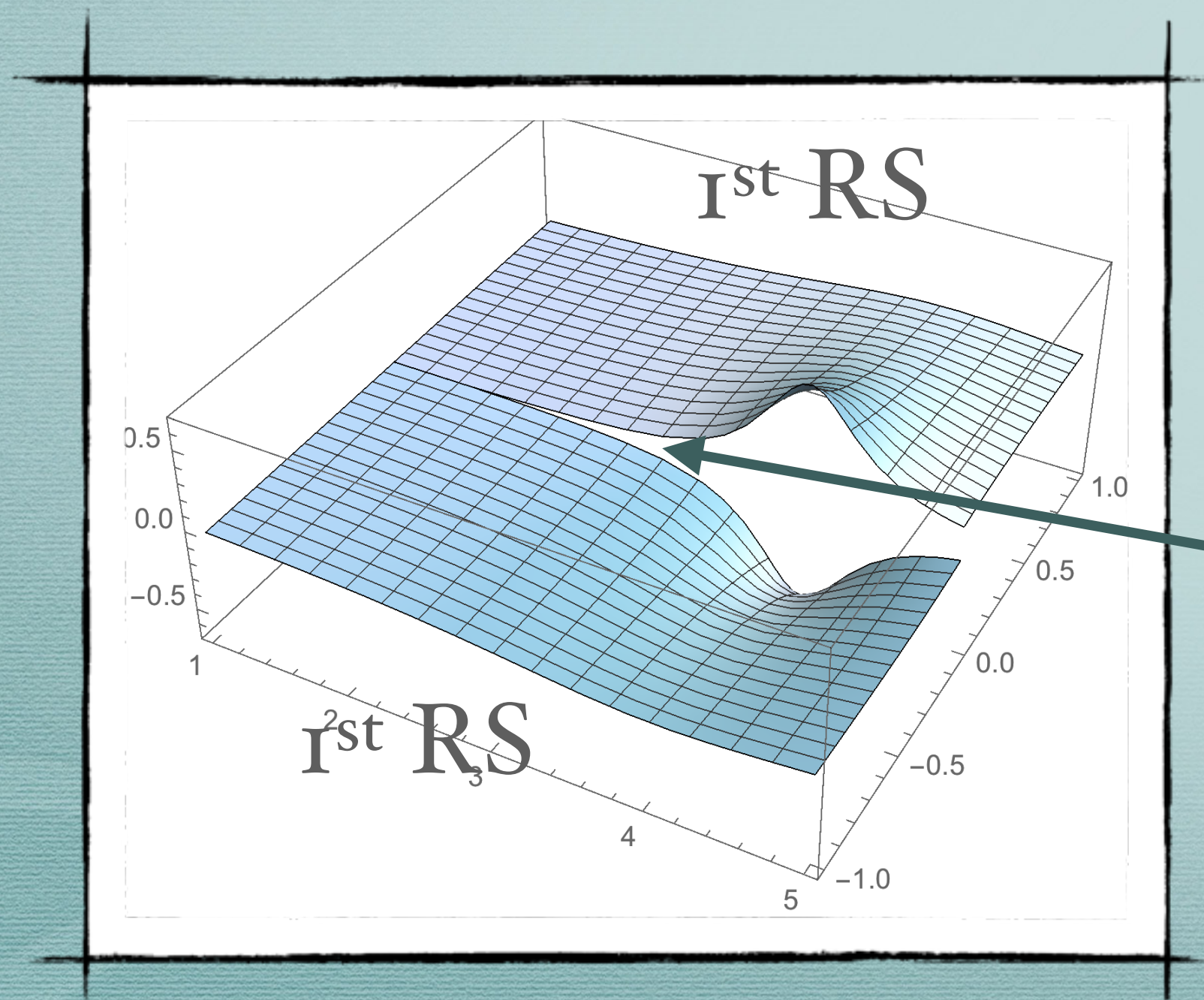
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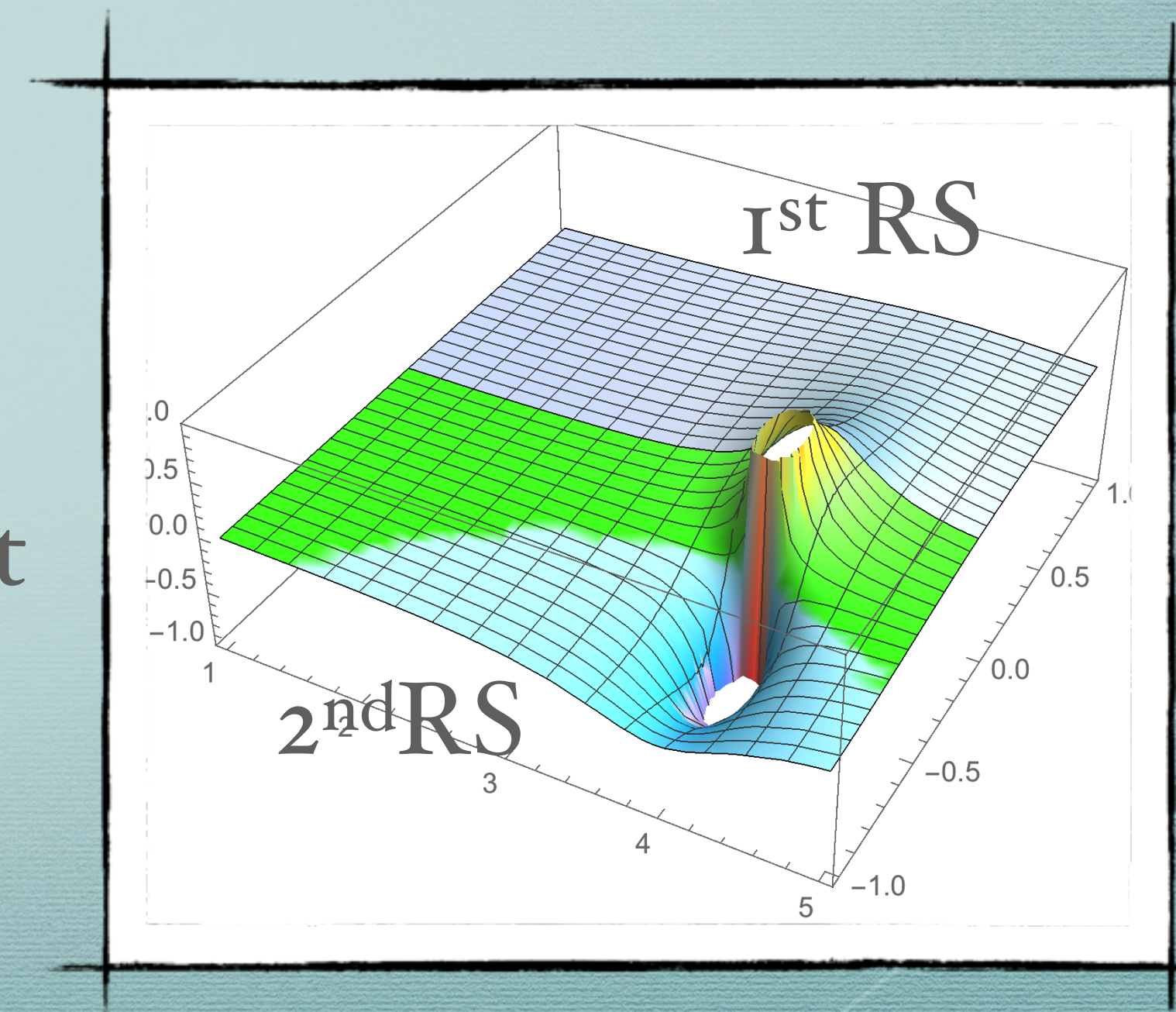
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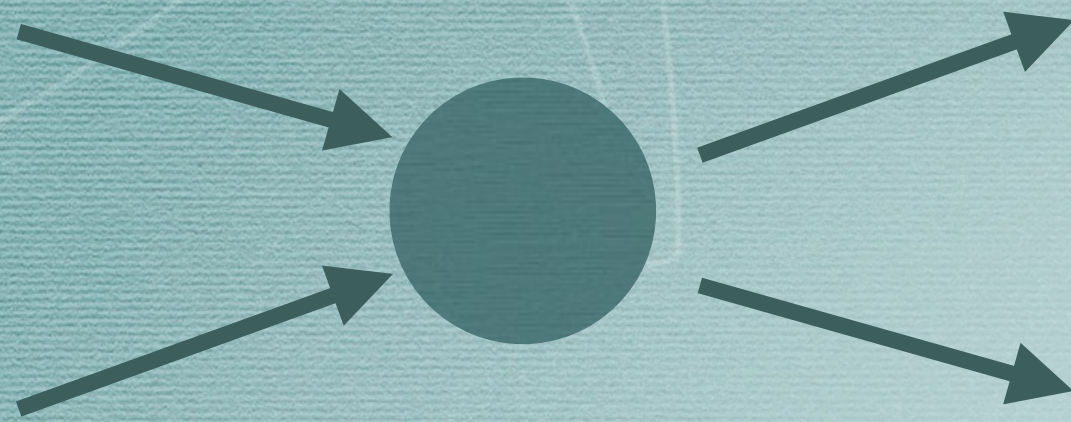


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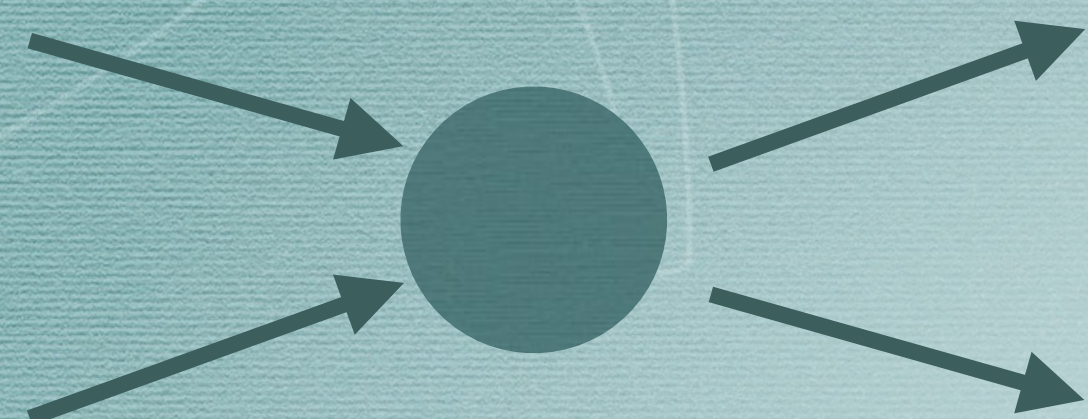


PWA



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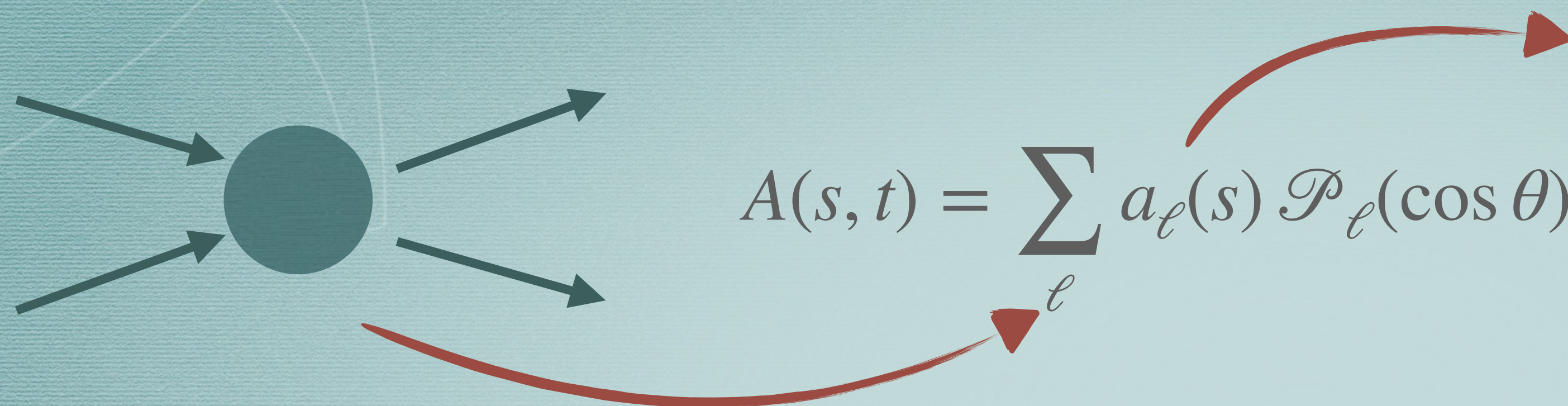
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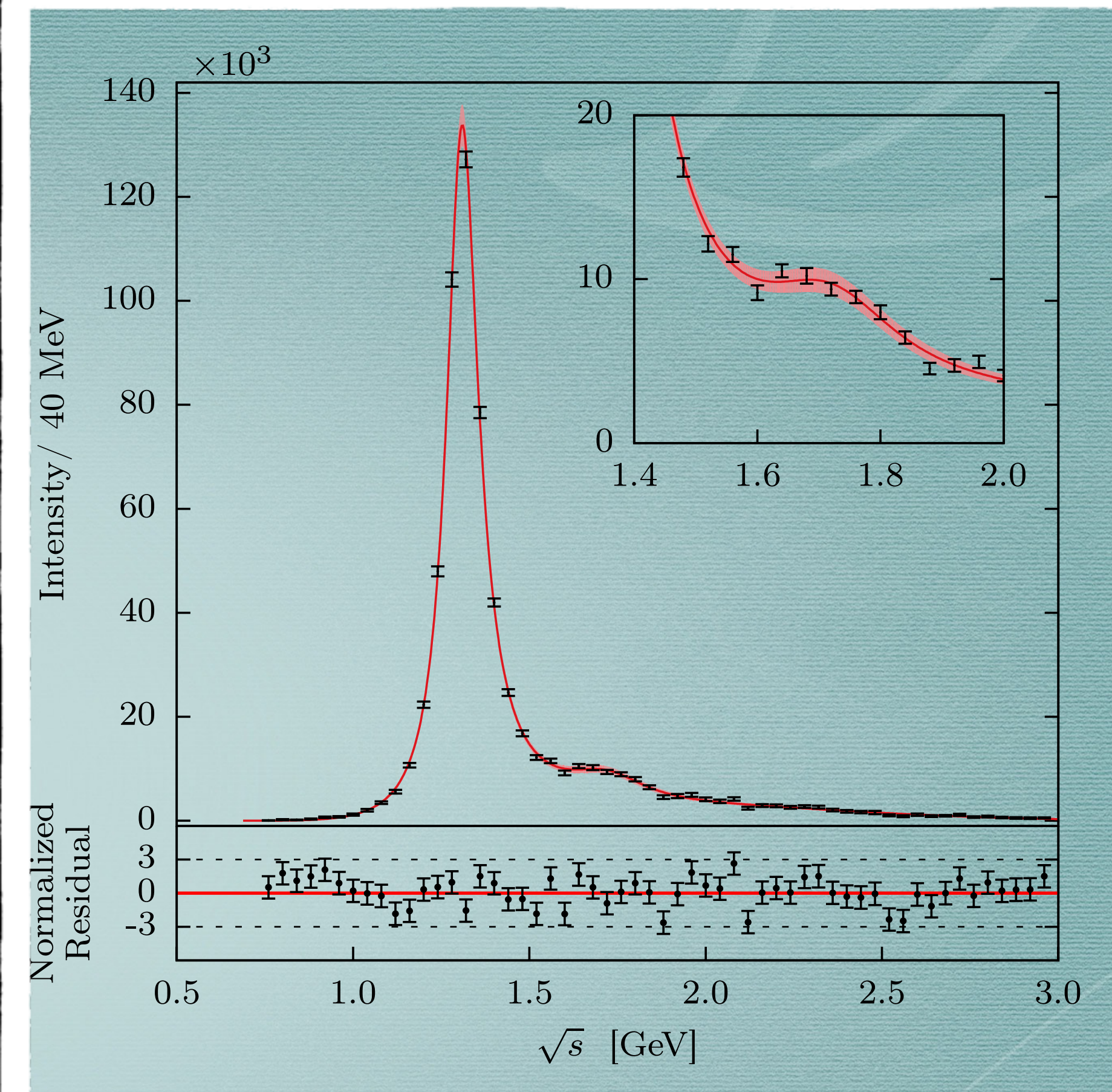
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Jackura et al. (JPAC & COMPASS), PLB 779 (2018) 464

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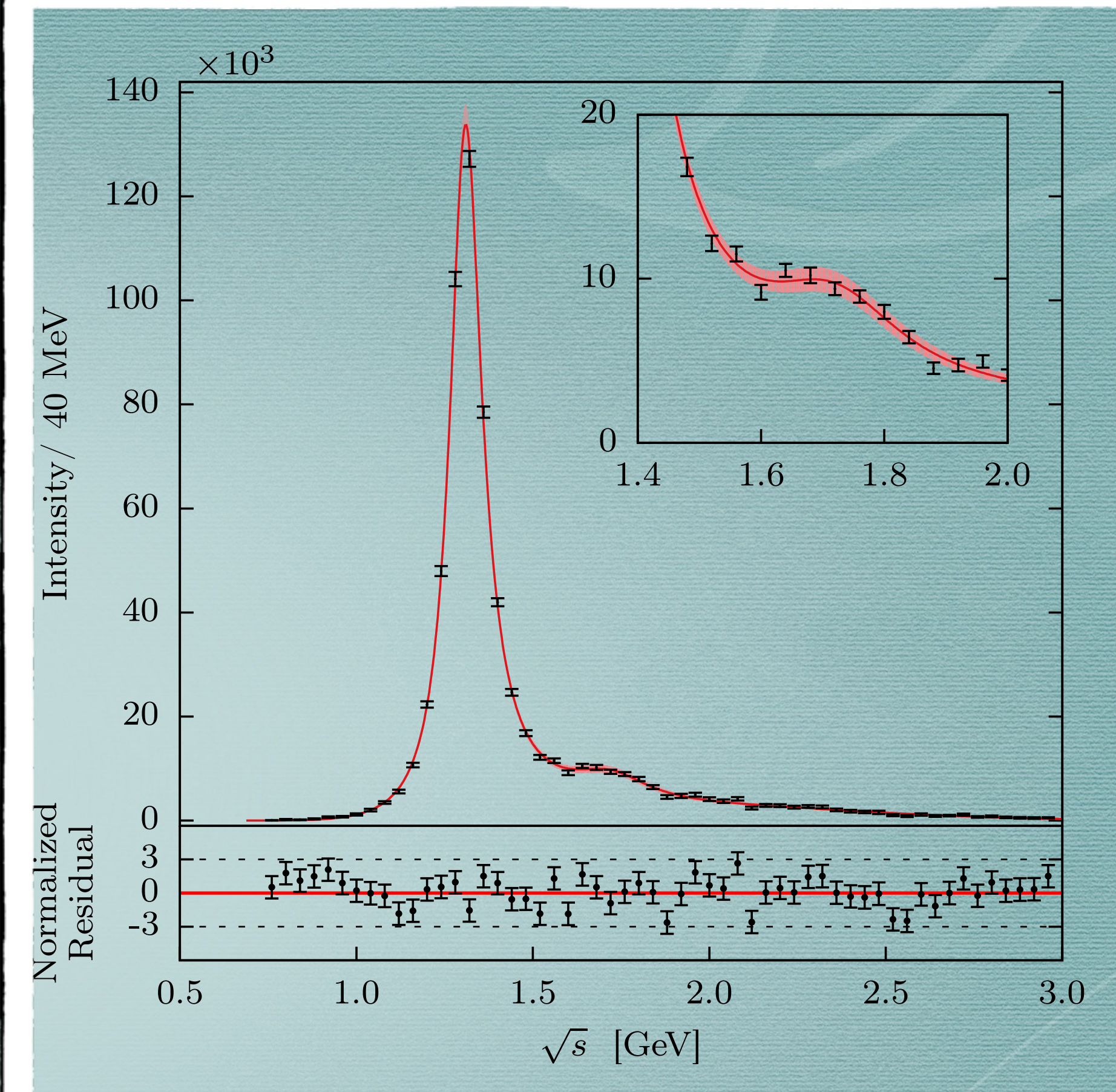
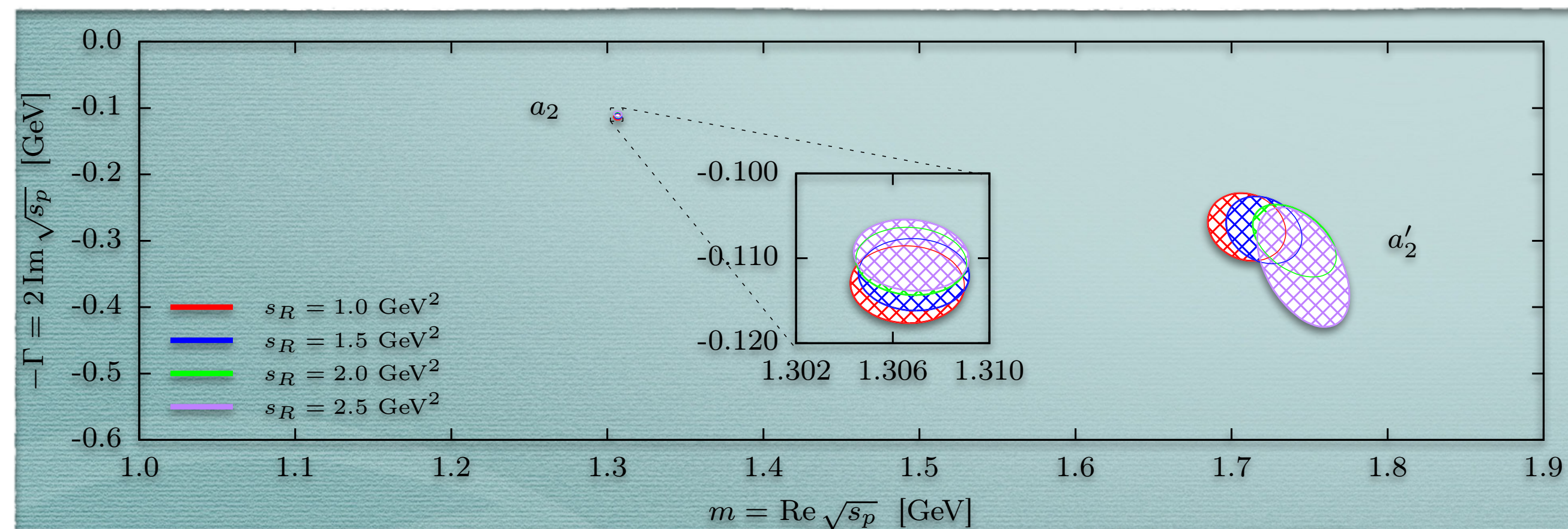
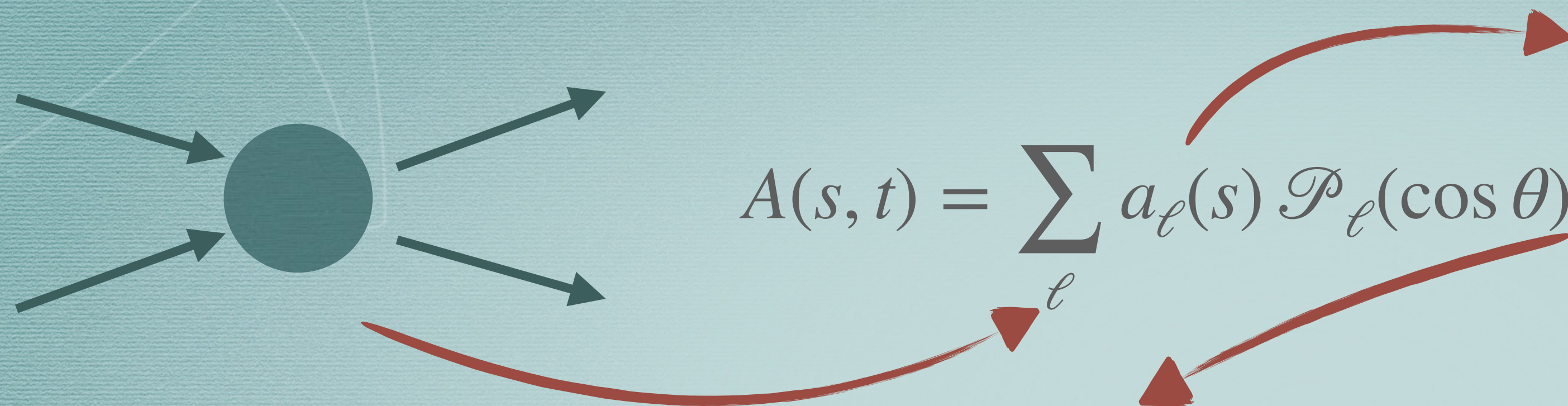


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Phase space is factored out, details of QCD dynamics are unknown

At a single pole

$$a_\ell(s_p) = \frac{A(s_p)}{B(s_p) - s - iA(s_p)\rho(s_p)} \quad (\text{this is exact})$$

$$M_p = \Re \left[\sqrt{B(s_p) + iA(s_p)\rho(s_p)} \right]$$

$$\Gamma_p = -2\Im \left[\sqrt{B(s_p) - iA(s_p)\rho(s_p)} \right]$$

- * If our model of the amplitude is a Breit-Wigner with constant parameters

$$A(s) = g^2 \qquad B(s) = M^2$$

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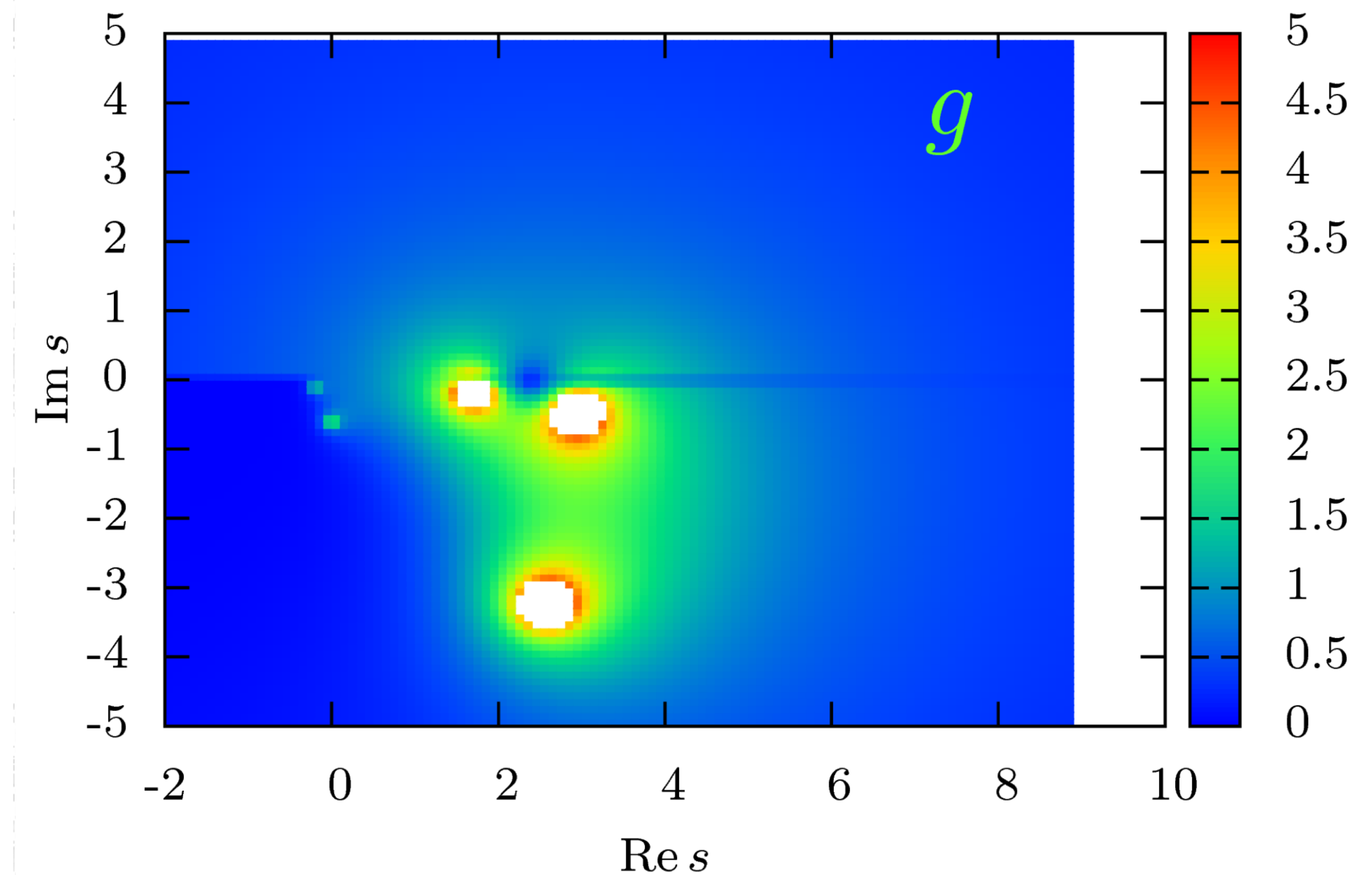
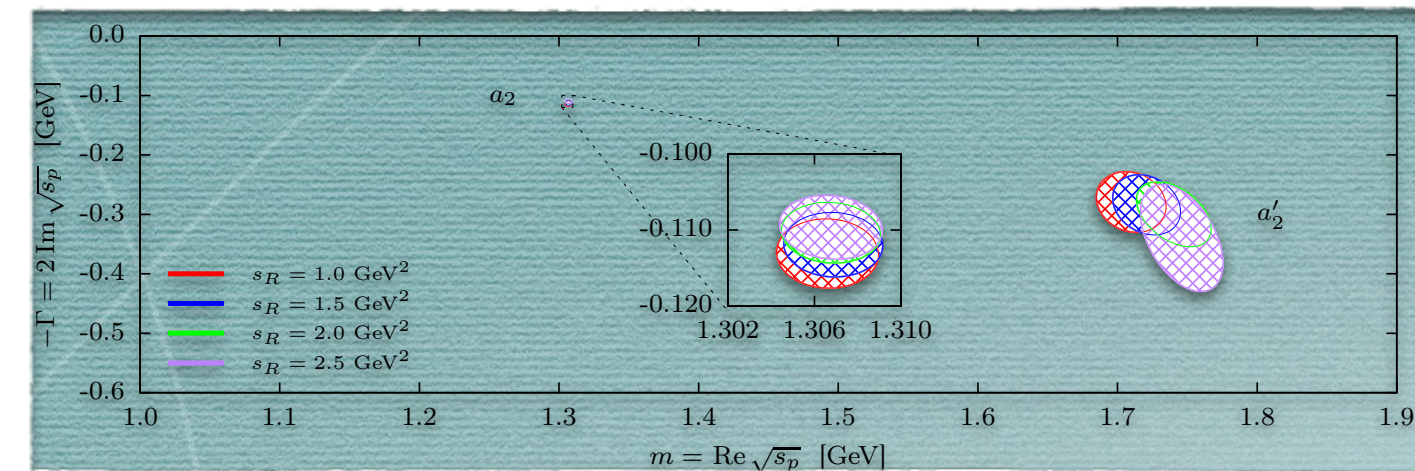
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If you have a more complicated model these ideas still hold

Spurious poles

Check out Alessssandro's
farewell seminar in the next
weeks for more on this



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Complex angular momentum theory

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through the complex angular momentum plane

by analytically continuing the Legendre polynomials

Signature

- * Actually amplitudes get classified in two different signatures according to the analytic continuation to the complex angular momentum plane
- * For example, pion-nucleon amplitudes get classified in two naturalities

$$\eta = +1, P = (-1)^{J-1/2}$$

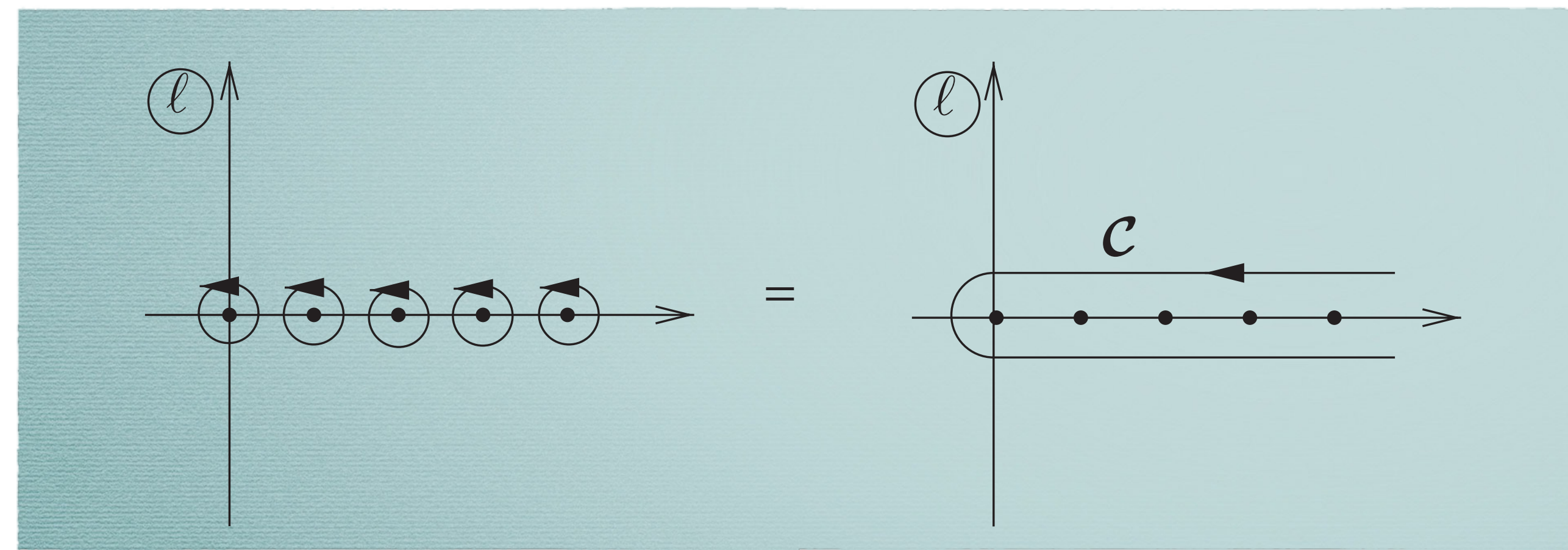
$$\eta = -1, P = -(-1)^{J-1/2}$$

- * And then two signatures

$$\eta = \tau P$$

Poles in the complex ang mom plane

- * In the complex angular momentum plane you have the Legendre polynomial poles. For fixed s



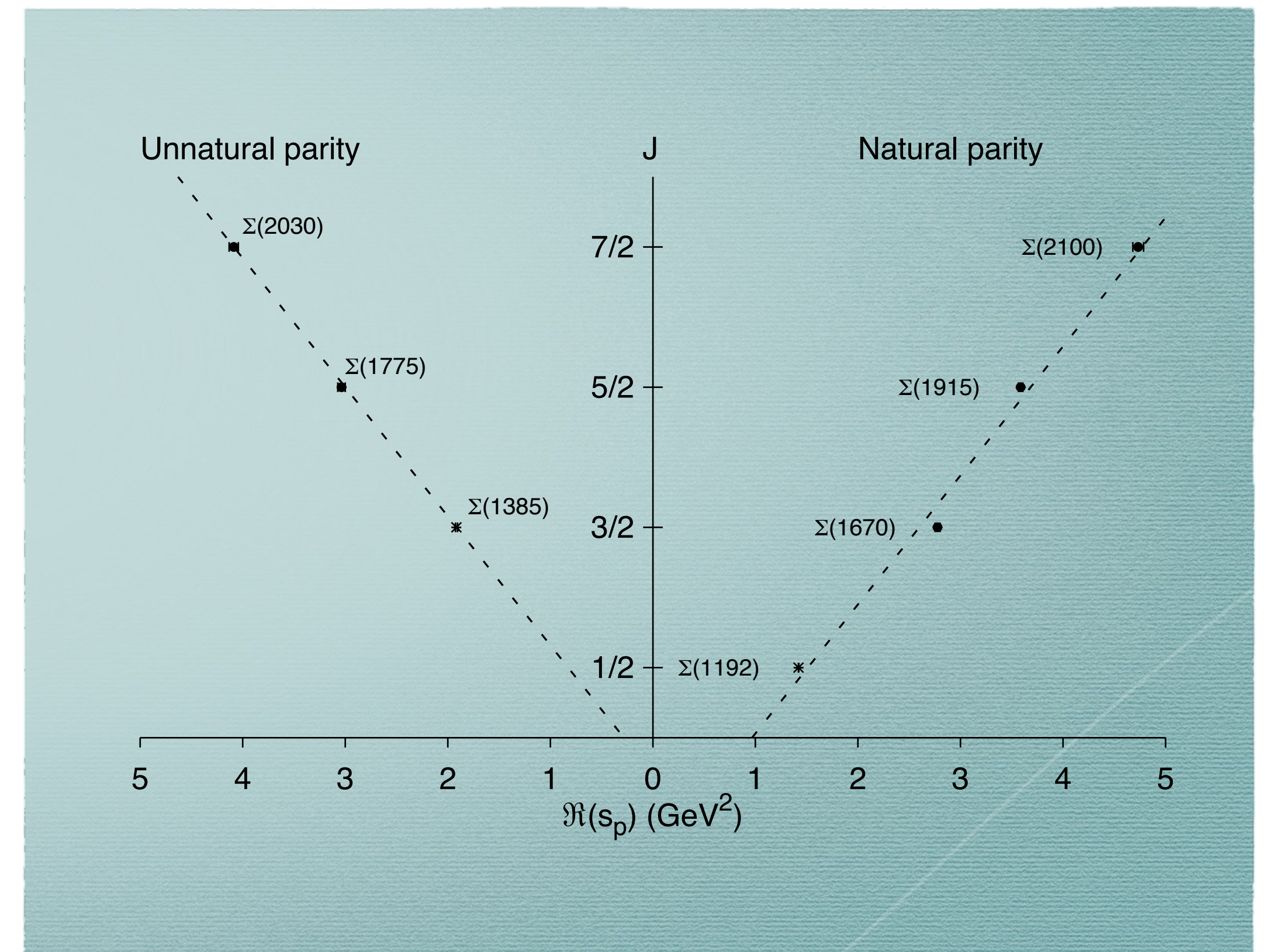
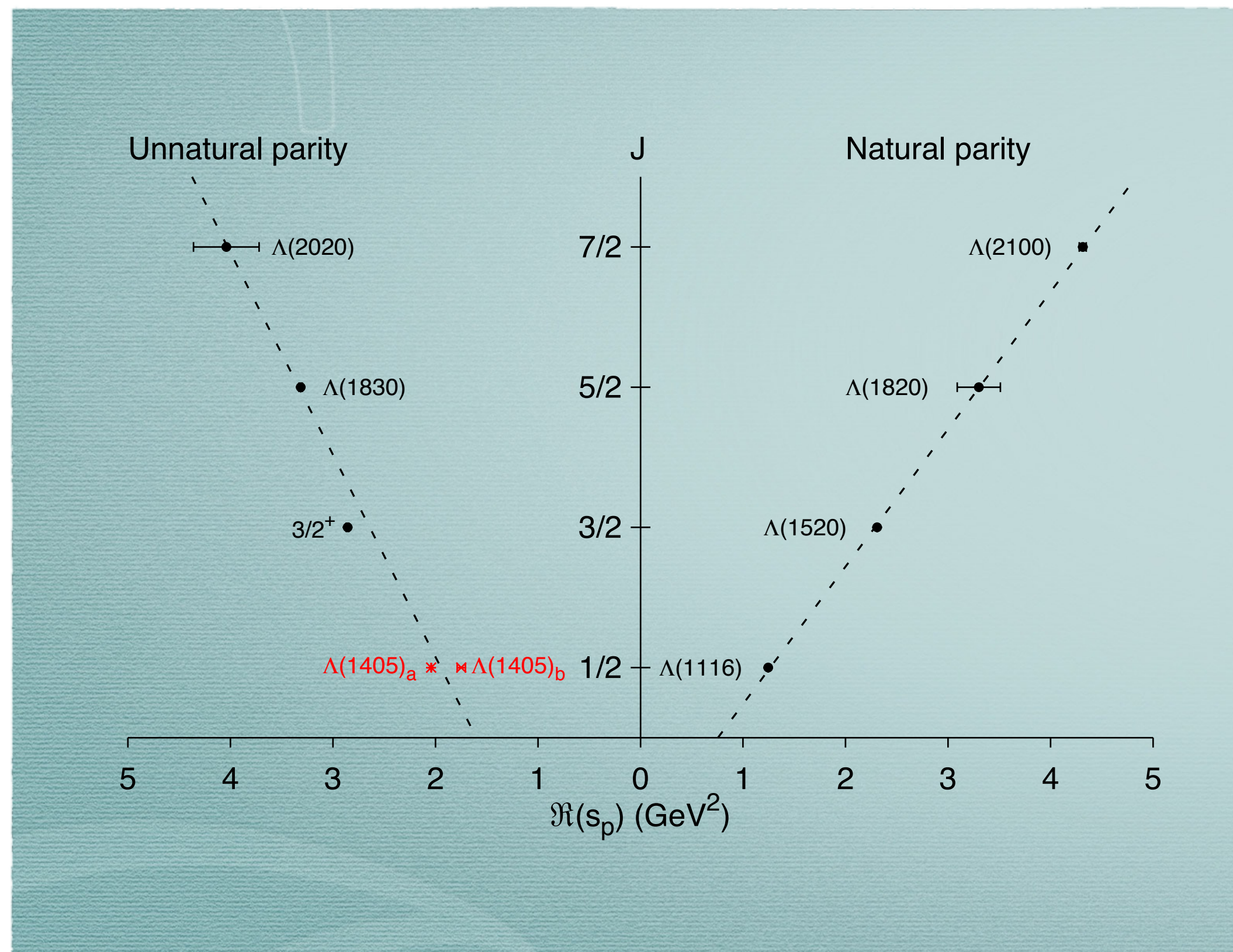
When you try to close the contour you hit complex angular momentum poles

$$a_{\ell}(s) = \frac{\beta(s)}{\ell - \alpha(s)}$$

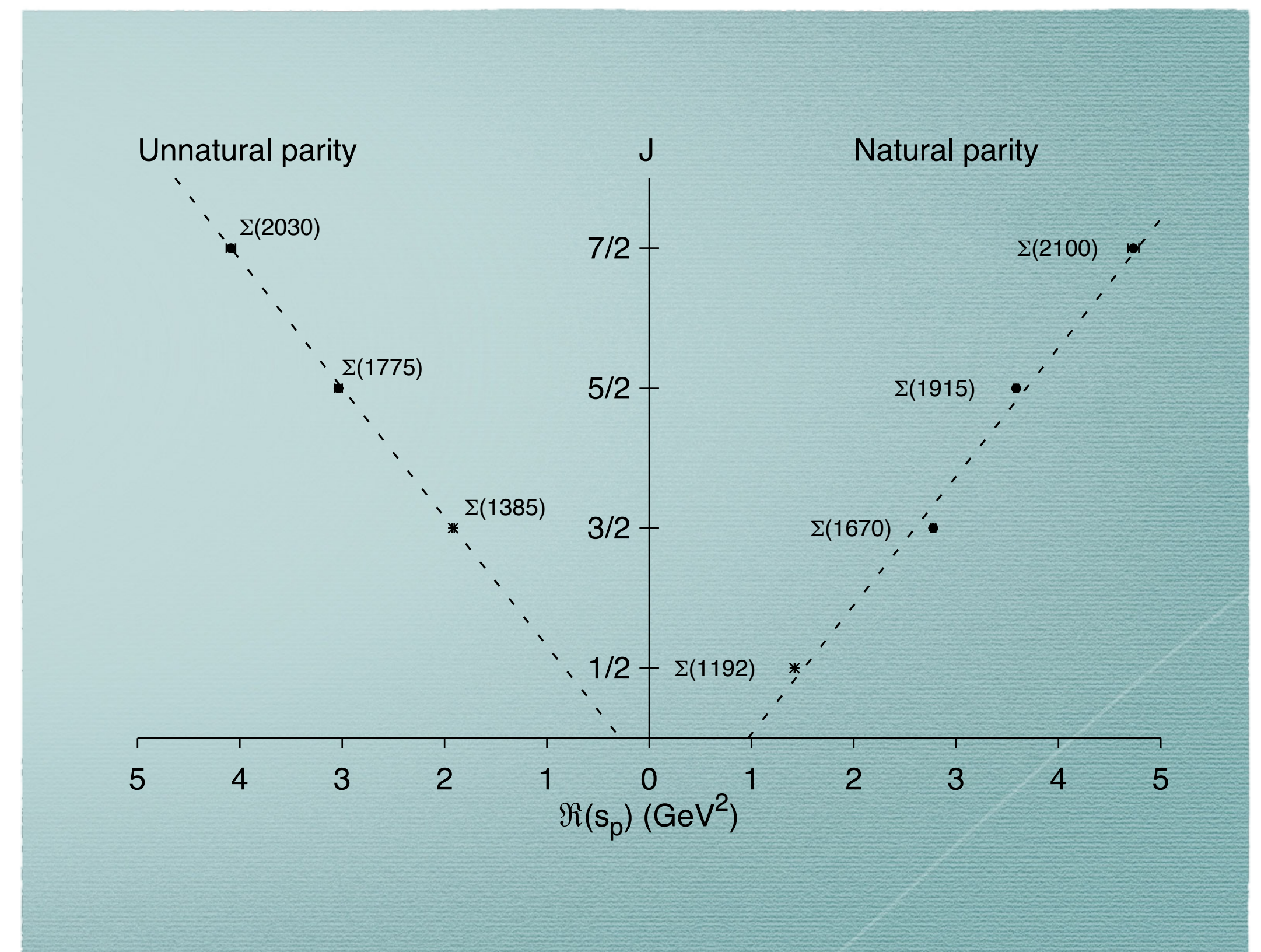
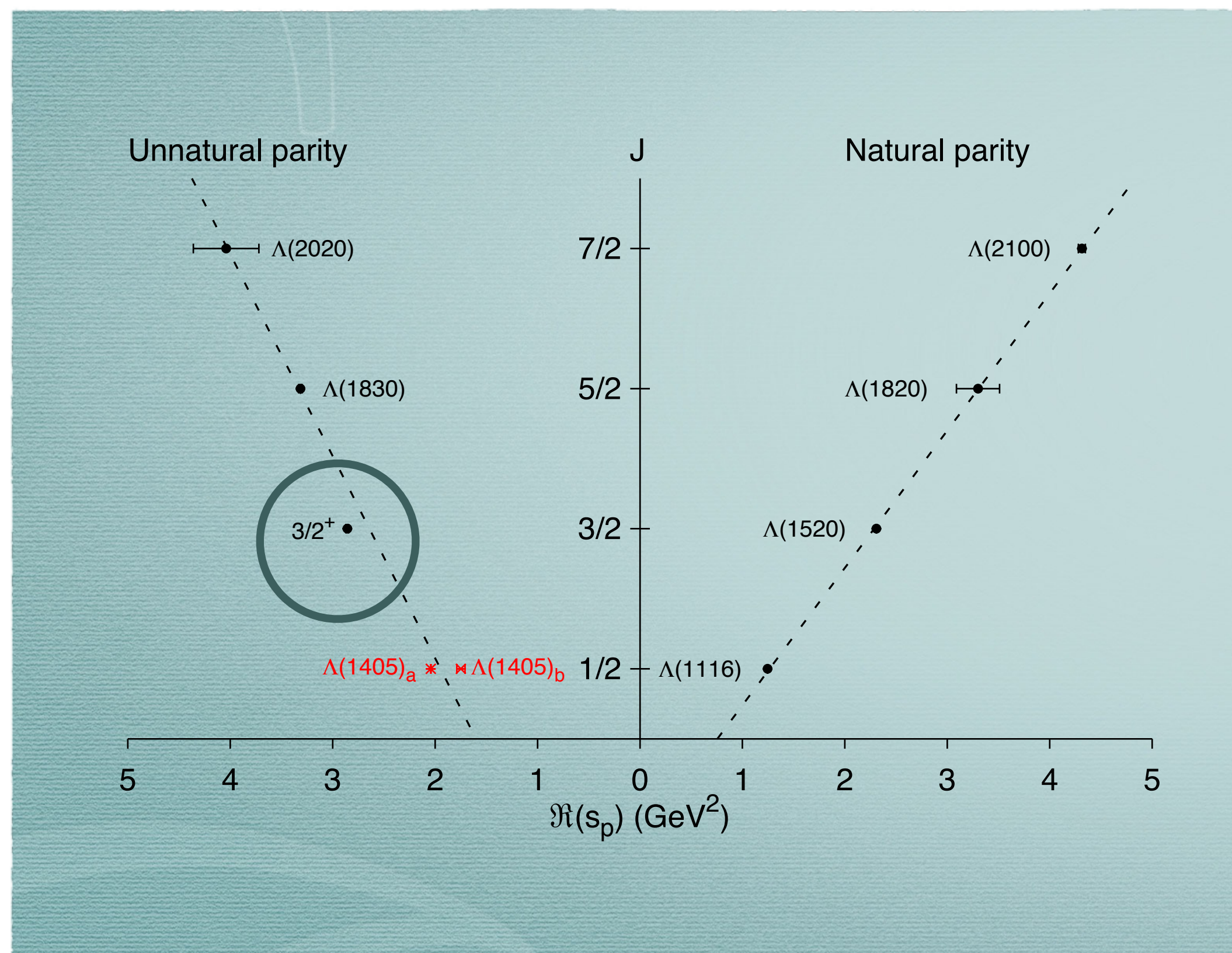
As you change s , the pole moves according to the Regge trajectory $\alpha(s)$

If $\ell = (\text{half-})\text{integer}$ you hit the resonances for the s_p , which is the pole in the complex- s plane

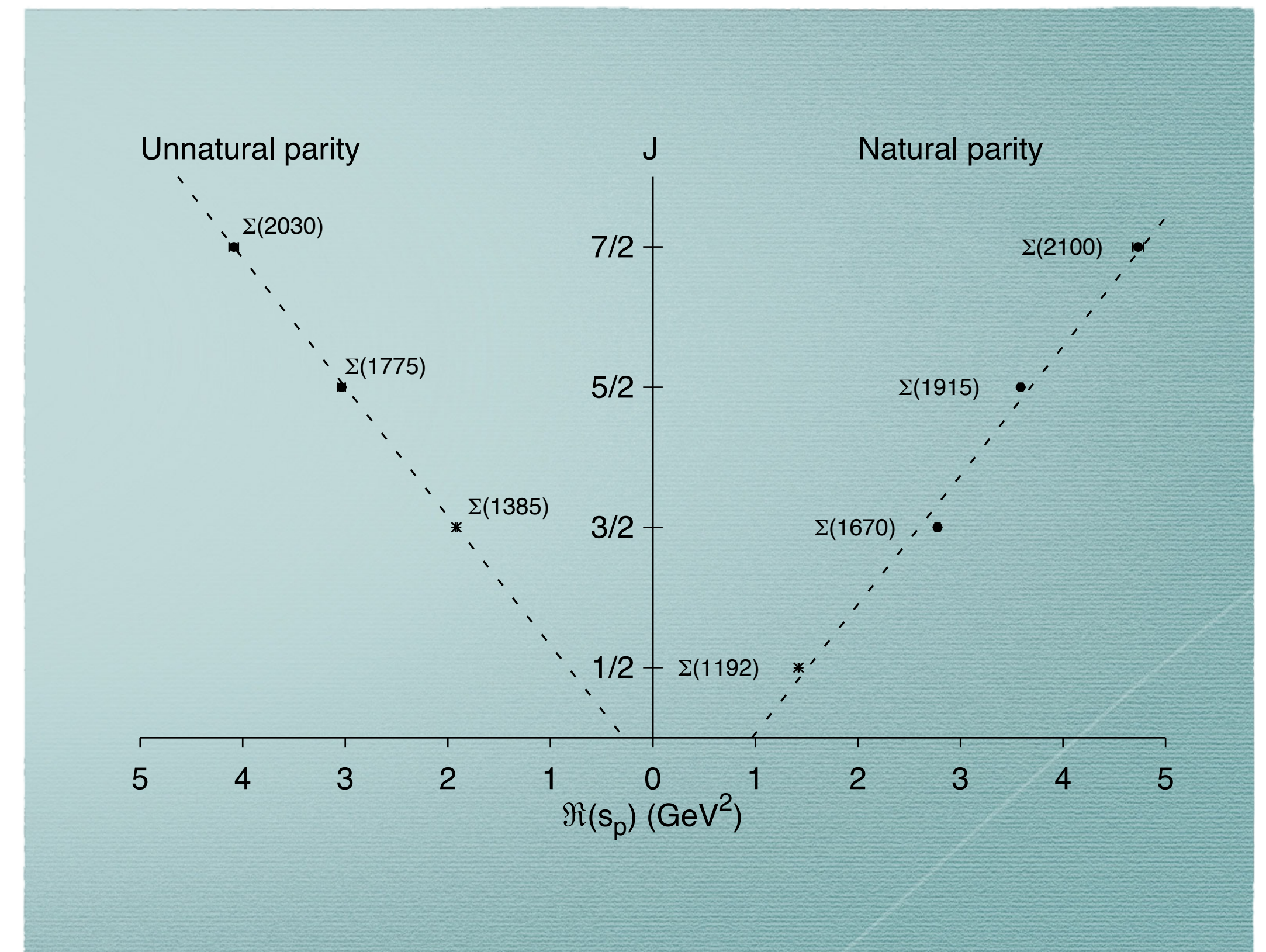
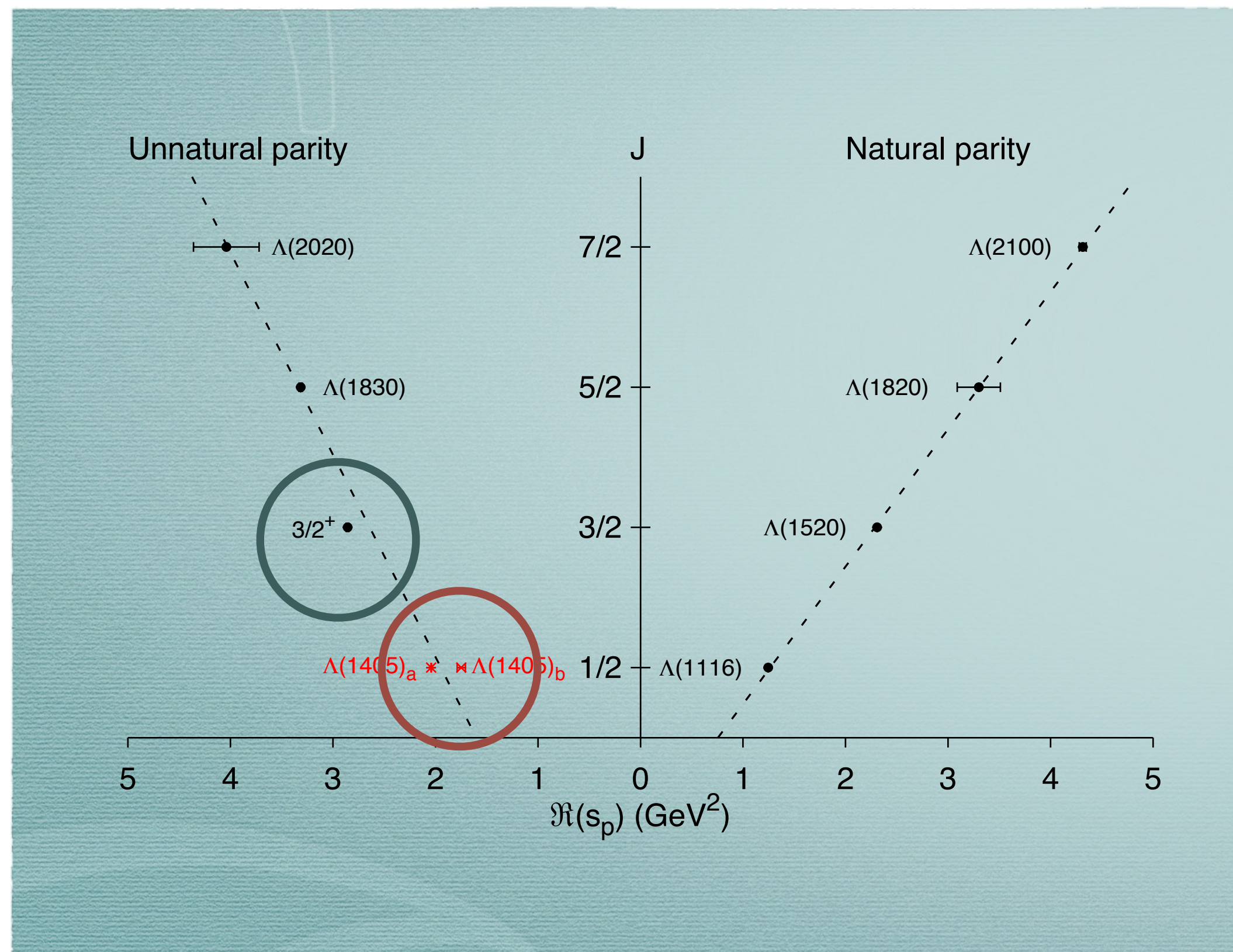
Λ and Σ spectra (Chew-Frautschi plots)



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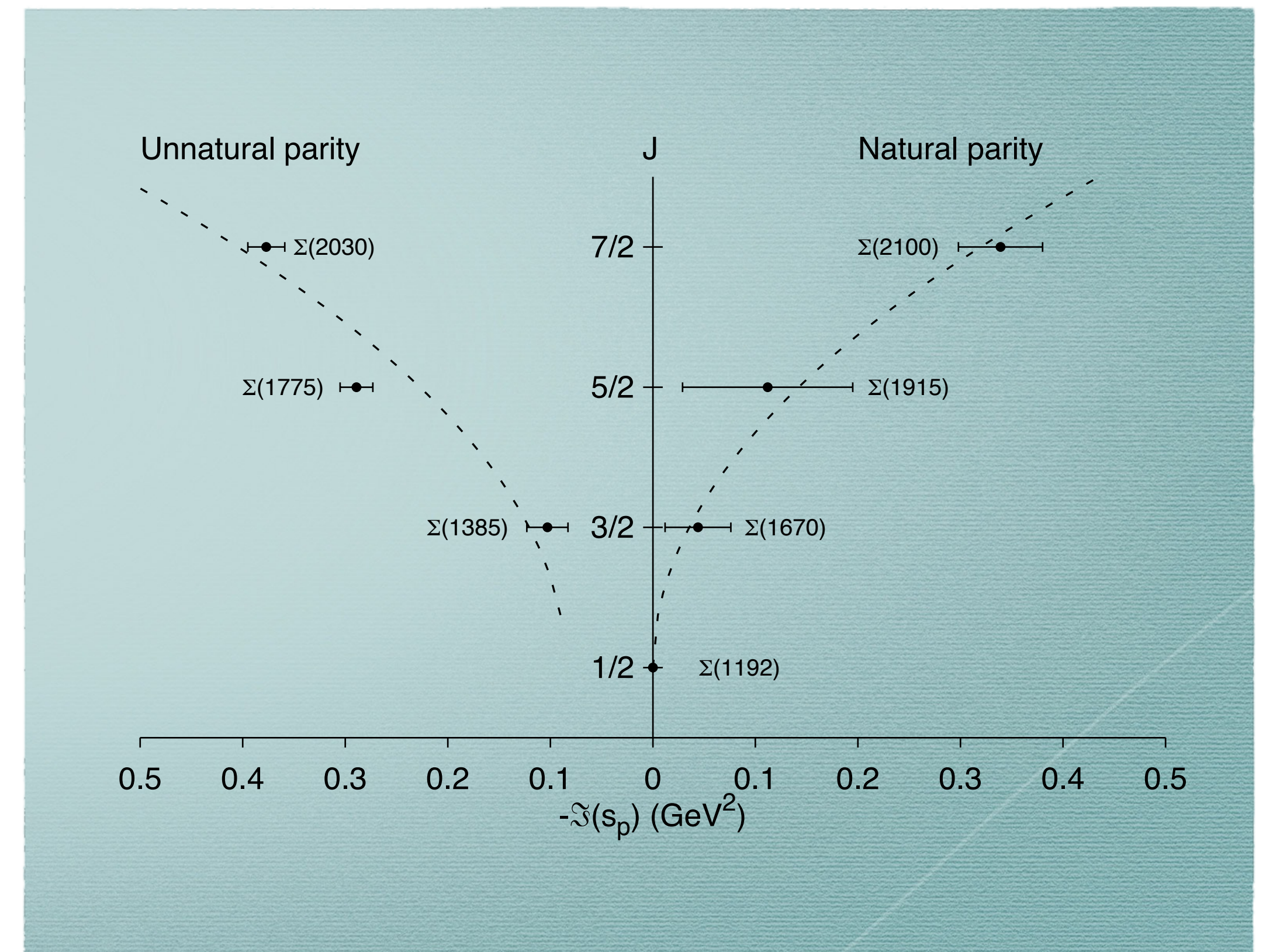
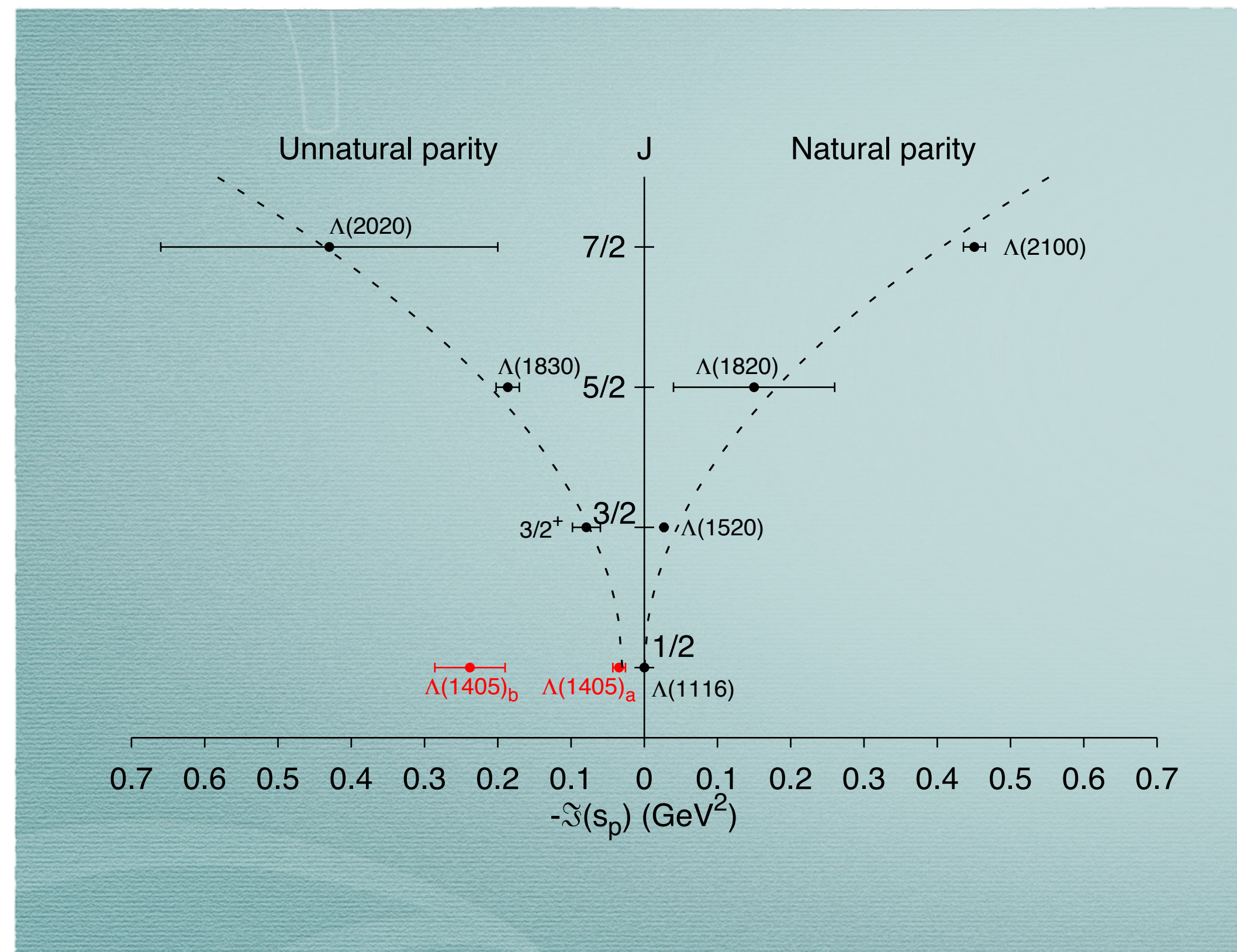


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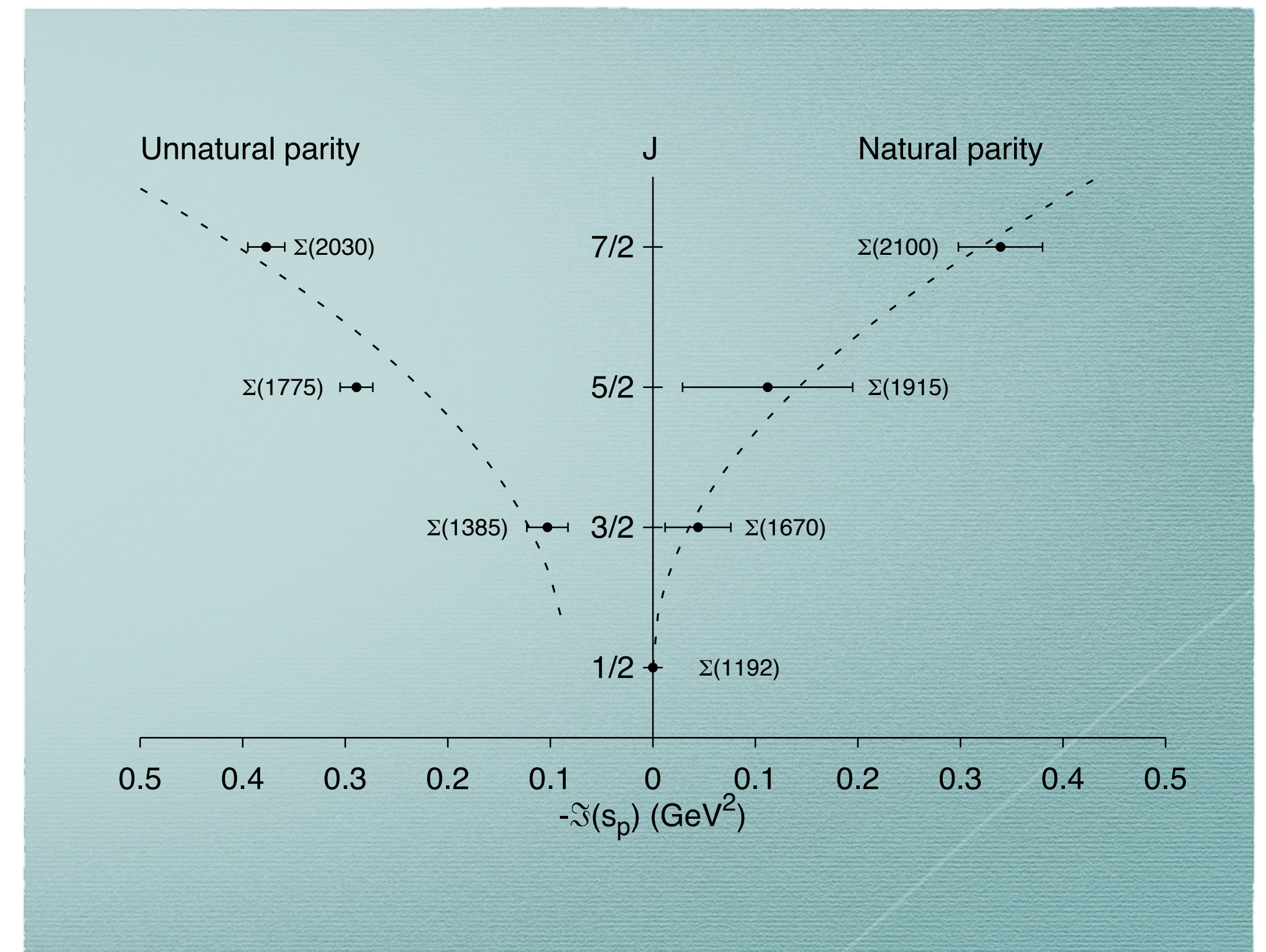
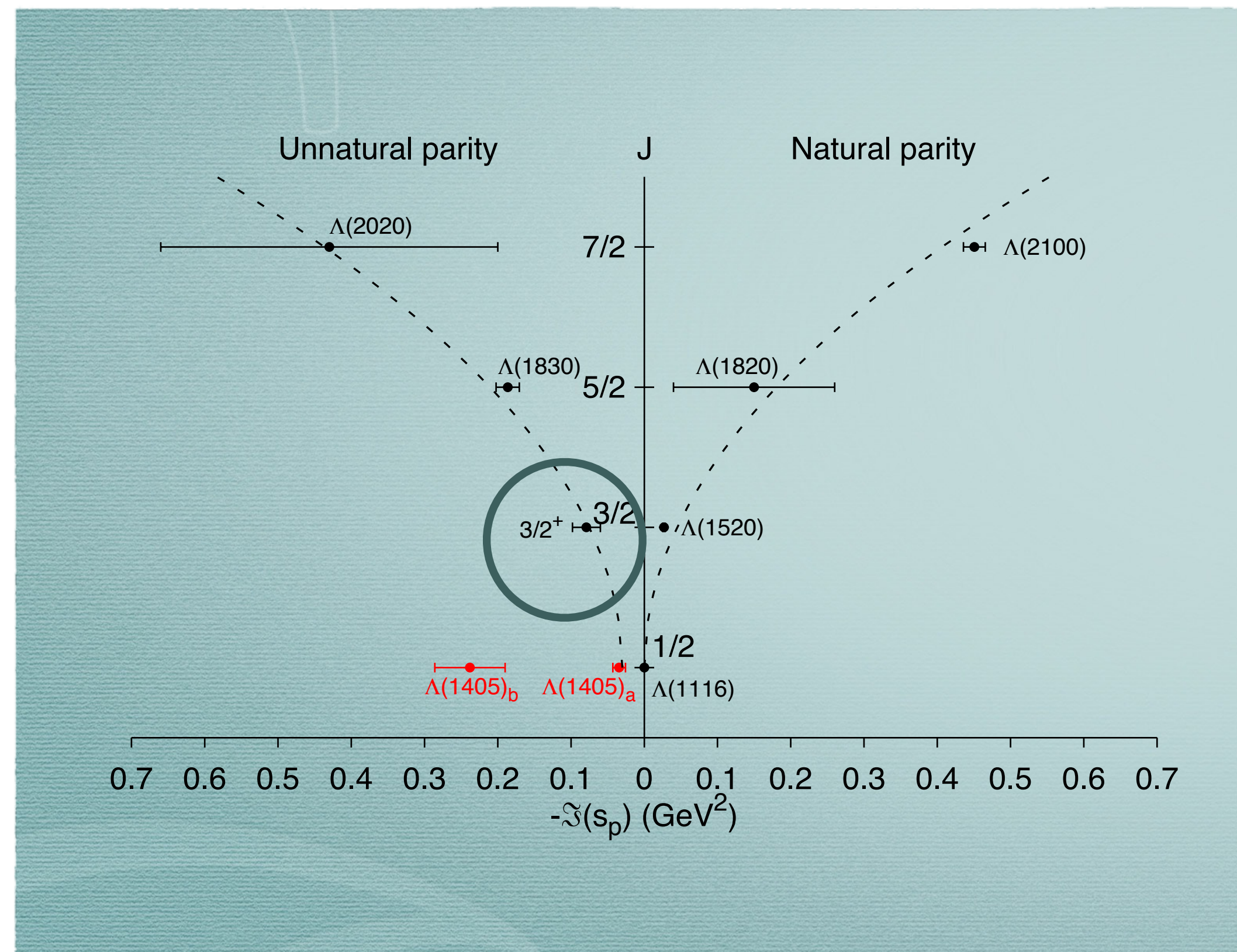
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($\text{Im } s_p$ vs J plots)



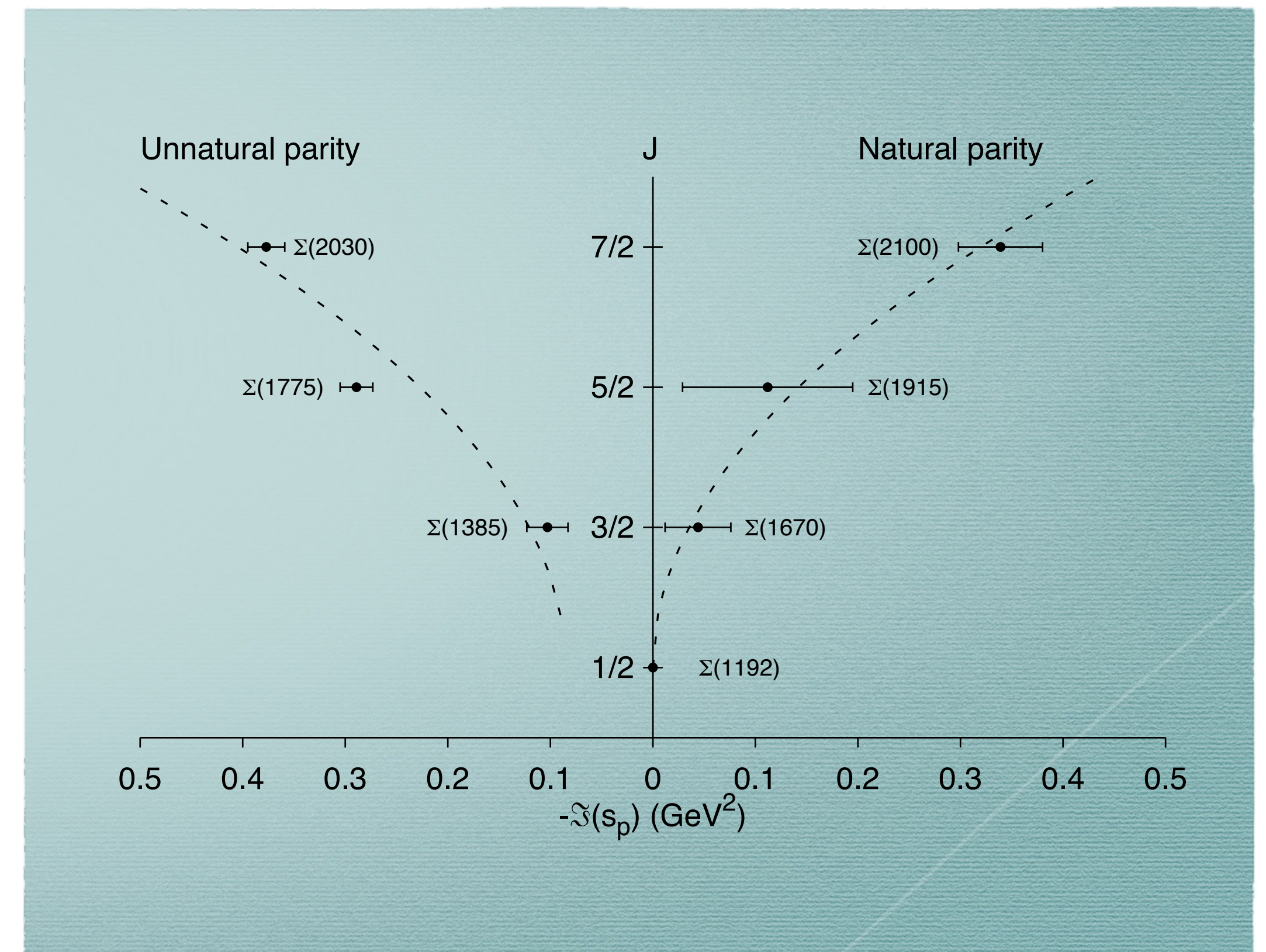
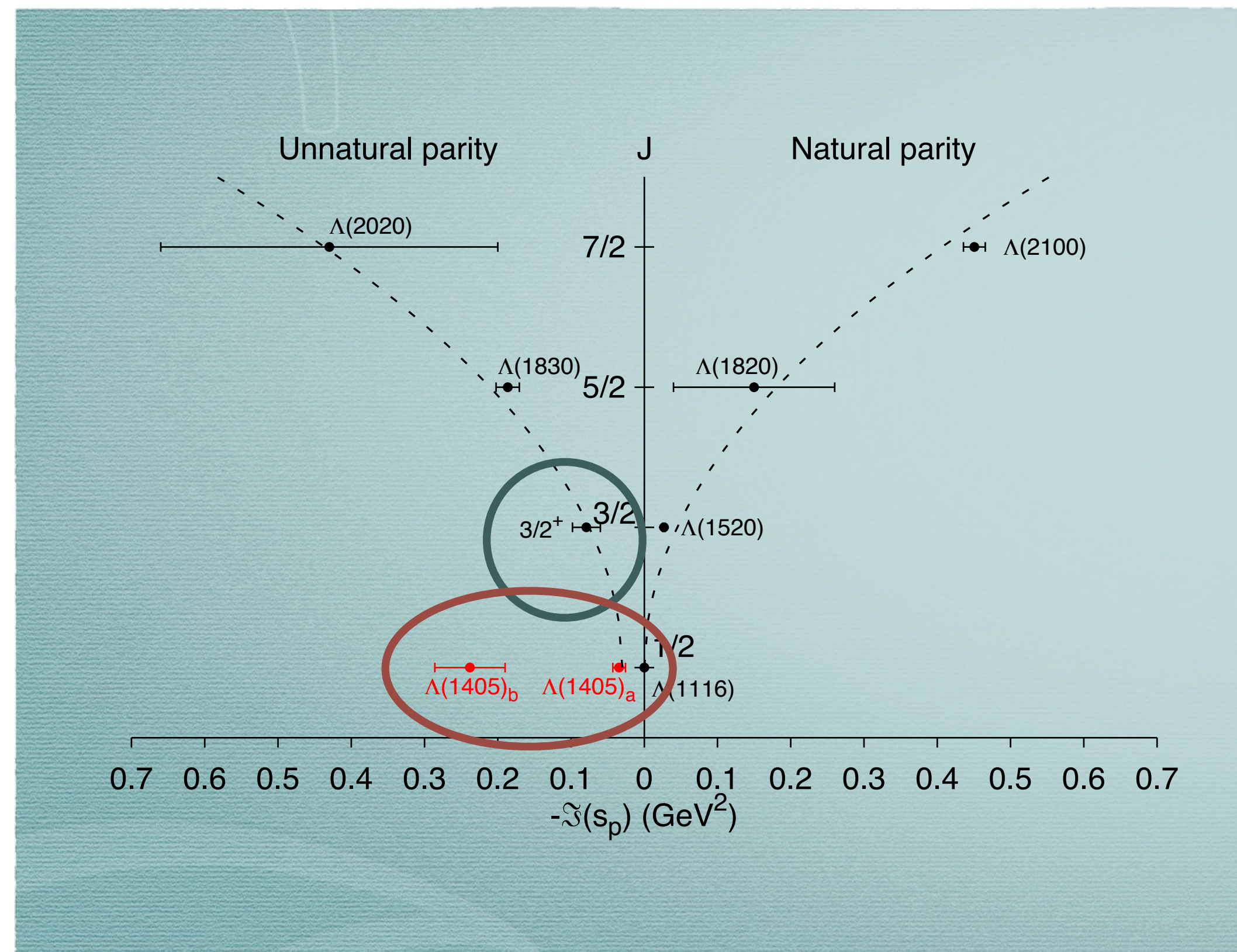
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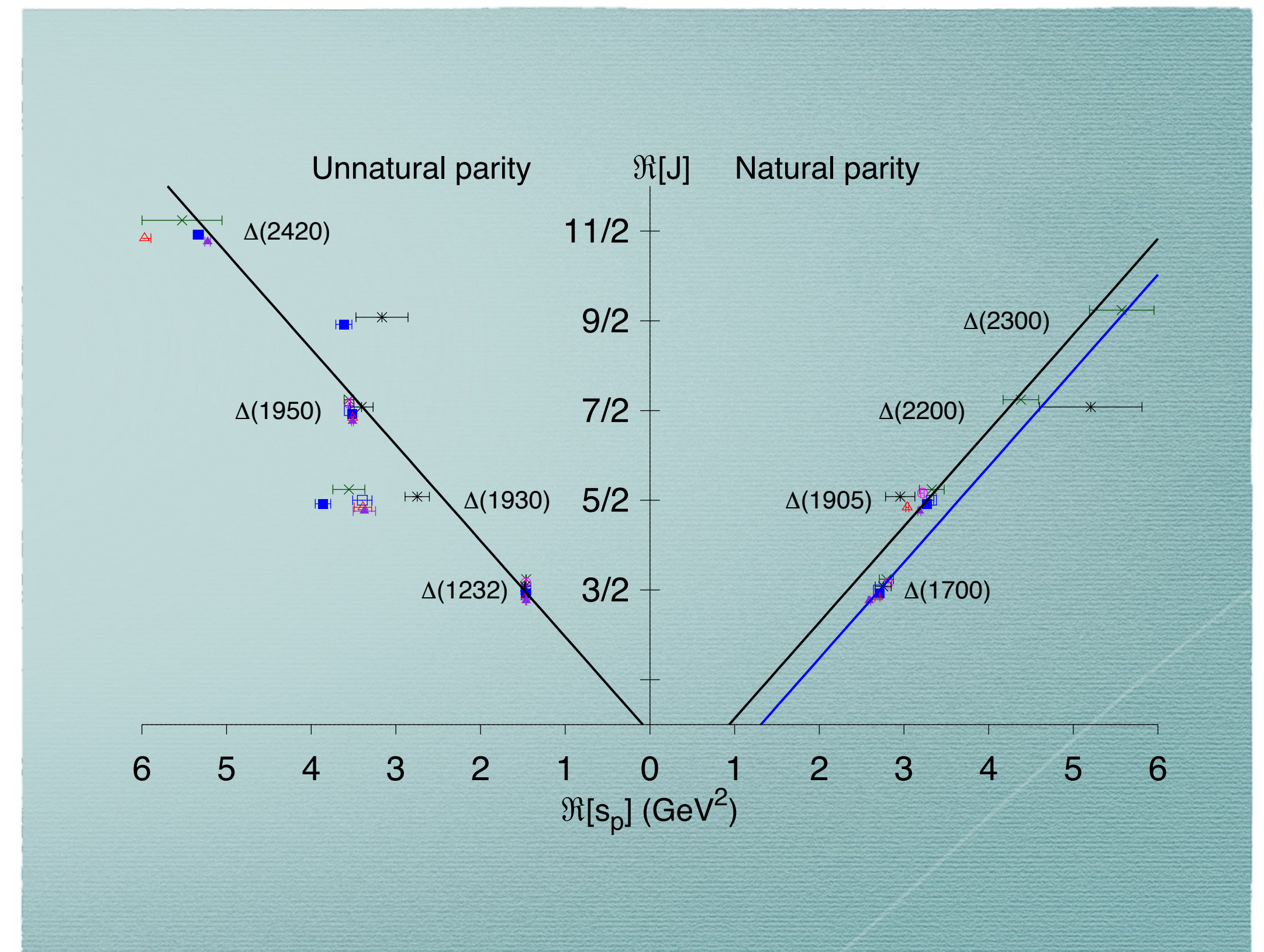
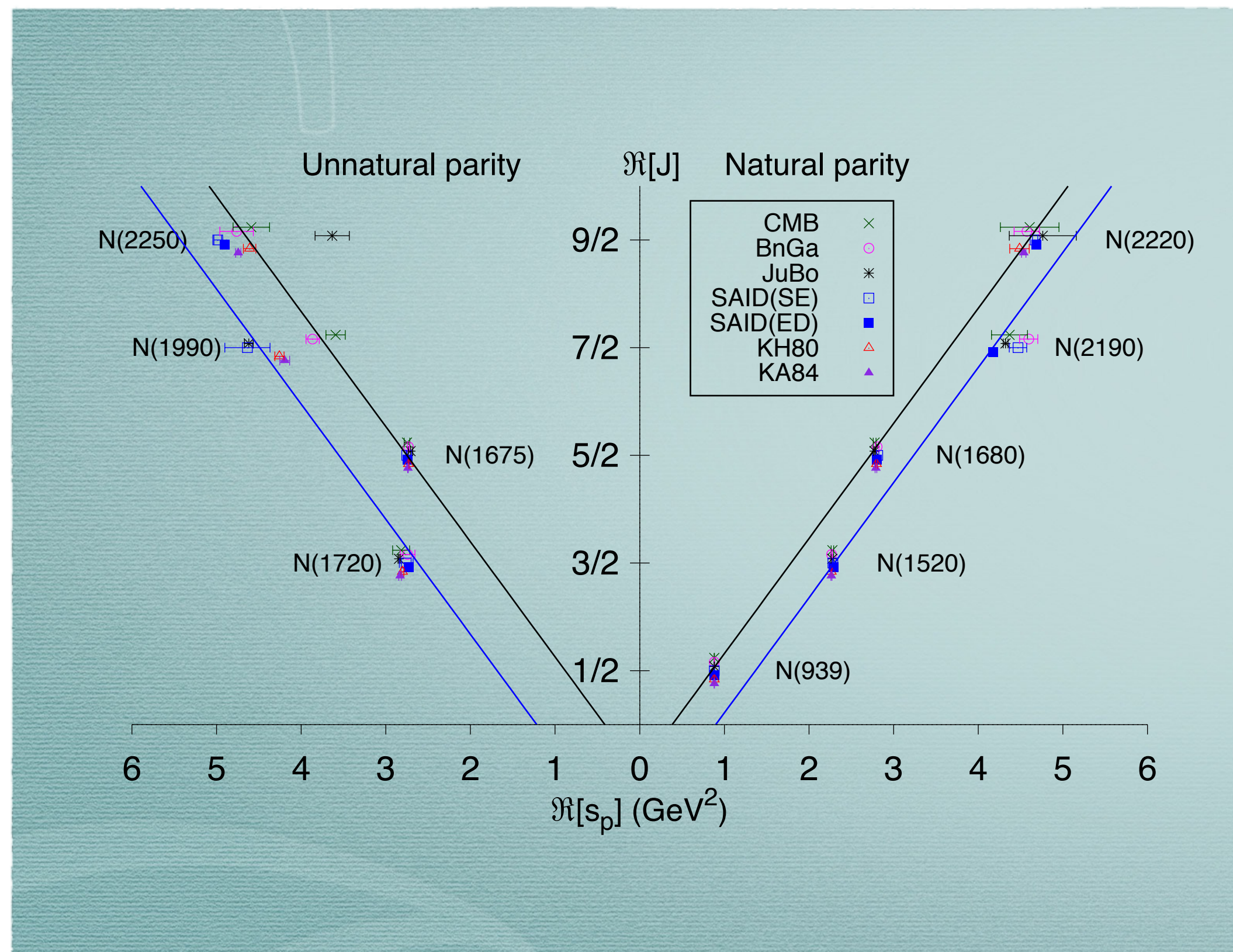


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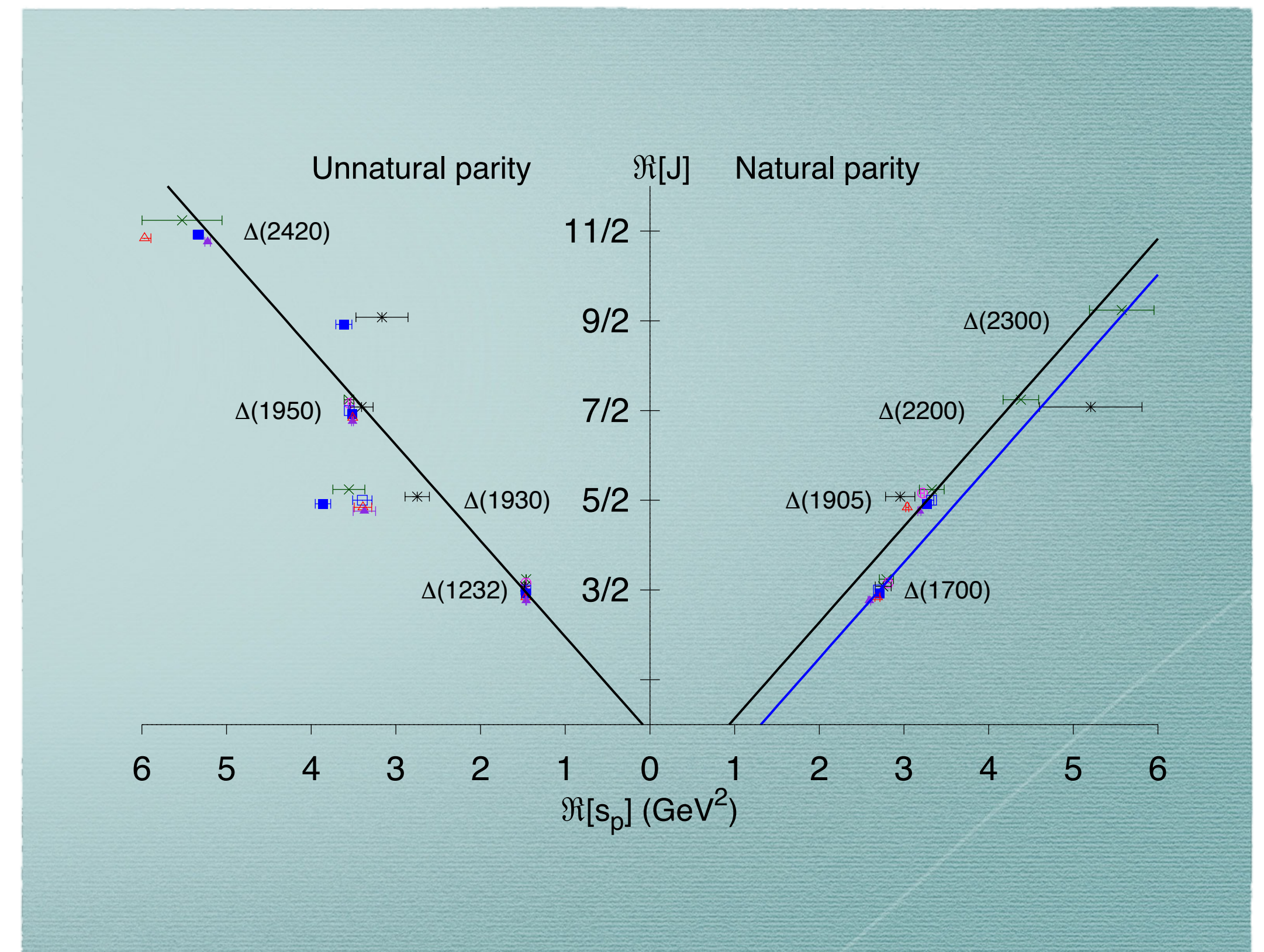
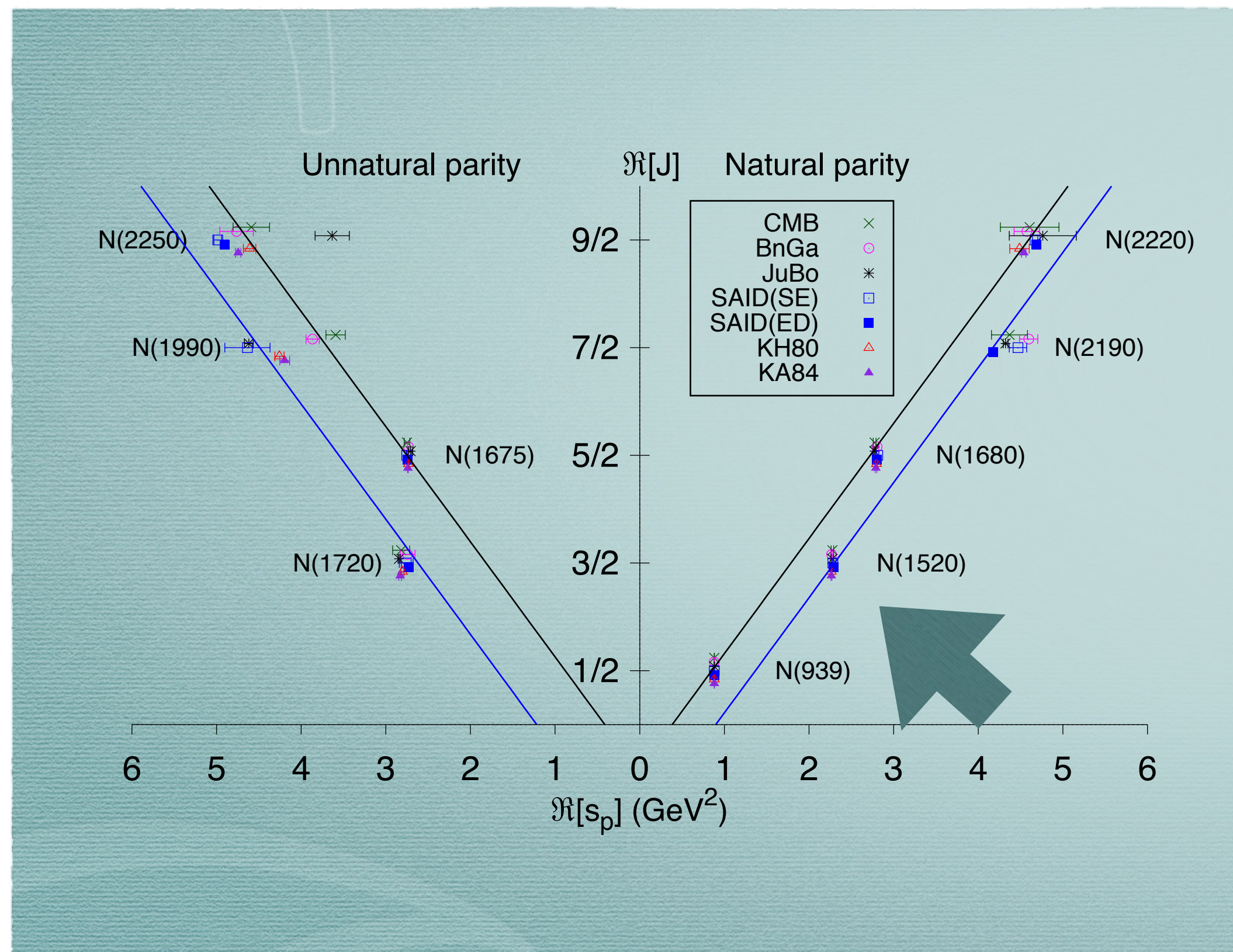
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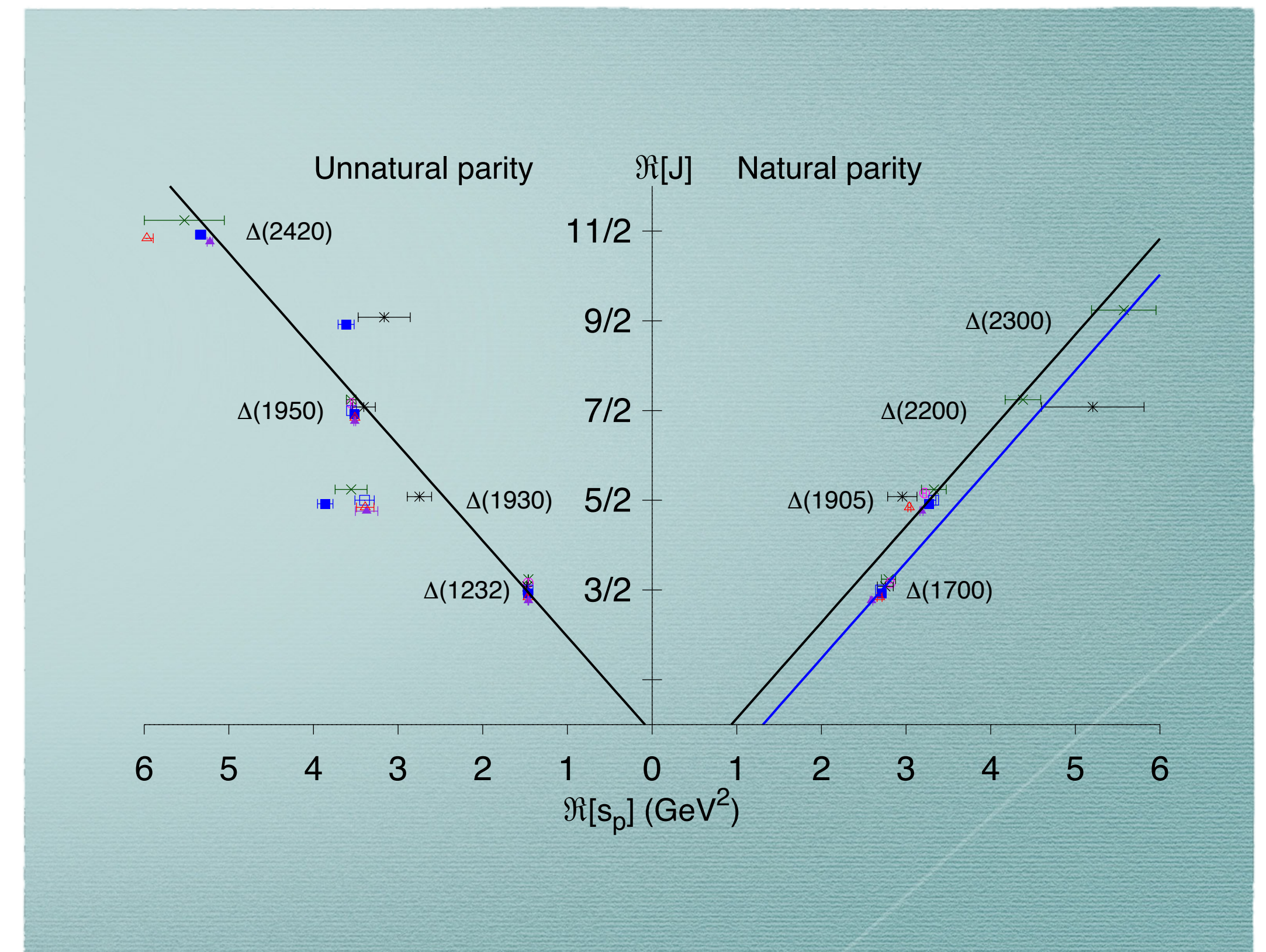
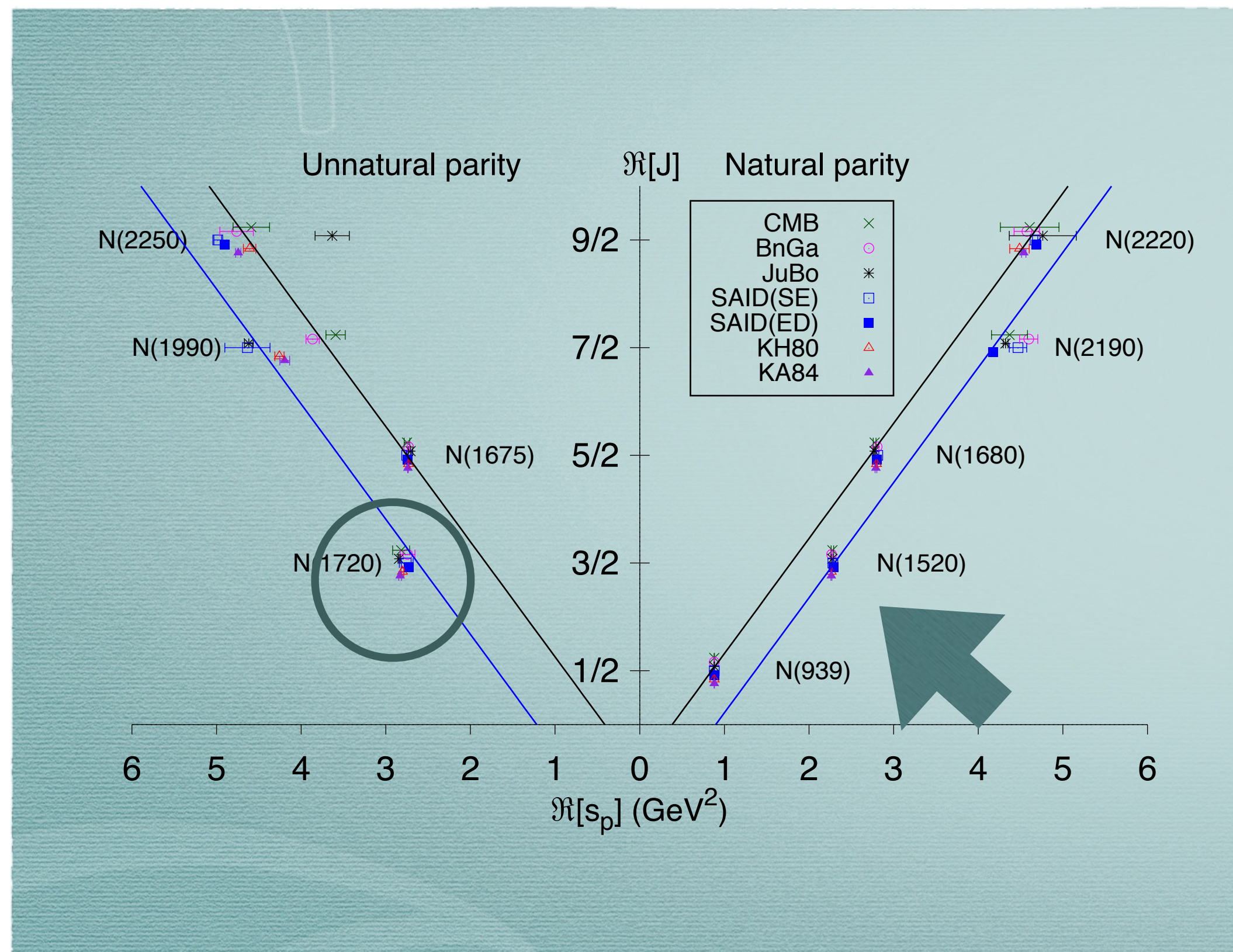
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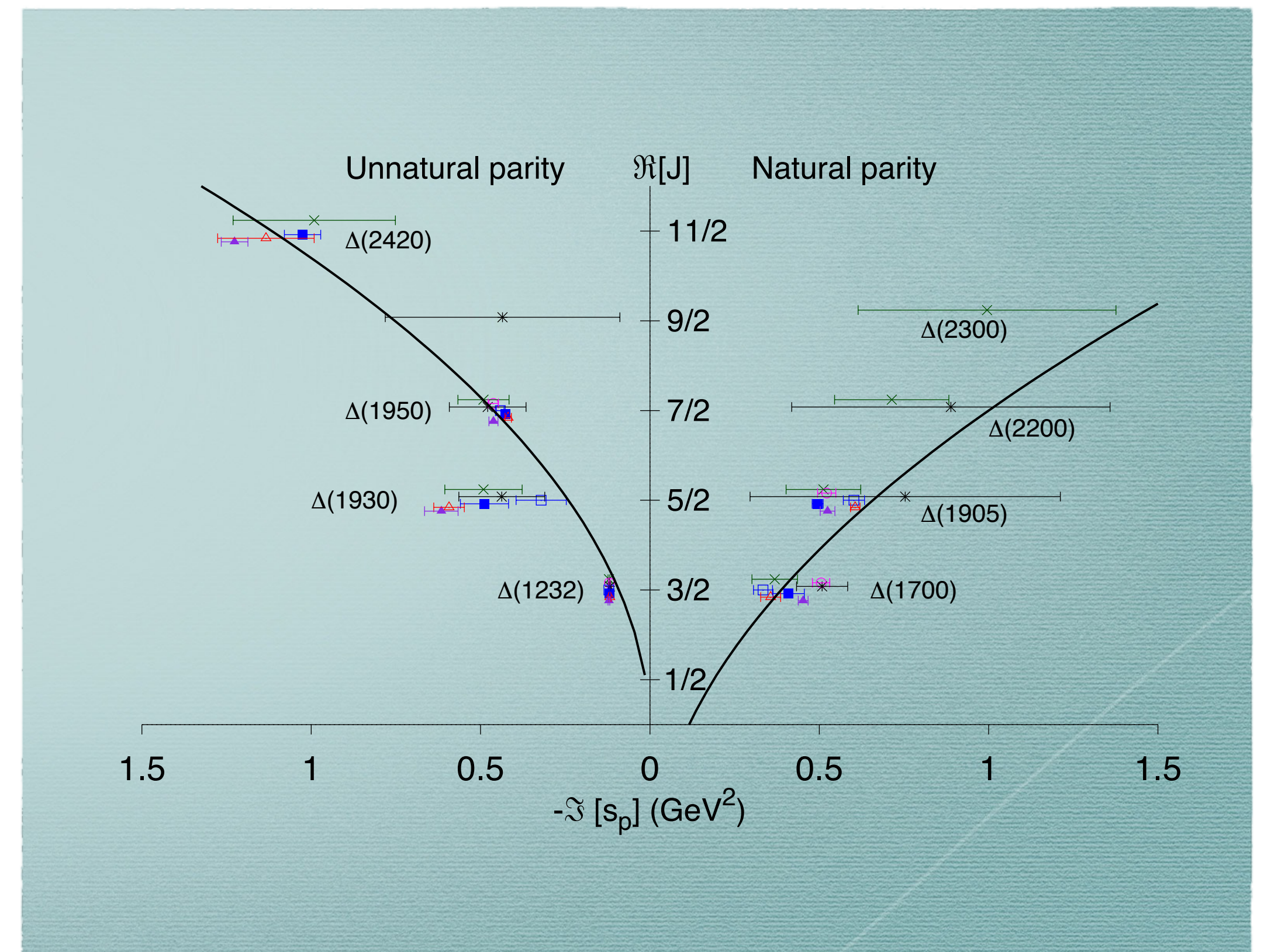
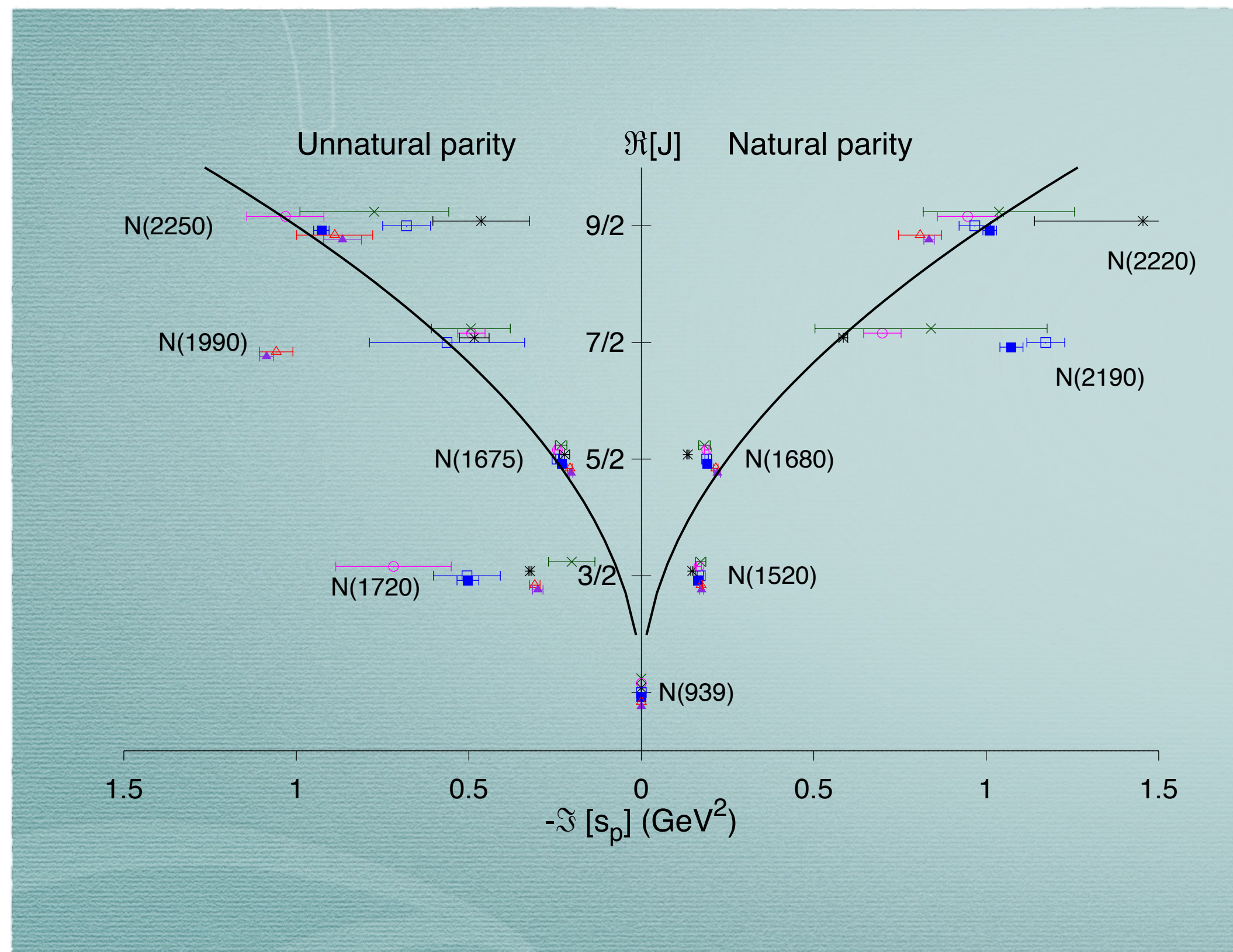
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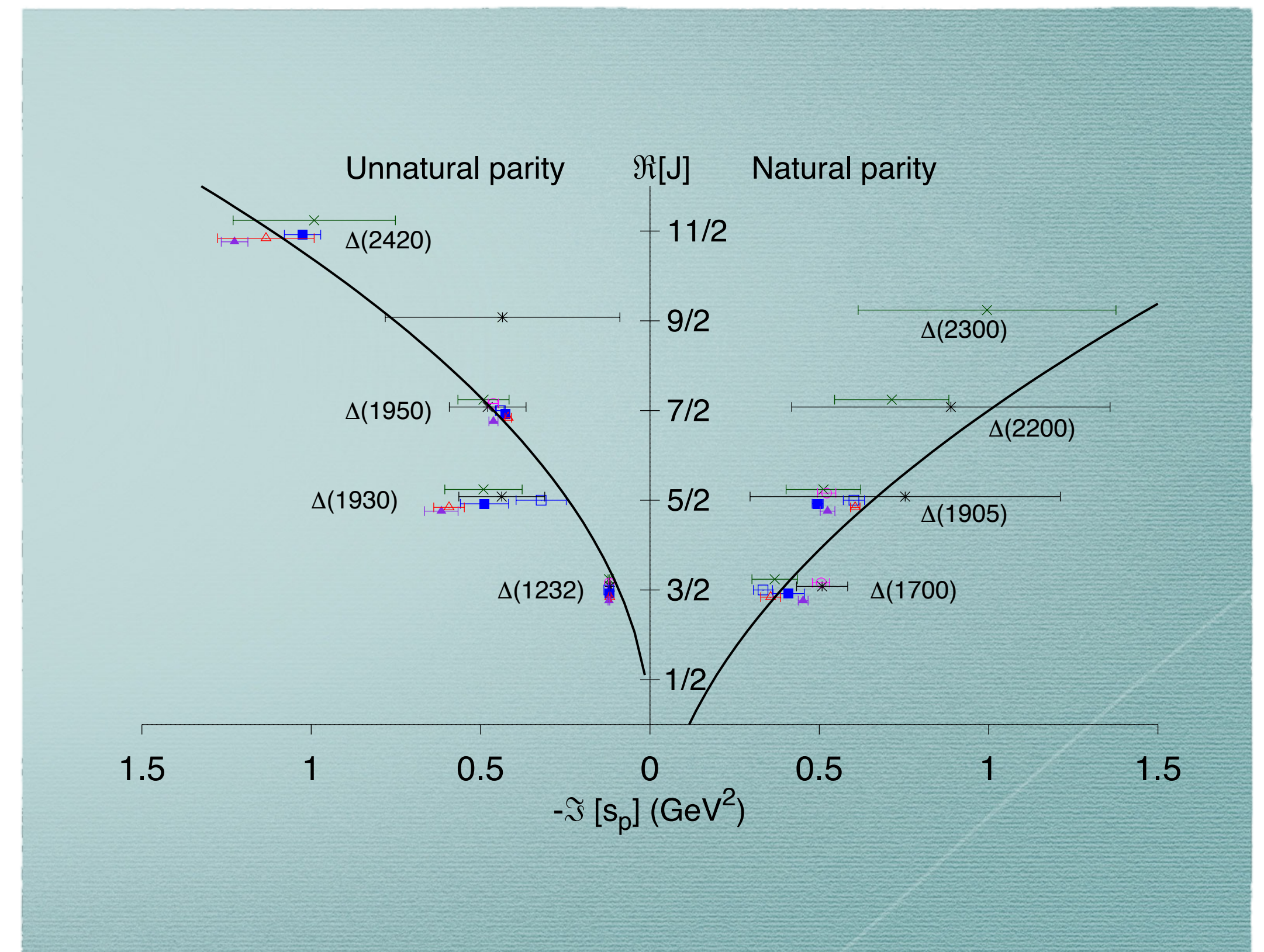
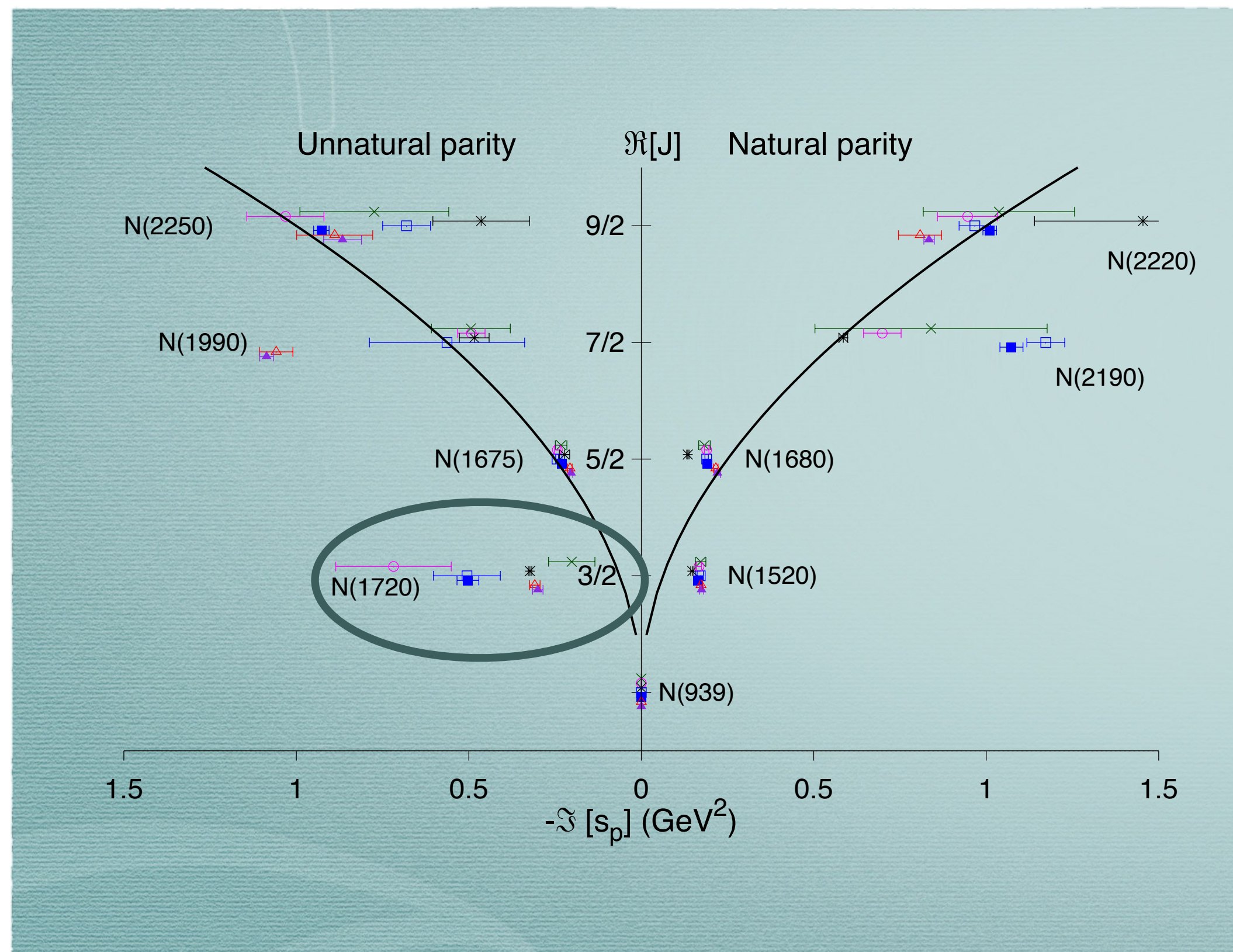
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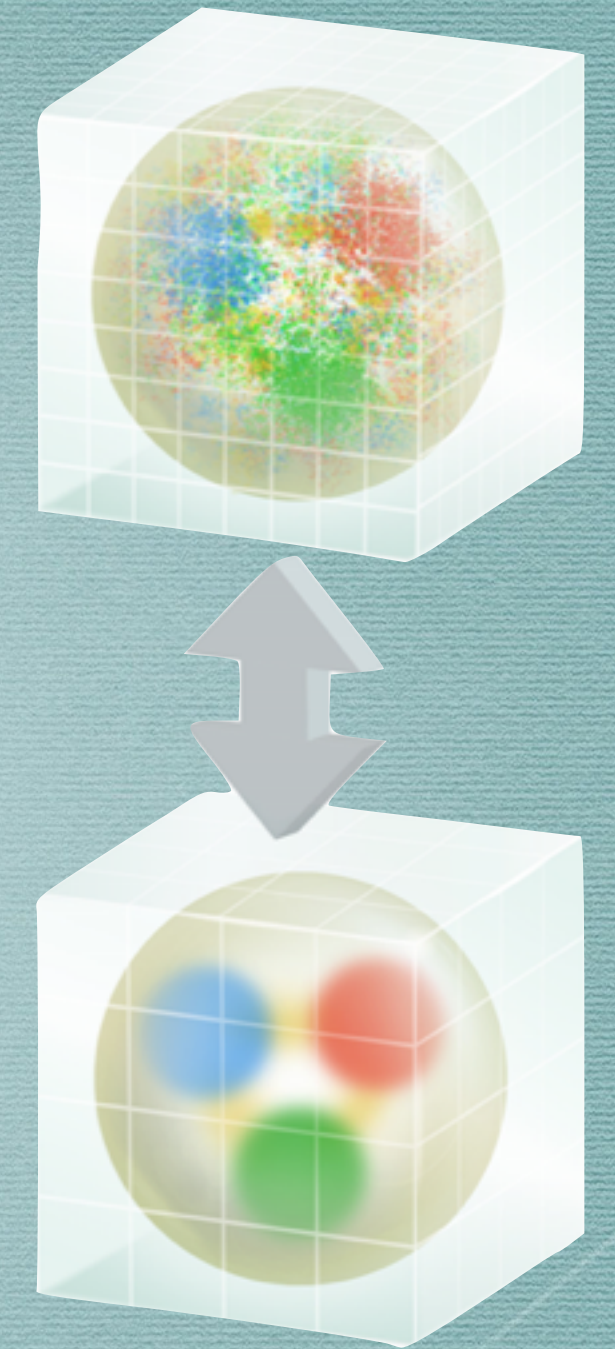


N and Δ spectra ($\text{Im } s_p$ vs J plots)



Interpretation

- * The linear Regge behavior is a signature of confinement
- * Quark models predict the approximate linear Regge behavior
- * What about the square-root-like behavior?
 - * Hypothesis: due to the phase space contribution
- * Quark model interpretation = Linear C-F + square-root-like in $\Im s_p$ vs J plots



Going further

$$t_\ell(s) \propto \frac{g^2}{M^2 - s - i g^2 \rho(s, s_t)}$$

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$$\alpha(s) = \alpha_0 + \alpha' s + i \gamma \phi(s, s_t) \quad \left\{ \begin{array}{l} i\phi_0(s, s_t) = 0 \\ i\phi_I(s, s_t) = i\sqrt{s - s_t} \\ i\phi_{II}(s, s_t) = \beta(s, s_t) + 2i\tau(s, s_t) \end{array} \right.$$

$$i\beta(s, s_t) = \frac{s - s_t}{\pi} \int_{s_t}^{\infty} \frac{\tau(s', s_t)}{s' - s_t} \frac{ds'}{s' - s} = \frac{2}{\pi} \frac{s - s_t}{\sqrt{s(s_t - s)}} \arctan \sqrt{\frac{s}{s_t - s}}$$

Fitting

$$d^2 = \sum_{poles} \{ \left[\Re[J] - \Re[\alpha(s_p)] \right]^2 + \left[\Im[J] - \Im[\alpha(s_p)] \right]^2 \}$$

Table I. Summary of pole positions M_p, Γ_p in MeV for $I^\eta = \frac{1}{2}^+$ states where $M_p = \Re \sqrt{s_p}$ and $\Gamma_p = -2 \Im \sqrt{s_p}$. I stands for isospin, η for naturality, J_p for spin, and P for parity. Naturality and parity are related by $\eta = \tau P$ where τ is the signature. For baryons, $\eta = +1$, natural parity, if $P = (-1)^{J_p-1/2}$, and $\eta = -1$, unnatural parity, if $P = -(-1)^{J_p-1/2}$.

Name	$N(939)$	$N(1520)$	$N(1680)$	$N(2190)$	$N(2220)$
Status	****	****	****	****	****
$I_{(\tau)}^\eta J_p^P$	$\frac{1}{2}_{(+)}^+ 1/2^+$	$\frac{1}{2}_{(-)}^+ 3/2^-$	$\frac{1}{2}_{(+)}^+ 5/2^+$	$\frac{1}{2}_{(-)}^+ 7/2^-$	$\frac{1}{2}_{(+)}^+ 9/2^+$
CMB	939(1), 0	1510(5), 114(10)	1667(5), 110(10)	2100(50), 400(160)	2160(80), 480(100)
JüBo	939(1), 0	1509(5), 98(3)	1666(4), 81(2)	2084(7), 281(6)	2207(89), 659(140)
BnGa	939(1), 0	1507(3), 111(5)	1676(6), 113(4)	2150(25), 325(25)	2150(35), 440(40)
SAID(SE)	939(1), 0	1512(2), 113(6)	1678(4), 113(3)	2132(24), 550(25)	2173(7), 445(21)
SAID(ED)	939(1), 0	1515(2), 109(4)	1674(3), 114(7)	2060(11), 521(16)	2177(4), 464(9)
KH80	939(1), 0	1506(2), 115(3)	1674(3), 129(4)	—	2127(27), 380(29)
KA84	939(1), 0	1506(2), 116(4)	1672(3), 132(5)	—	2139(6), 390(7)

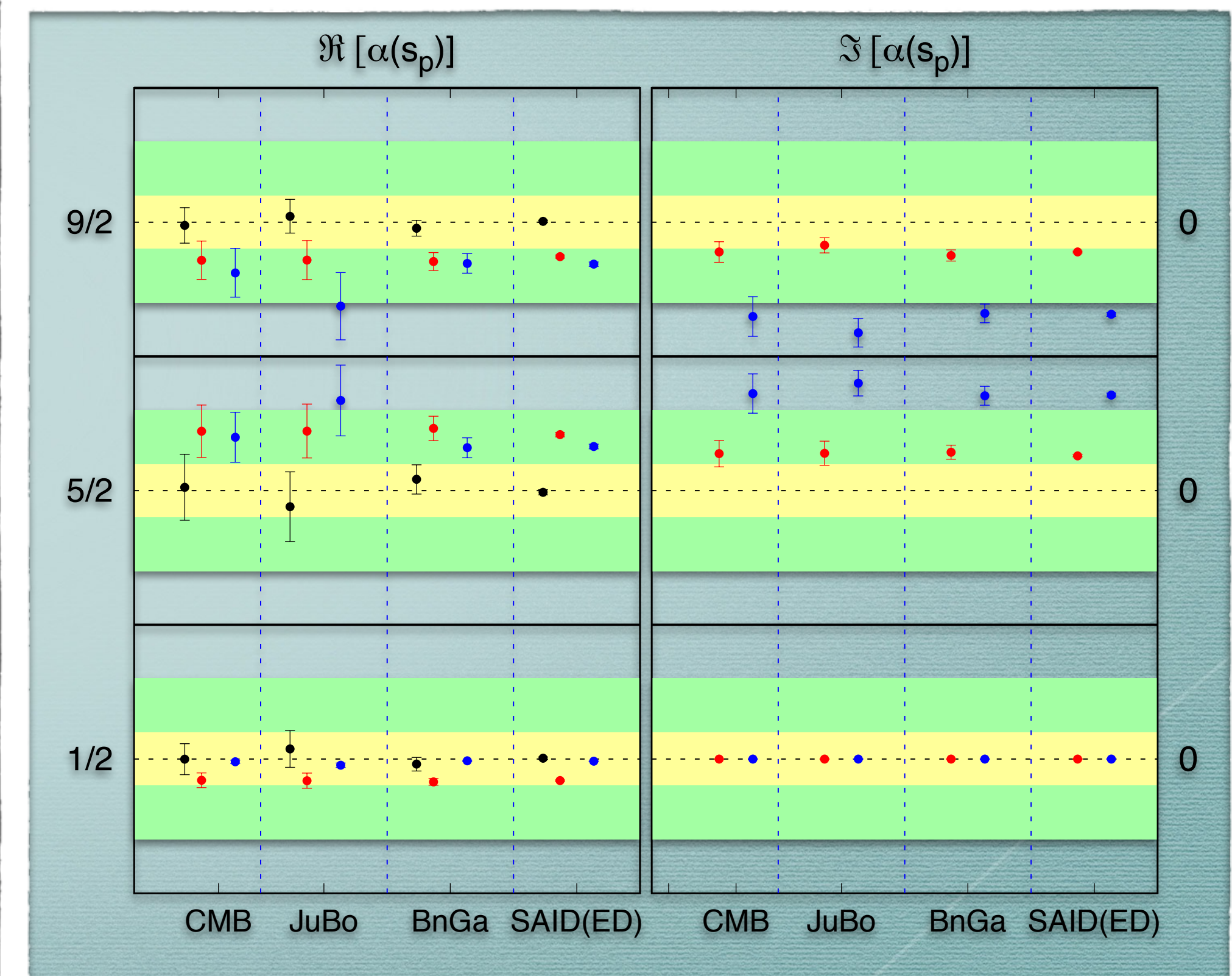
Consistency check N

- * Deviations are what we care about
- * N(1680) shows an interesting behavior

black model 0 (linear)

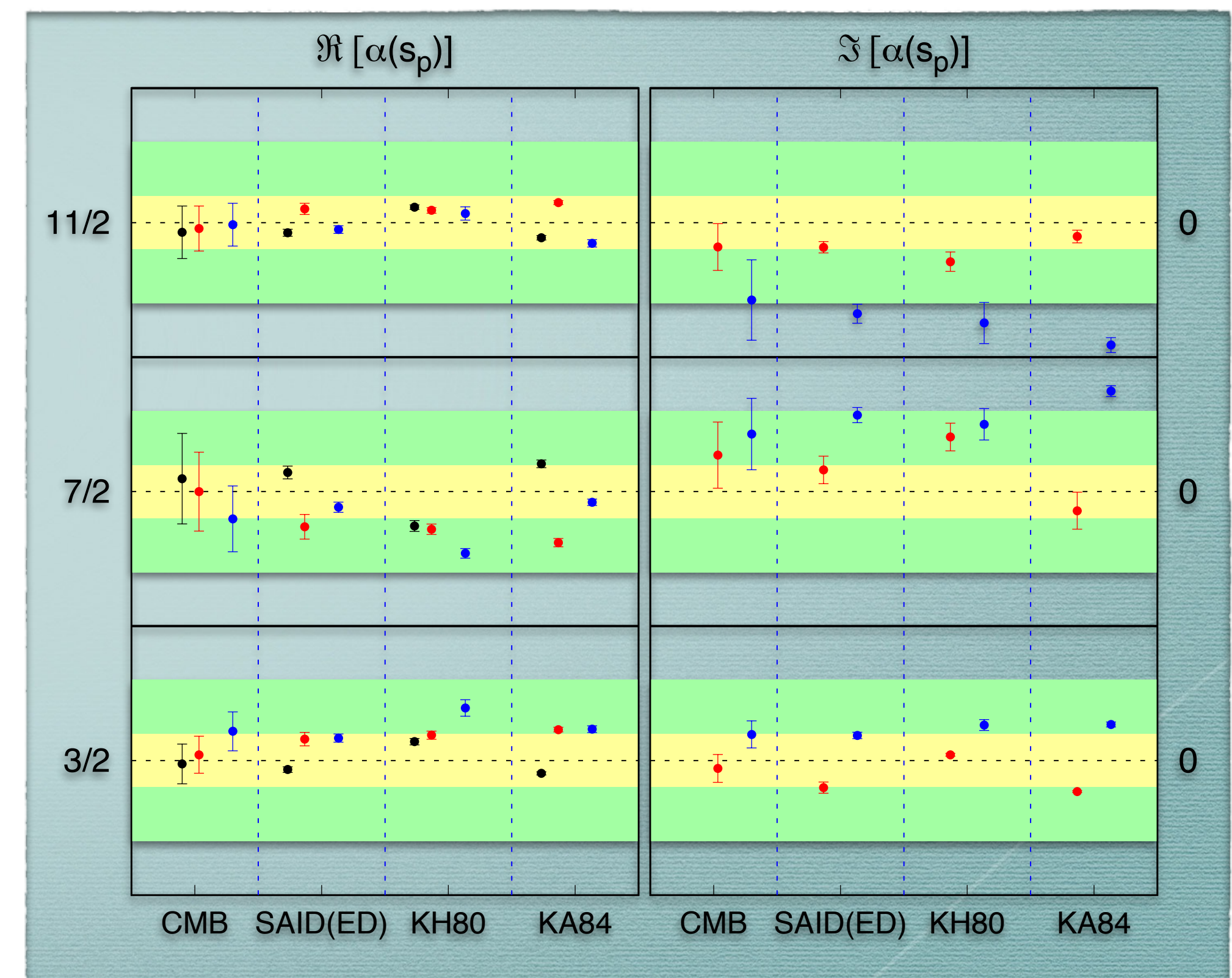
red model I (square root)

blue model II (dispersive)



Consistency check Δ

* Deviations are what we care about



Regge trajectory parameters (dispersive)

$$\alpha'_{\frac{1}{2}(+)} = 0.99 \pm 0.12 \text{ GeV}^2$$

$$\alpha_{0,\frac{1}{2}(+)} = 0.21 \pm 0.38$$

$$\gamma_{\frac{1}{2}(+)} = 0.651 \pm 0.040$$

$$s_{t,\frac{1}{2}(+)} = 1.02 \pm 0.13 \text{ GeV}^2$$

$$\alpha'_{\frac{3}{2}(-)} = 1.21 \pm 0.15 \text{ GeV}^2$$

$$\alpha_{0,\frac{3}{2}(-)} = -0.45 \pm 0.44$$

$$\gamma_{\frac{3}{2}(-)} = 0.86 \pm 0.22$$

$$s_{t,\frac{3}{2}(-)} = 1.52 \pm 0.12 \text{ GeV}^2$$

They can be used for proton-antiproton annihilation at PANDA

Summary

- * To study the hadron spectrum as a whole provides insight
- * Poles are complex quantities and should be taken into account when we look at Regge trajectories (phenomenology & complex angular momentum theory)
- * Gaps and deviations are useful to guide amplitude analysis and pole extractions
- * Phase space is most of the contribution to the width $\rightarrow$ Deviations signal dynamics
- * Quark model interpretation = Linear CF + square-root-like in $\Im s_p$ vs J plots, approximate linear behavior alone is not enough